

Industrial Applications of Computational Mechanics

Shear Walls and Fluids

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FEM - Reminder

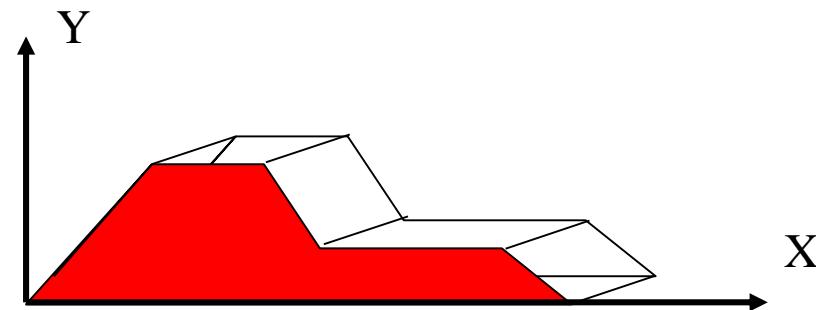
- A mathematical method
- The real (continuous) world is mapped on to a discrete (finite) one.
- We restrict the space of solutions.
- We calculate the optimal solution within that space on a global minimum principle
- Don't expect local precision

Plates (Slabs and shear walls)

- Classical plate bending solution (Kirchhoff)
 $K \Delta\Delta w = p$
- Classical solution for shear walls (Airy stress function F)
 $\Delta\Delta F = 0$
- FE / Variational approach for bending plates
 $\Pi = \frac{1}{2} \int \kappa D \kappa dV = \text{Minimum}$
- FE / Variational approach for shear walls
 $\Pi = \frac{1}{2} \int \varepsilon D \varepsilon dV = \text{Minimum}$

Shear Walls

- Unknowns:
Displacements $u=u_x$ and $v=u_y$
Rotation φ_z is not defined (see Cosserat)
- Strains
 $\varepsilon_x = \partial u / \partial x ; \varepsilon_y = \partial v / \partial y$
 $2 \varepsilon_{xy} = \gamma_{xy} = \partial v / \partial x + \partial u / \partial y$
- stresses
 $\sigma_x, \sigma_y, \tau_{xy}, [\sigma_z]$
- Plane Strain Condition $\varepsilon_z = 0$
- Plane Stress Condition $\sigma_z = 0$





Plane Stress Condition



$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \frac{E}{1-\mu^2} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & (1-\mu)/2 \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$$

$$\begin{bmatrix} n_x \\ n_y \\ n_{xy} \end{bmatrix} = \frac{E \cdot t}{1-\mu^2} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & (1-\mu)/2 \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$$

Extended Formulation

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \sigma_z \end{bmatrix} = \frac{E}{(1+\mu)(1-2\mu)} \begin{bmatrix} 1-\mu & \mu & 0 & \mu \\ \mu & 1-\mu & 0 & \mu \\ 0 & 0 & (1-2\mu)/2 & 0 \\ \mu & \mu & 0 & 1-\mu \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \varepsilon_z \end{bmatrix}$$

- Shear modulus $G = E/(2*(1+m))$ at Position 3,3
- Plane Strain $\varepsilon_z = 0$
- Axisymmetric condition $\varepsilon_z = u/r$

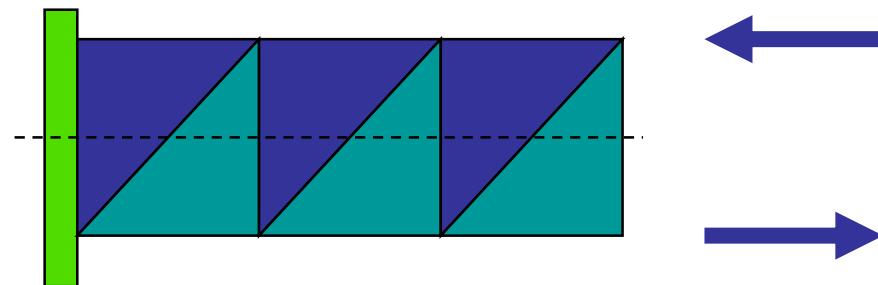
Remarks

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \sigma_z \end{bmatrix} = \frac{E}{(1+\mu)(1-2\mu)} \begin{bmatrix} 1-\mu & \mu & 0 & \mu \\ \mu & 1-\mu & 0 & \mu \\ 0 & 0 & (1-2\mu)/2 & 0 \\ \mu & \mu & 0 & 1-\mu \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \varepsilon_z \end{bmatrix}$$

- Incompressible limit
 - For $\mu = 0.5$ the matrix becomes singular
- Extensions for anisotropic behaviour via inverse matrix
 - E-Modulus in fibre direction $\varepsilon_x = \sigma_x/E_x + \mu_{xy} \cdot \sigma_y/E_y$
 - E-Modulus transverse to fibre direction (E_{90}) analogue
 - Poisson ratio for off diagonal term is not uniquely defined
 - Rotation of axis of Isotropy creates a fully populated matrix
 - Special effects for foams possible

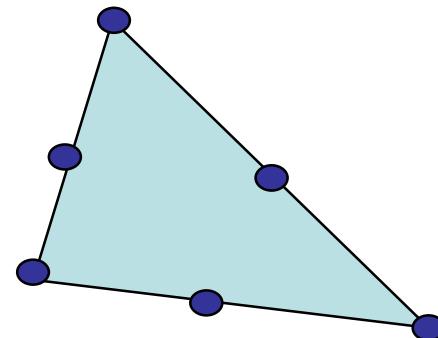
Membrane element

- constant strain triangular elements CST
 - Linear displacements
 - Constant stress



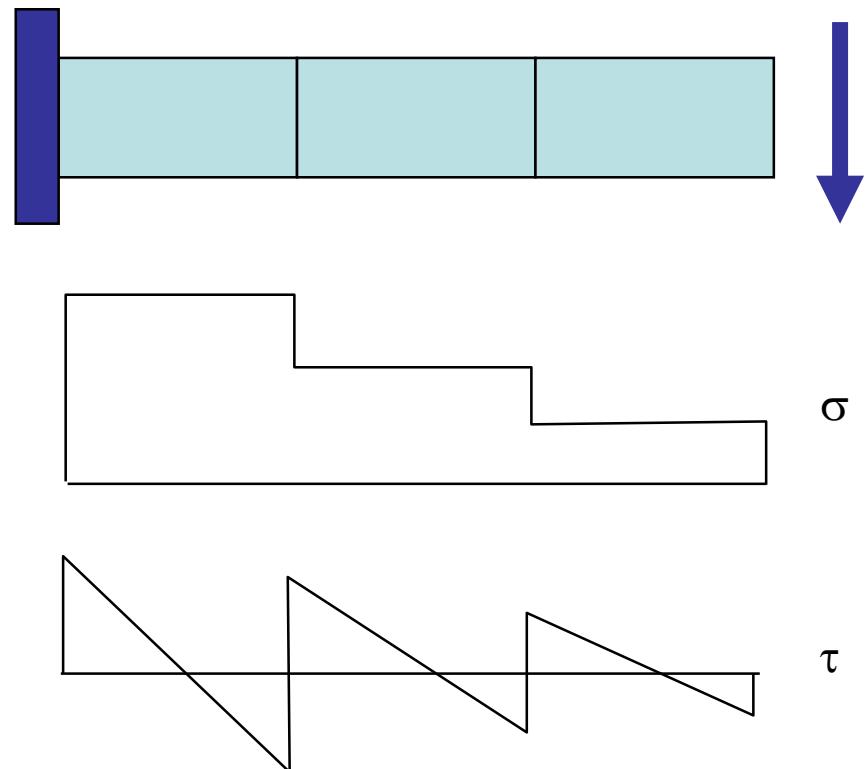
Enhanced triangular elements

- LST – Element with 6 nodes
 - Complete quadratic function space
 - Drilling-Degrees of Freedom
 - The displacements of the mid nodes are calculated from the end nodes including the rotation



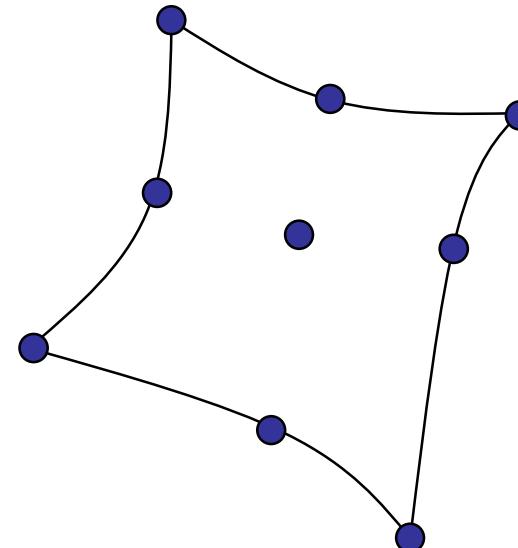
Membrane Elements

- Quadrilateral bilinear elements
 - Linear Displacements
 - Constant stress
 - Shear stress may become spurious



Enhancements

- Quadratic Shape functions
 - Lagrange Elements (nine noded)
 - Serendipity (without central node)
 - „Isoparametric“
 - “Isogeometric”



Disadvantages

- Uniform loadings creates nodal loads as:
 $1/6$, $2/3$, $1/6$
- Thus coupling with beam elements is difficult,
i.e. we need also isoparametric beam elements
- Special coupling conditions (friction, no tension etc.) also
difficult, i.e. we need isoparametric interface elements

Drilling Degrees I

- Same principle as with the triangular element
- Further mathematical tricks required to reduce the space of the shape functions compared to the Serendipity-Element
- Moments as nodal loads not easy to understand / handle
- Advantages for folded structures or shells expected
- My own benchmarks showed poor quality of results.

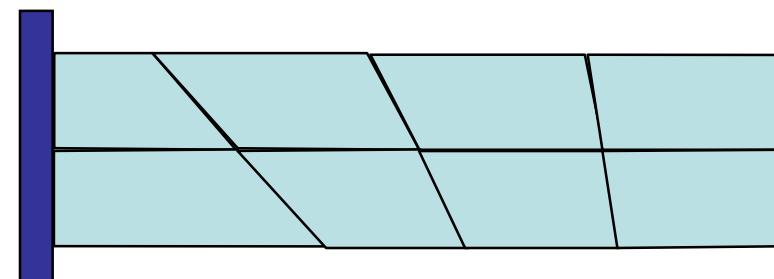
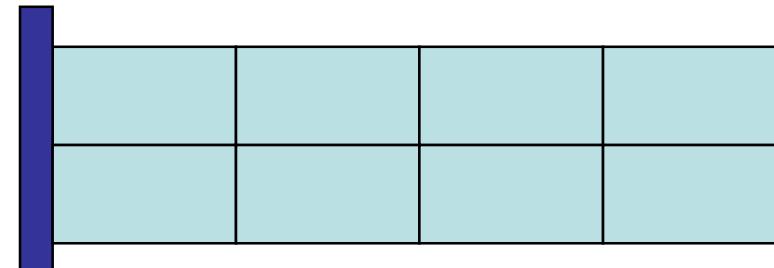


Enhancements

- Bilinear non conforming (Wilson)
$$u = \dots + (1-s^2) q_1 + (1-t^2) q_2$$

$$v = \dots + (1-s^2) q_1 + (1-t^2) q_2$$
- May model constant curvatures exactly
- Static Condensation
- Patch-Test fulfilled with a trick
(Jacobi-Determinant is treated as constant)
- Newer approach: Assumed strains

Patchtest



Patchtest (σ)

| Moment | constant | | linear | | quadratic | |
|-----------|----------|-------|--------|-------|-----------|-------|
| Mesh | x=0 | x=l/2 | x=0 | x=l/2 | x=0 | x=l/2 |
| Reference | 1500 | 1500 | 1200 | 600 | 1200 | 300 |
| R Q4+2 | 1500 | 1500 | 1051 | 600 | 940 | 337 |
| V Q4+2 | 1322 | 1422 | 1422 | 701 | 773 | 452 |
| R Q4 | 1072 | 1072 | 1072 | 428 | 659 | 240 |
| V Q4 | 687 | 578 | 578 | 187 | 393 | 172 |

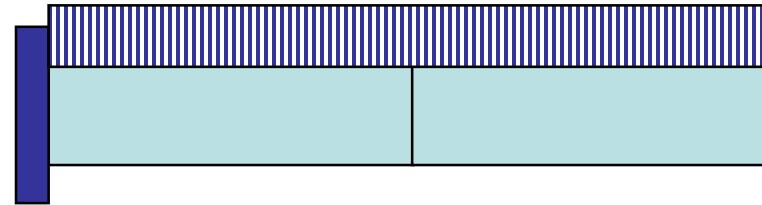


Patchtest (τ)



| Moment | constant | | linear | | quadratic | |
|-----------|----------|-------|--------|-------|-----------|-------|
| Mesh | x=0 | x=l/2 | x=0 | x=l/2 | x=0 | x=l/2 |
| Reference | 0 | 0 | 50 | 50 | 100 | 50 |
| R Q4+2 | 0 | 0 | 50 | 50 | 87.5 | 50 |
| V Q4+2 | 58 | 28 | 65 | 80 | 130 | 73 |
| R Q4 | 438 | 0 | 364 | 8 | 376 | 8 |
| V Q4 | 502 | 220 | 380 | 294 | 366 | 11 |

Convergence of displacements



| Mesh | u (Q4) | u (Q4+2) |
|---------------|----------|------------|
| 1×8 | 0.715 | 1.035 |
| 2×16 | 0.939 | 1.036 |
| 4×32 | 1.010 | 1.038 |
| 8×80 | 1.021 | 1.039 |

Drilling Degrees II

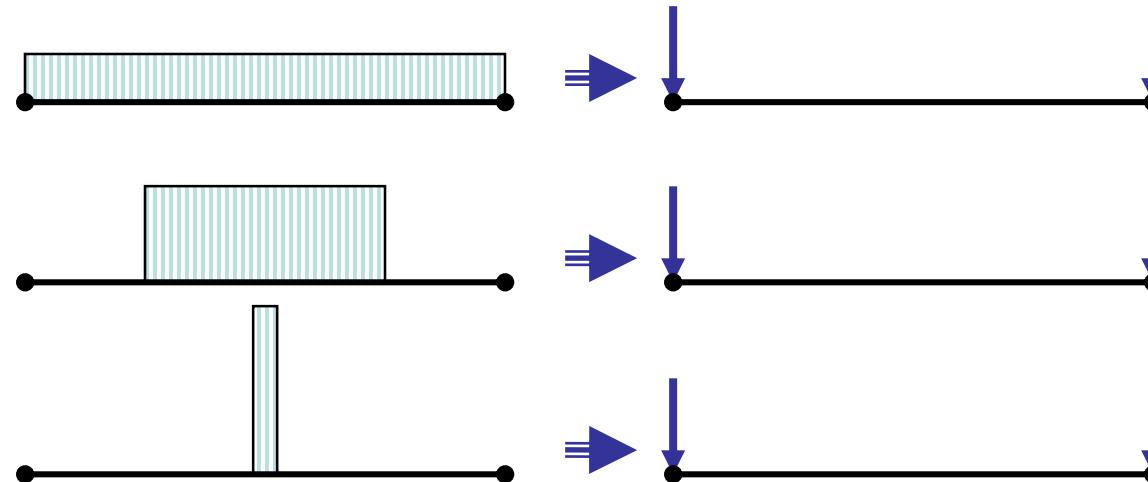
- Better approach with a strain field (Hughes/Brezzi)

$$\Pi = \int (\text{symm grad } v) \cdot c \cdot (\text{symm grad } v) d\Omega + \\ \gamma \cdot \int |\text{skew grad } v - \omega|^2 d\Omega$$

- Constraint about the rigid body rotations
- Adding deformation energy makes the element stiffer
- Combination with nonconforming modes is recommended

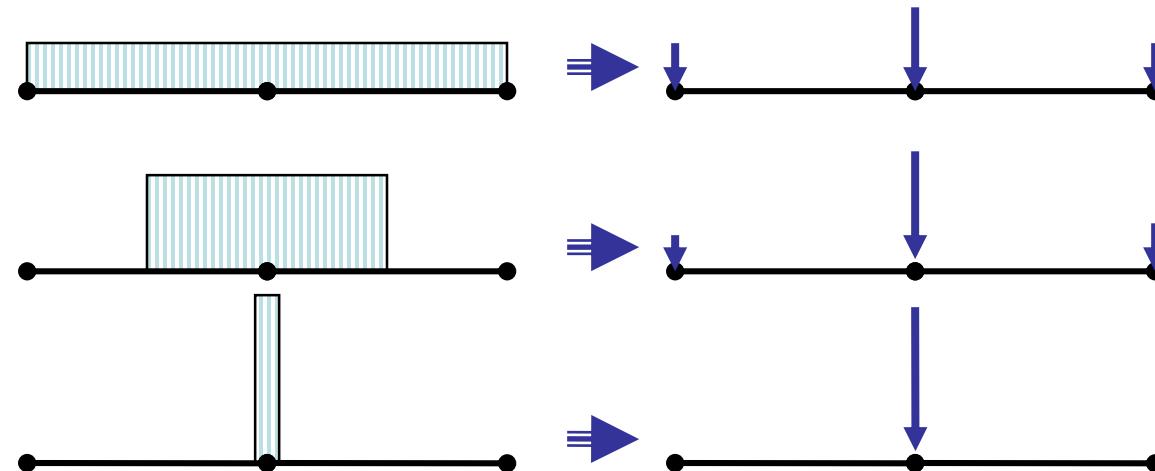
Loads

- Nodal loads are no point loads



Loads

- Resolution of a mesh for loads

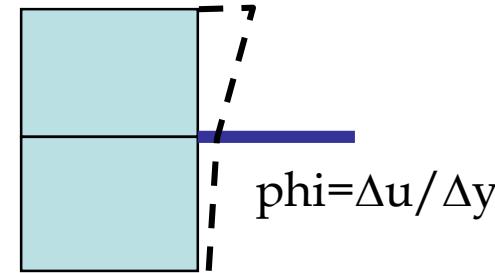
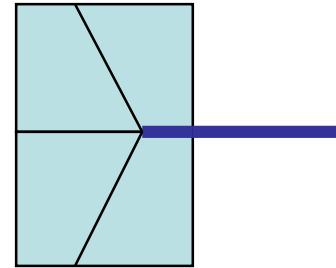




Moments



- There is no degree for that !
- Possibilities
 - Use more than one node
 - Kinematic Constraints, EST-Conditions
 - Drilling degrees of freedom





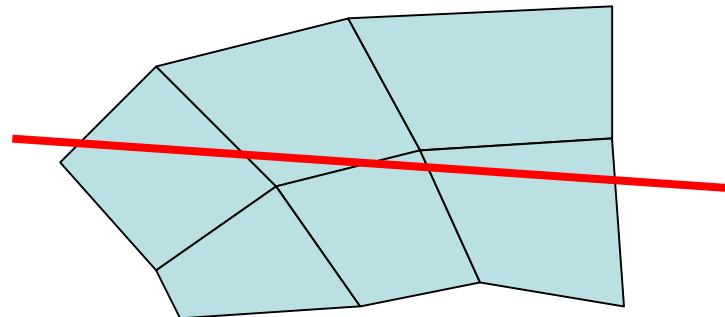
Result-Evaluation



- Direct solution: Displacements u
- Support Forces (Residuals)
(exact for all degrees of freedom!) $f = K u - p$
- Stresses in elements
 - Centre (Mean value = Super convergent Point)
 - Gauss-Points
 - Nodes of elements (Extrapolation!)
- Mean values in nodes
- Error estimates

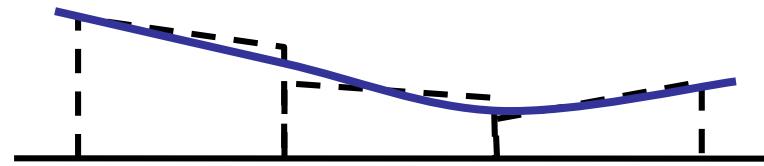
Equilibrium

- As it is the base for our solution it should be fulfilled for the residual forces even if system and loadings are completely garbage.
- It is not fulfilled within the elements
- It is not fulfilled at the edges of the elements
- It is not fulfilled within general cuts across the elements



Nodal stresses

- Stresses are discontinuous between elements

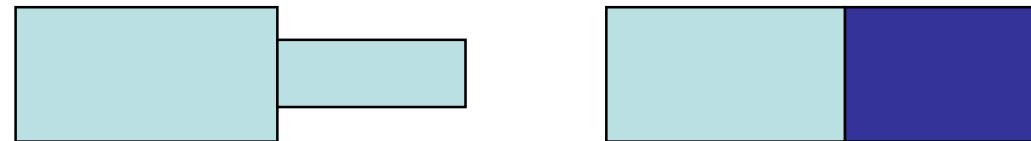


- The jump value of the stresses is a measure for the quality (e.g. error) of the solution for that mesh

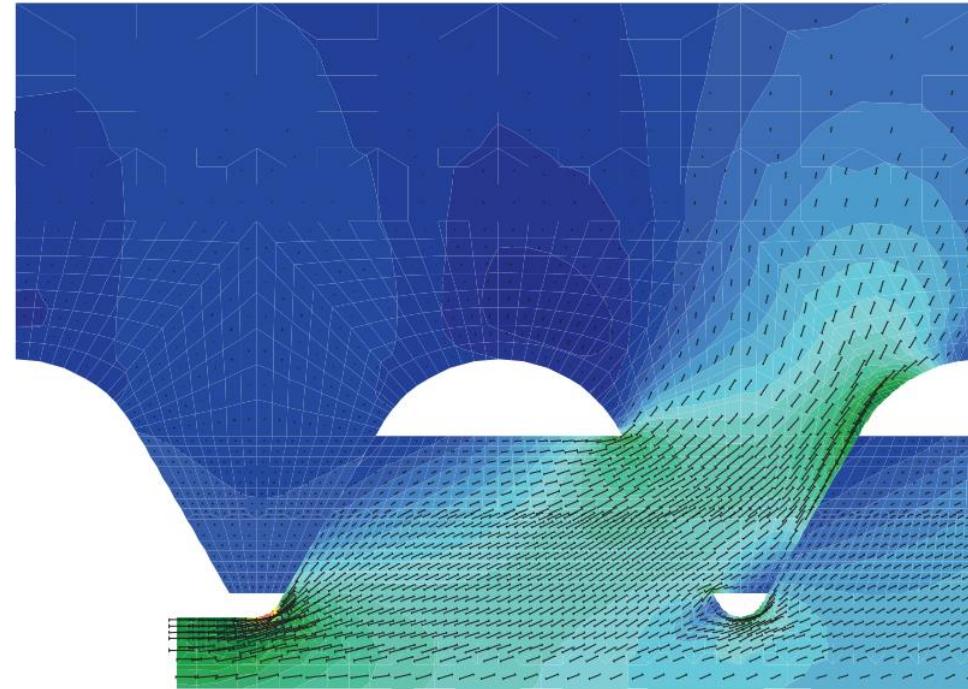
Nodal stresses



- Mean value of stresses in nodes to obtain „nicer“ pictures
 - Discontinuity of Thickness
 - Discontinuity of E-Modulus
 - Discontinuity of Geometry

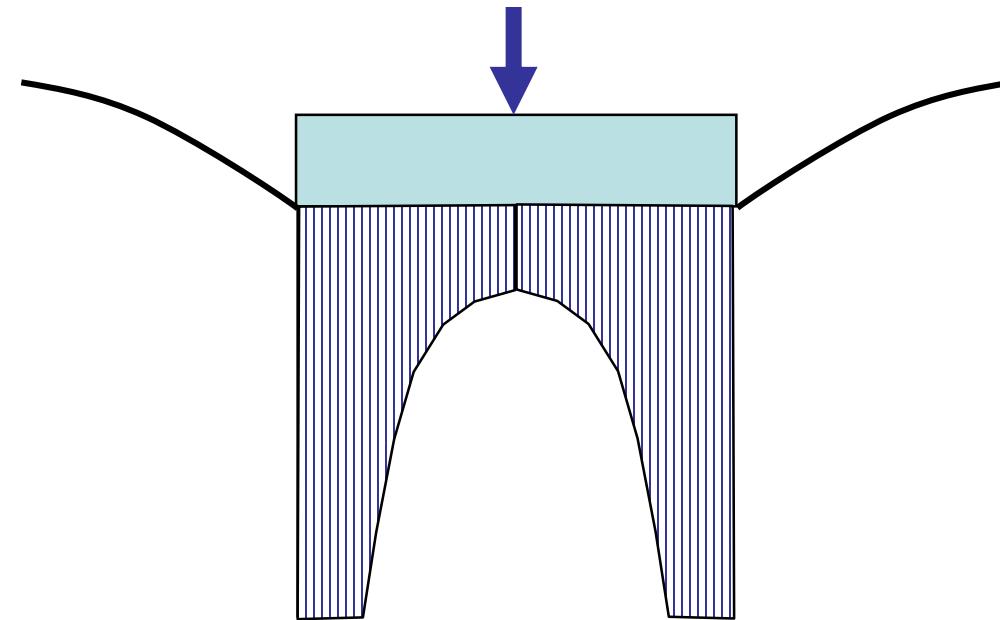


Colours do not show all



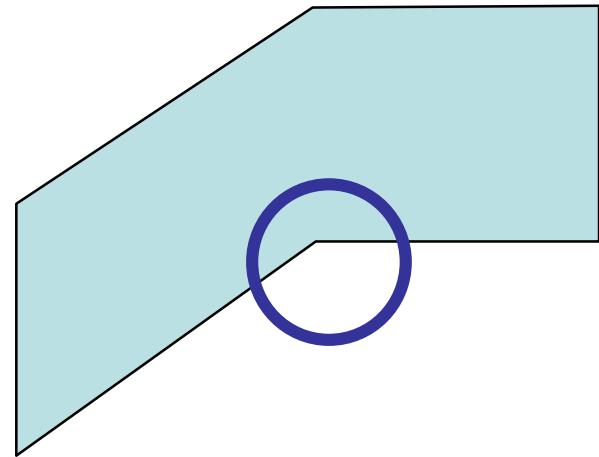
Singularities

- Rigid Footing on a half space



Singularities

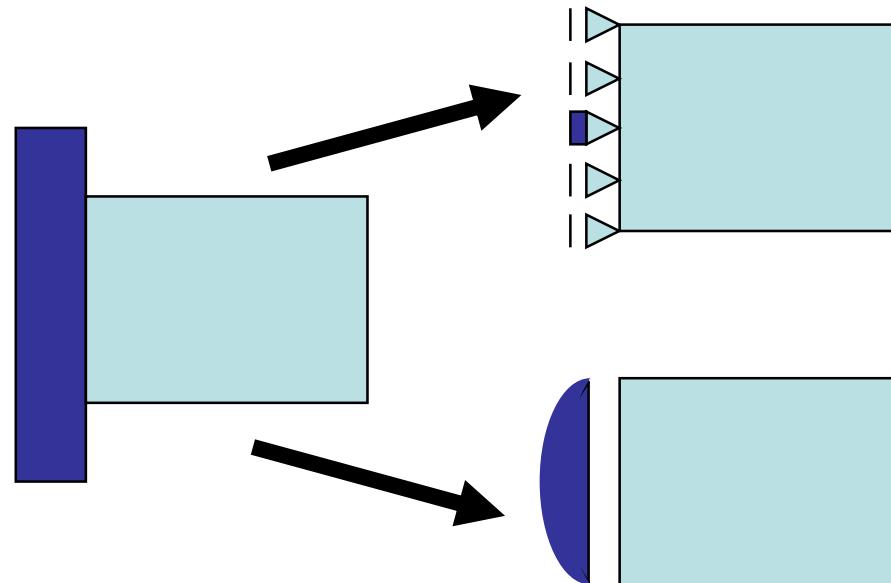
- Re-Entrant Corners



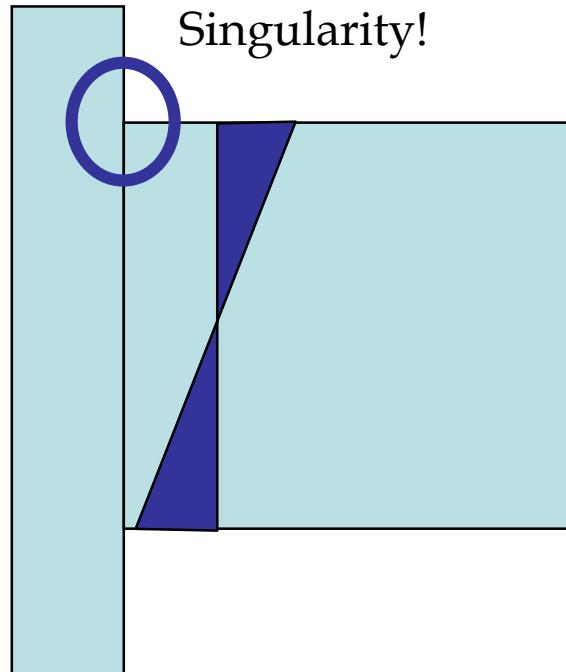
Adaptive Mesh Refinement

- At those locations where we have a large error estimate we refine the mesh either geometrically (h-Version) or we increase the Polynomial degree (p_Version) or both (hp-Version).
- Strong advantages compared to a uniform refinement
- Loads are not allowed to be defined for nodes or elements, but are required in a more general geometric way.
- For any design purpose we need results for all load cases at the same location.
=> a mesh for every load case makes life not easier.

Detail of supports



Detail of supports



- Element stress is discontinuous
- Nodal stresses as mean values are not correct at this point
- Separate the nodal stresses in groups

Remarks

□

- Axissymmetric case
 - The strains are neither constant nor linear nor quadratic
 - None of the classical elements may describe this exactly
 - Integral of loads has to include the radius
 - But not the nodal loads !
- 3D case
 - Most of the plain strain issues are also valid
 - Two more shear stresses
 - Elements as Hexahedra or Tetrahedra or something in between
 - Mesh generation is a complex topic !



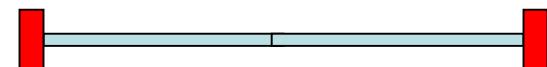
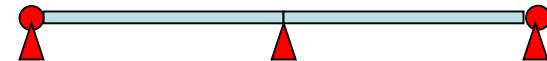
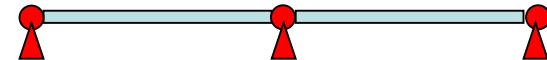
Construction Stages



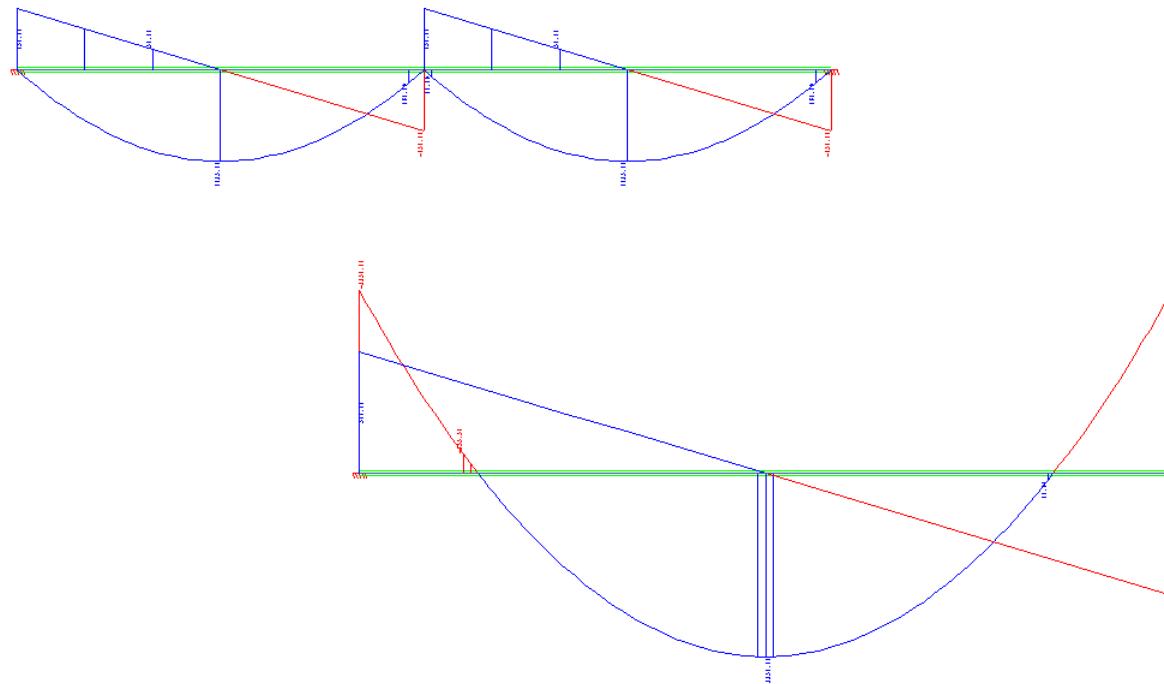
- Classical Approach: We build and then we switch gravity on ?
- There are many cases, especially with non linearity where the simulation of the construction process becomes essential
 - Dam construction
 - Tunnelling
 - Bridges
- Effects to consider:
 - Stress path
 - Change of forces due to creep
 - Adding or removing parts of the structure

A simple beam example

- Two single span beams
- Connected to a continuous beam
=> Creep will change the forces towards the continuous case
- Changed to a single span beam
=> Moment distribution will be different



Removal of central support



How is this done best ?

- First Principle: Strain increments!
- Consider each load case not in total but as difference to the primary state before
- New stresses are old stresses + tangential stiffness times strain increments

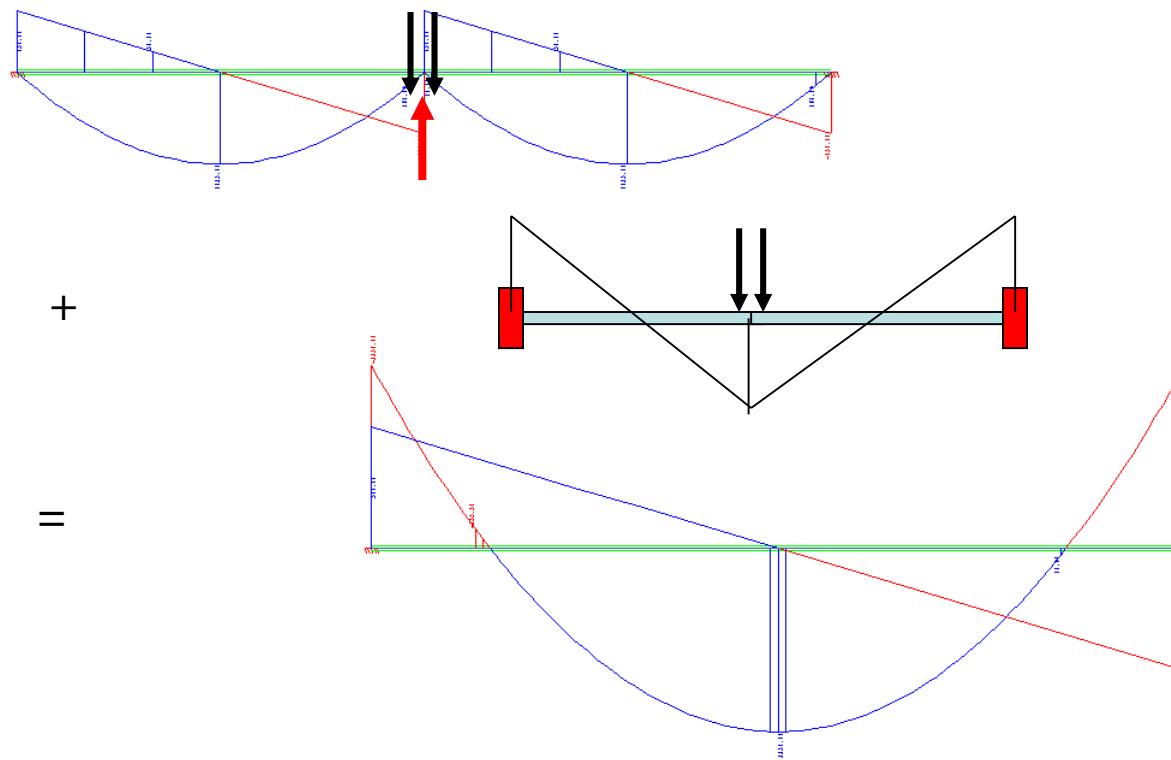
$$\sigma_{new} = \sigma_{old} + E_t \cdot \Delta \varepsilon$$

Incremental loads

- The stresses of the primary load case are in equilibrium with the loadings and support forces of the primary state
- The load vector of the new case is the difference between the total load vector and the residual load vector of the primary stresses.

$$\Delta P = P_{total} - \int B^T \sigma_{primary} dV$$

How it works



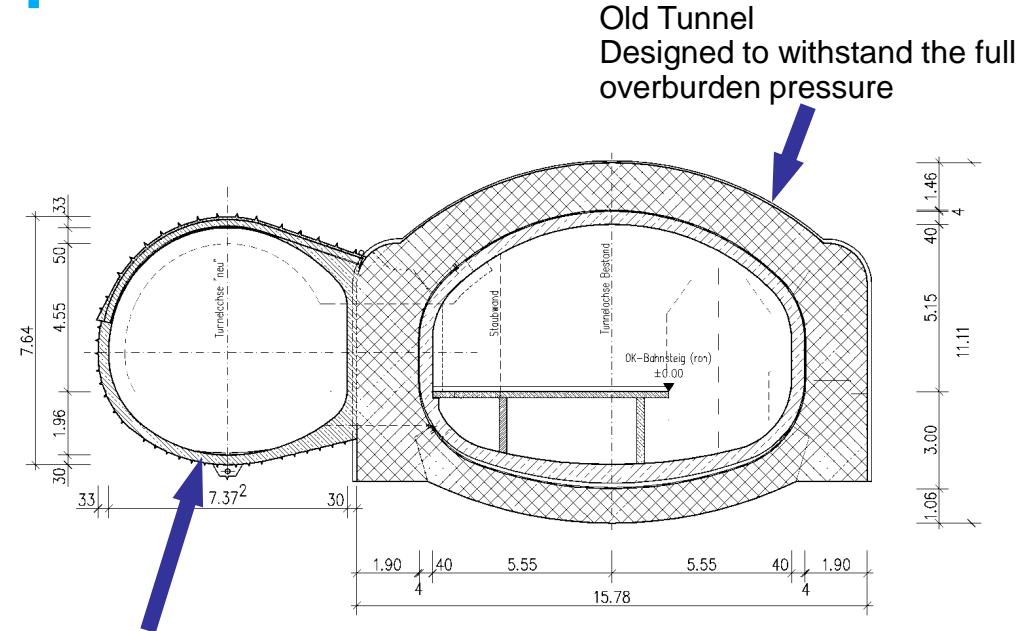


Last not least



- Partially removed or activated systems
 - Shotcrete hardening
 - Tunnelling
 - Loss of strength due to many effects
 - Icing and deicing a soil
- The full set of tools
 - Factor for Stiffness
 - Factor for primary stress
 - Factor for primary loading

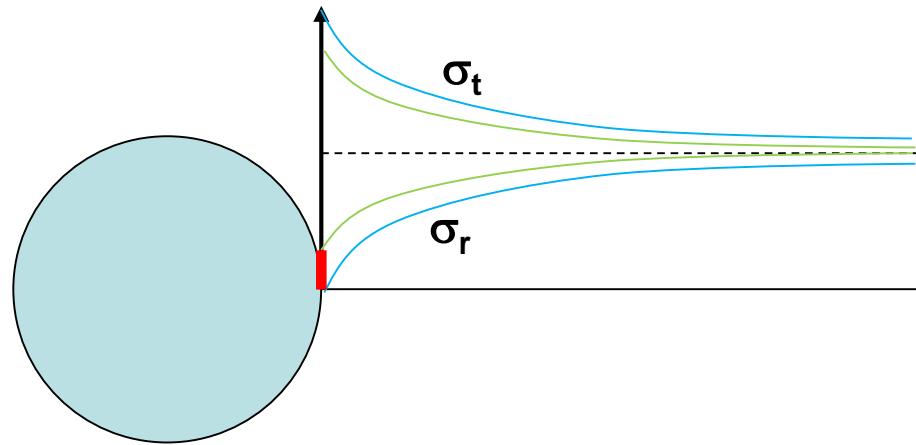
Enlargement of Metro Station Marienplatz Munich



New Tunnel
Designed to support/
mobilize the soil's load
carrying capacity

Old Tunnel
Designed to withstand the full
overburden pressure

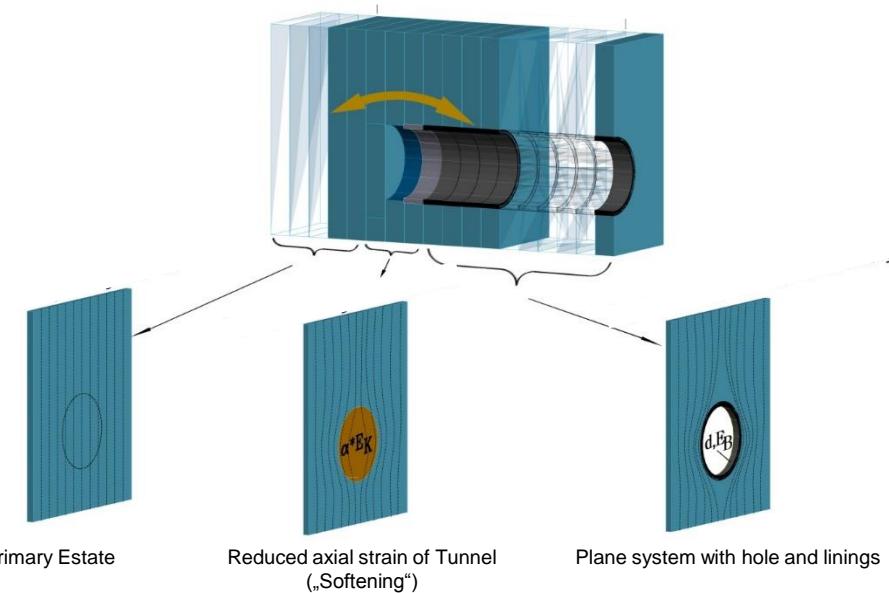
„New“ Austrian Tunneling Method



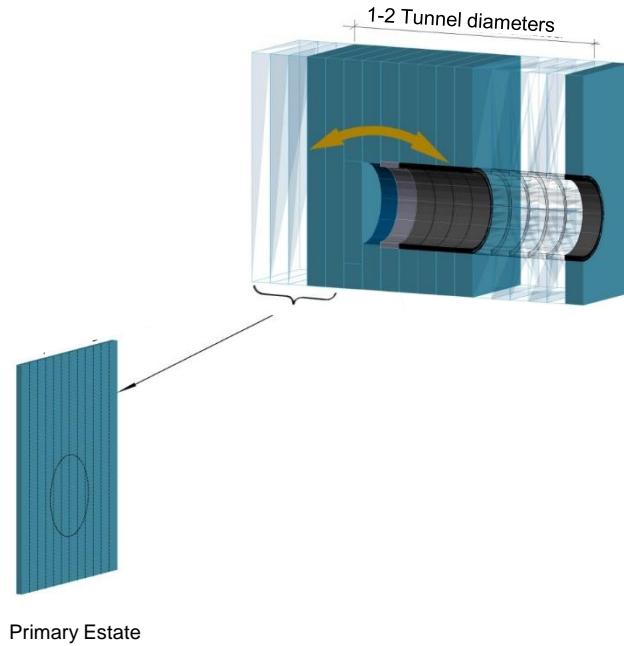
- Stress differences exceeding the strength yield plastic deformations
- A small outward pressure of the lining will inhibit this
- This pressure is obtained if the lining is in place before deformation starts

Analytical Model

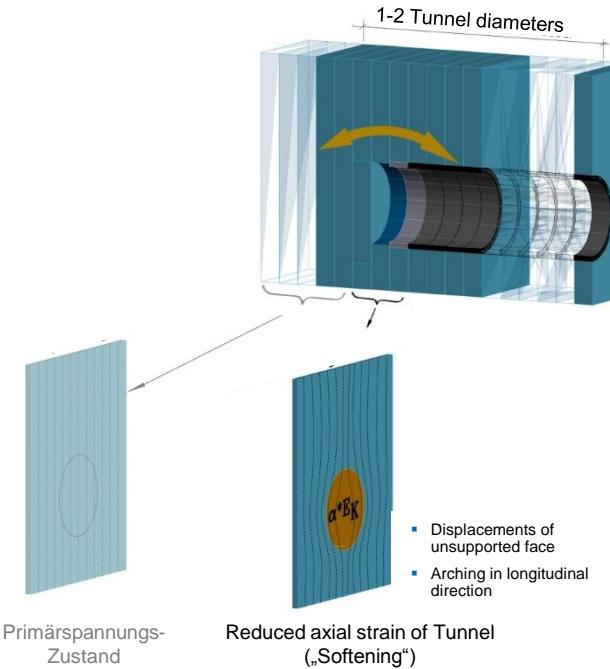
- Analysis at a set of representative cross-section slices under plane strain conditions
- Incorporating 3D stress redistribution effects by
⇒ stiffness reduction method (α -Method)



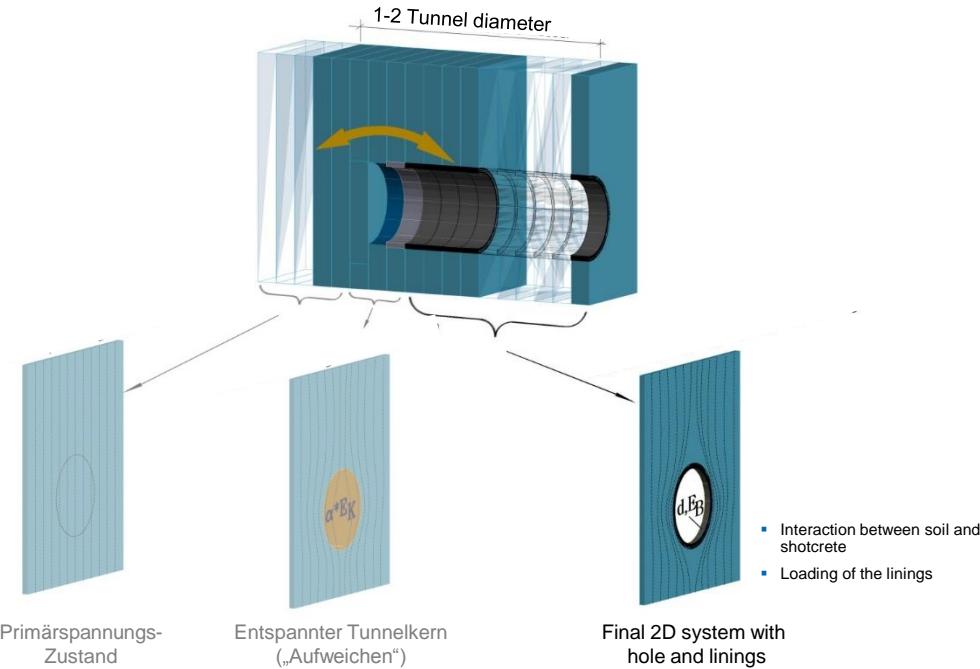
Stiffness reduction method (α -Method)



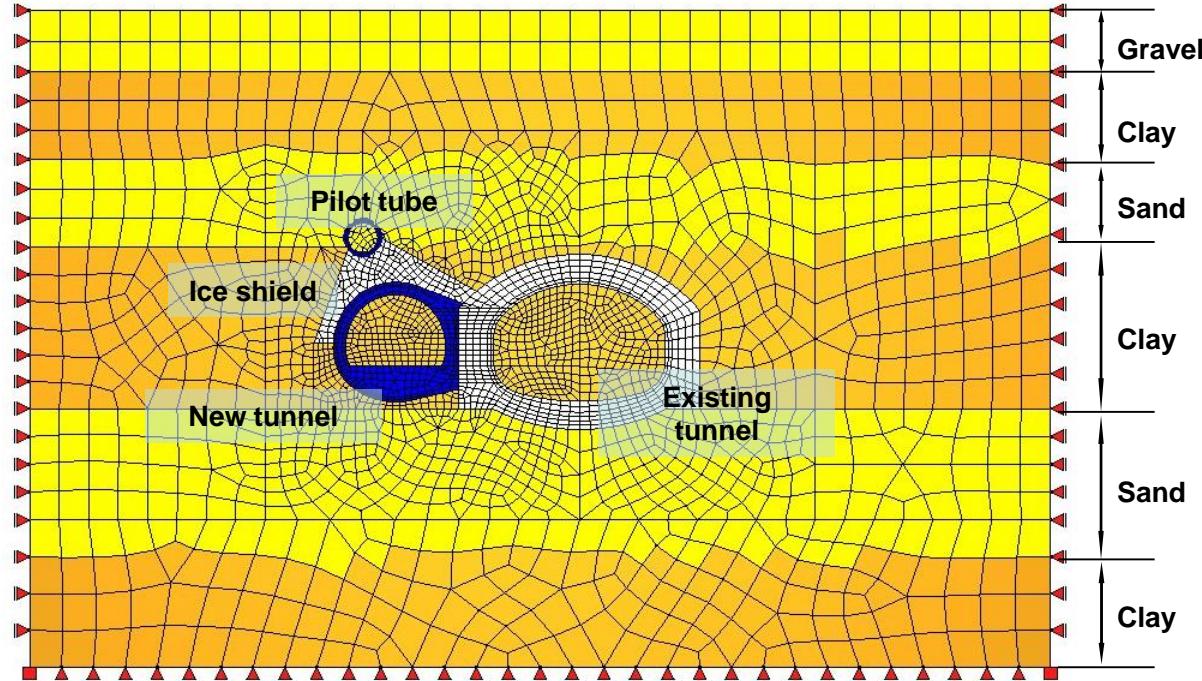
Stiffness reduction method (α -Method)



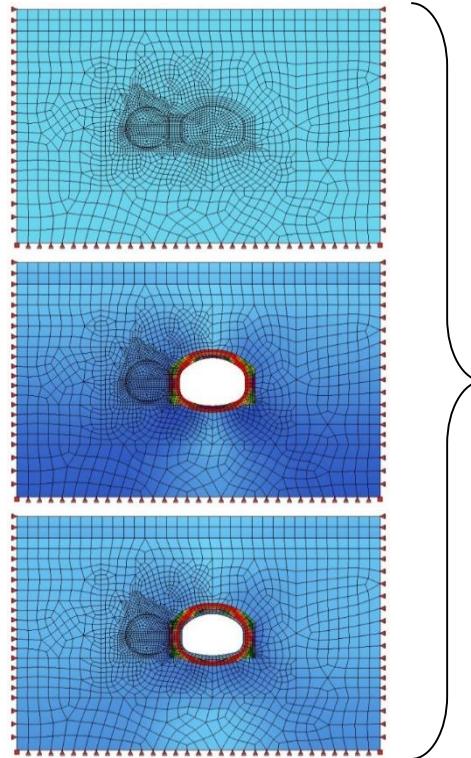
Stiffness reduction method (α -Method)



Finite element model



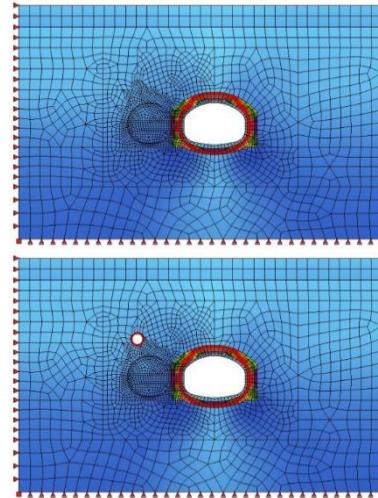
Simulation 1st stage: primary stress state



Simulating the historic construction process

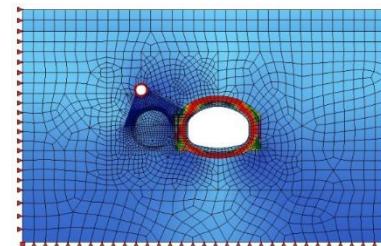
⇒ generating a model loading state that reflects the situation prior to construction activity

Simulation 2nd stage: preparatory steps

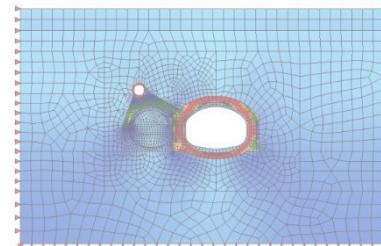


- Drainage of lower aquifer
- Installation of pilot tunnel

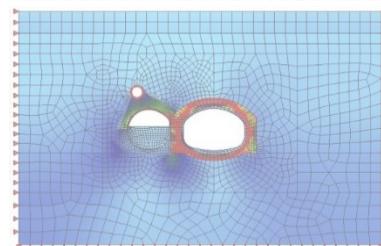
Simulation 3rd stage: tunnelling process



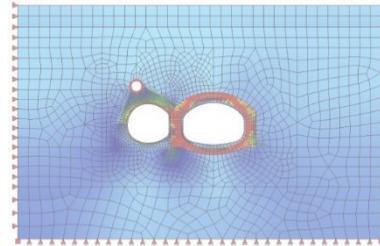
- Soil freezing



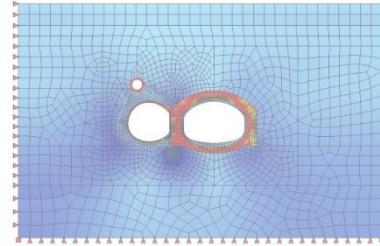
- Relaxation of calotte region



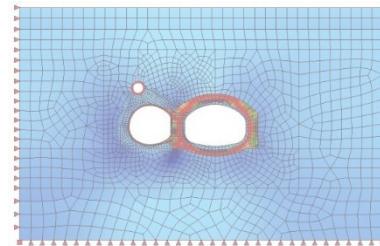
- Excavation calotte
- Installation shotcrete lining



- Excavation base
- Installation shotcrete lining

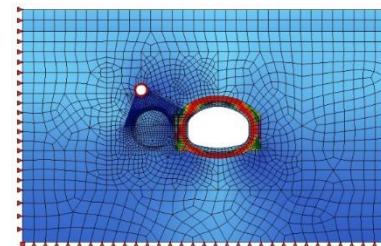


- Defrosting

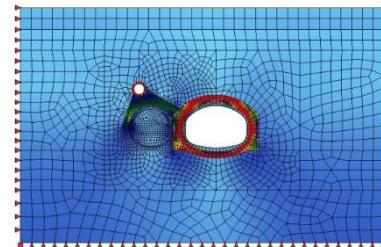


- Drainage turning-off

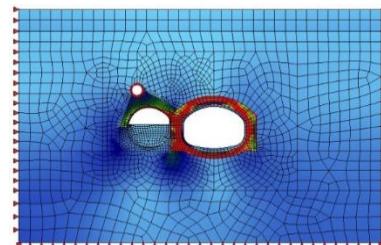
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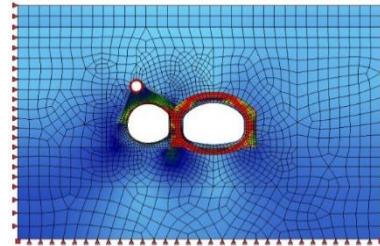
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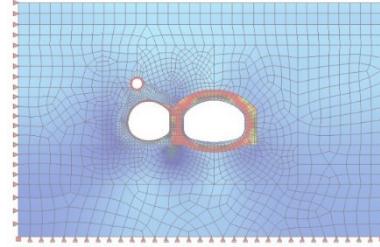
- Relaxation of calotte region



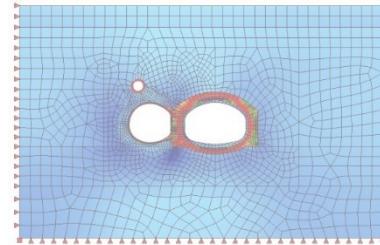
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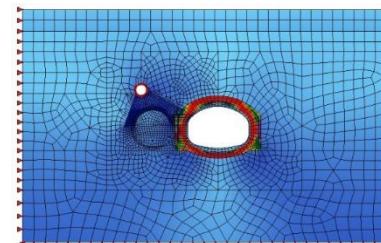


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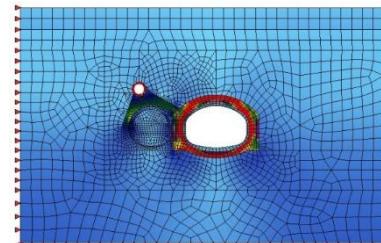


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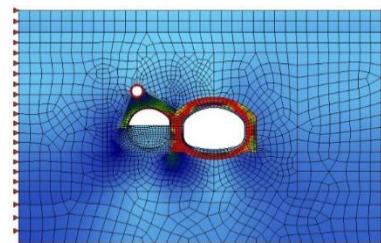
Simulation 3rd stage: tunnelling process



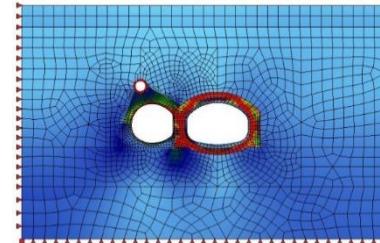
- Soil freezing



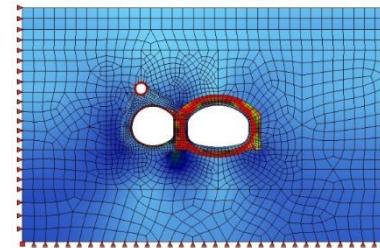
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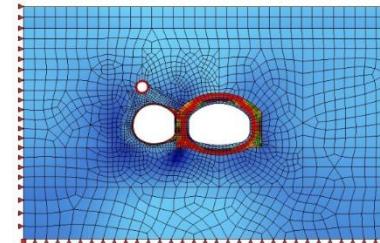
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- Excavation base
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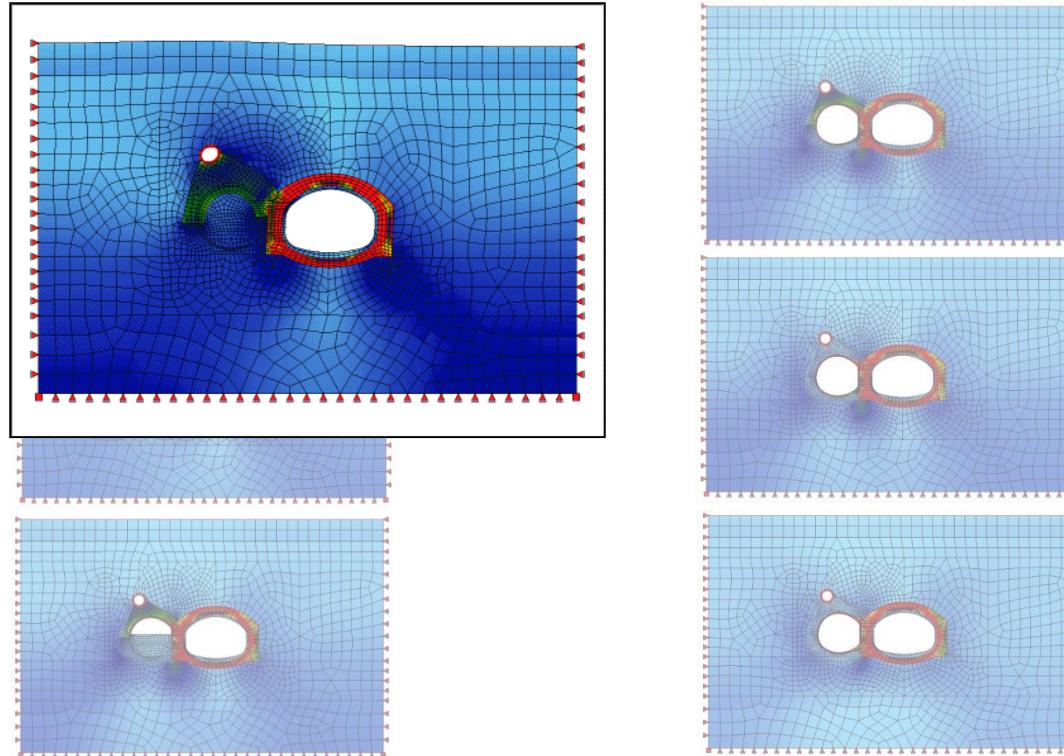


- Defrosting



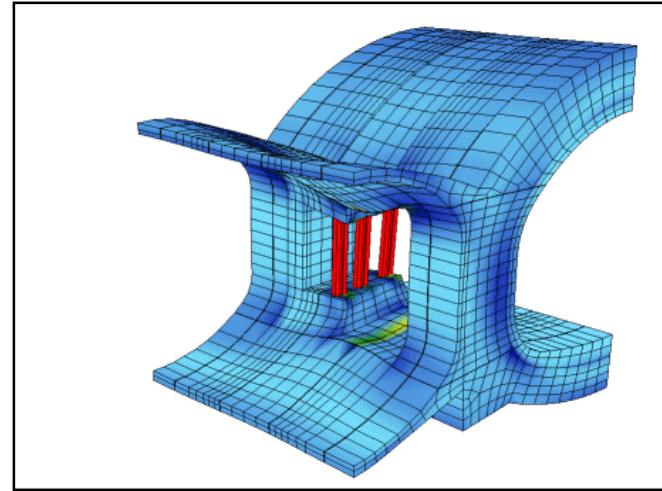
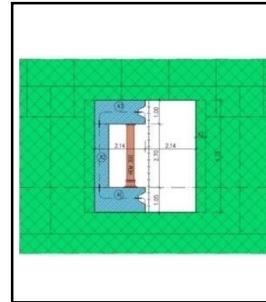
- Drainage turning-off

Soil freezing



Simulating the 3D cross cutting process

- There are some more phases in 3D

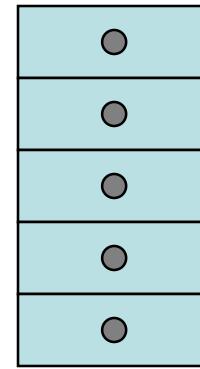




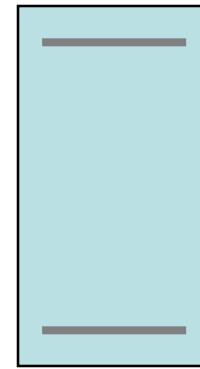
Design

- Finite Element Analysis is linear in general
- Design is based on ultimate loads and plasticity
- The real ultimate loading depends on all elements within the structure.
- Thus, you may be either
 - Not economical
 - Not save

Example of a Beam



5 “membrane” elements
Each obtaining its individual
reinforcement



Classical
beam element

M = 250 kN

| Element | Stress | | Reinforcement | |
|--|----------|--------|---------------|------|
| | Theoret. | FE | Classical | FE |
| 1 | -11.11 | -11.03 | | 485 |
| 2 | -5.56 | -5.51 | | 106 |
| 3 | 0.00 | 0.00 | | 0 |
| 4 | 5.55 | 5.51 | | 694 |
| 5 | 11.11 | 11.03 | 1973 | 1391 |
| Sum | | | 1973 | 2676 |
| Beam design with distribution from FE-Results | | | | 2658 |

M=250 kNm, N= -500 kN

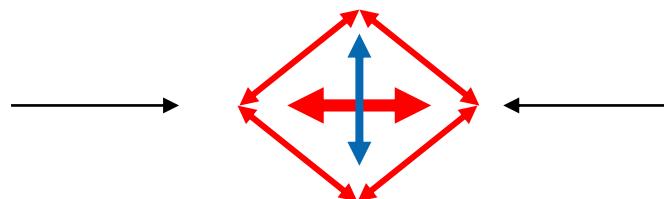
| Element | Stress | | Reinforcement | |
|---|----------|--------|---------------|------|
| | Theoret. | FE | Classical | FE |
| 1 | -13.89 | -13.81 | 540 | 985 |
| 2 | -8.33 | -8.29 | | 160 |
| 3 | -2.79 | -2.78 | | 54 |
| 4 | 2.79 | 2.73 | | 344 |
| 5 | 8.33 | 8.29 | 1262 | 1040 |
| Sum | | | 1802 | 2583 |
| Beam design with distribution from FE-Results | | | | 2244 |

M=250 kN, N= -1000 kN

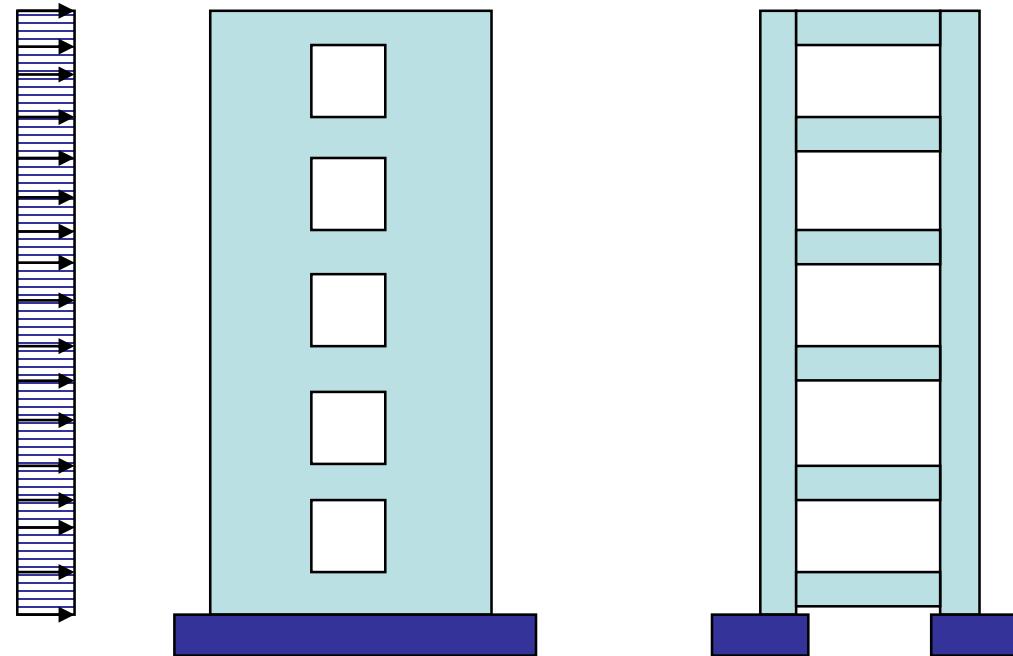
| Element | Stress | | Reinforcement | |
|---|----------|--------|---------------|------|
| | Theoret. | FE | Classical | FE |
| 1 | -16.67 | -16.58 | 1359 | 1486 |
| 2 | -11.11 | -11.06 | | 492 |
| 3 | -5.56 | -5.56 | | 107 |
| 4 | 0.00 | 0.05 | | 0 |
| 5 | 5.56 | 5.48 | 1359 | 691 |
| Sum | | | 2718 | 2776 |
| Beam design with distribution from FE-Results | | | | 2260 |

Design of Shear Walls

- Direction of principal stresses
- Direction of reinforcements
- Direction of cracks
- Models available from Baumann / Leonhardt or Stiglat/Wippel
- Analysis based on minimum of deformation work
- A bunch of detailed problems - be careful e.g. inclined compressive reinforcement



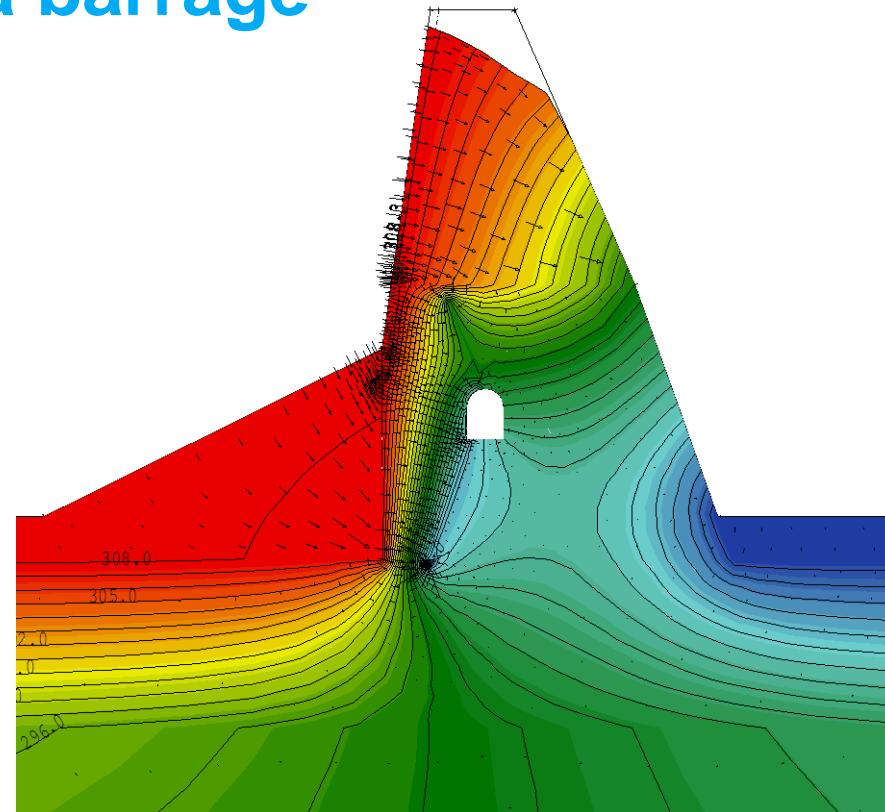
Structured shear walls



Structured Shear Walls

| | Beams | Mod. Stiff. in nodes | Rigid Nodes | FE Solution |
|-----------|-------|-------------------------|----------------|----------------|
| N (left) | 25.6 | 133.2 | 133.2 | 117.8 |
| V (left) | 200.1 | 187.7 | 195.2 | 191.8 |
| M (left) | -2818 | -2216 | -2251 | -2316 |
| N (right) | -25.6 | -133.2 | -133.2 | -117.8 |
| V (right) | 137.4 | 149.8 | 142.3 | 145.5 |
| M (right) | -2612 | -2146 | -2112 | -2199 |

Example of a barrage

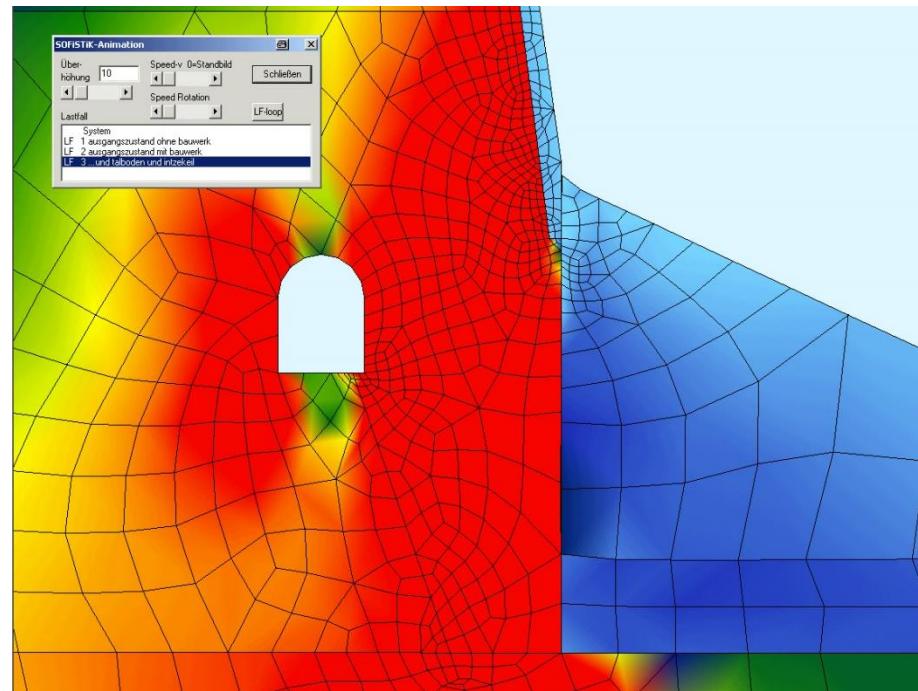
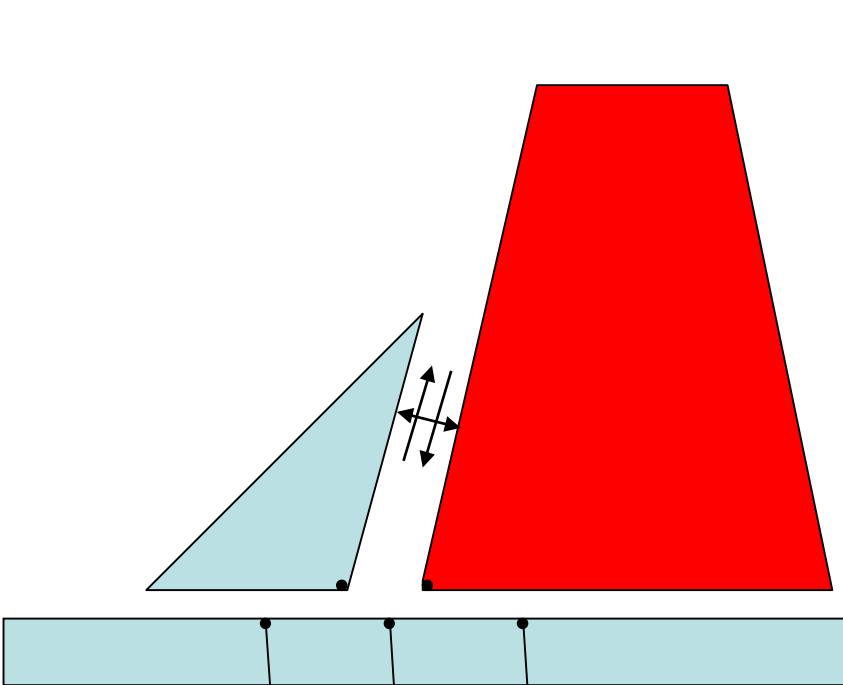


Problem

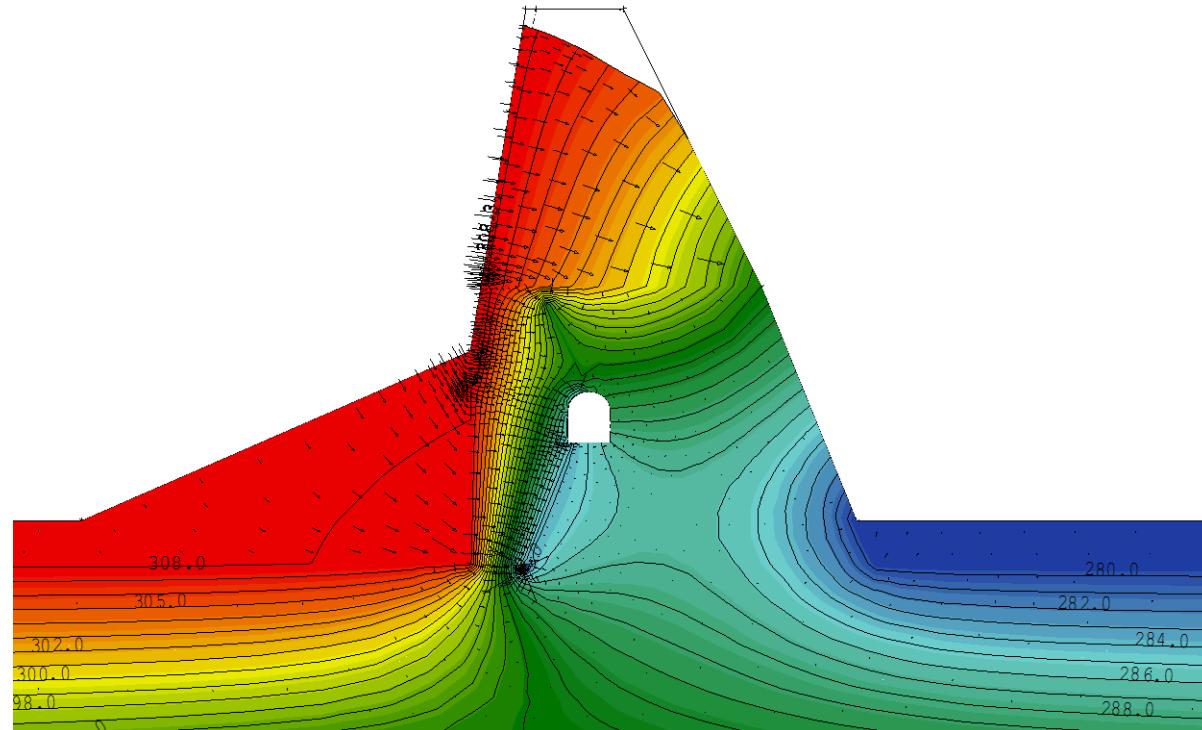
- Stability of a barrage (height 32,50 m / width 21 m)
- Load cases to be considered:
 - Seepage
 - Temperature
 - Ice pressure
 - Earthquake
- On the water side there is an additional brickwork and a so called „Intze-Keil“ to ensure water tightness
- Non linear material for rock and dam

Friction between Soil and Dam

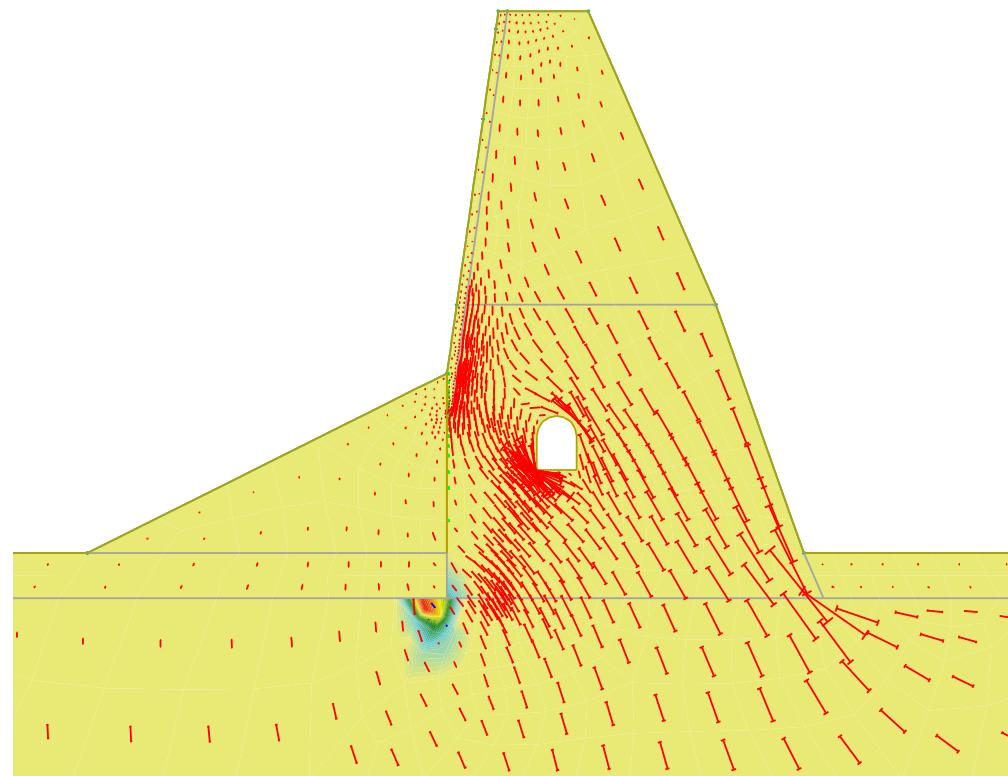
L



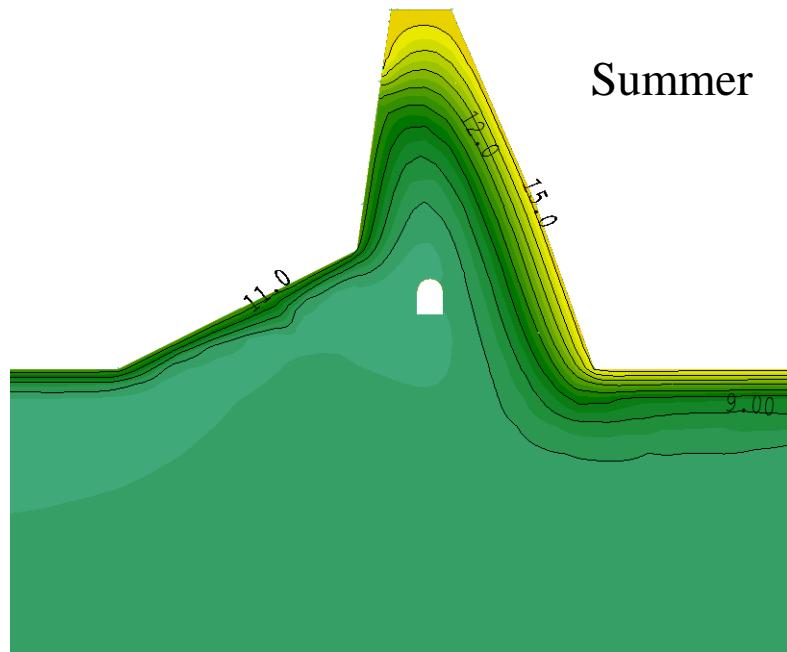
Seepage through dam



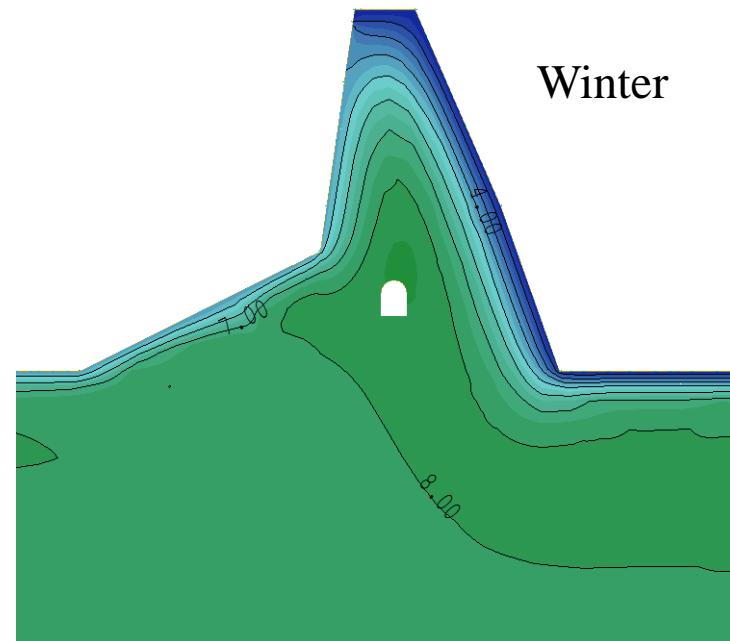
Stresses including seepage



Transient Temperatures



Summer

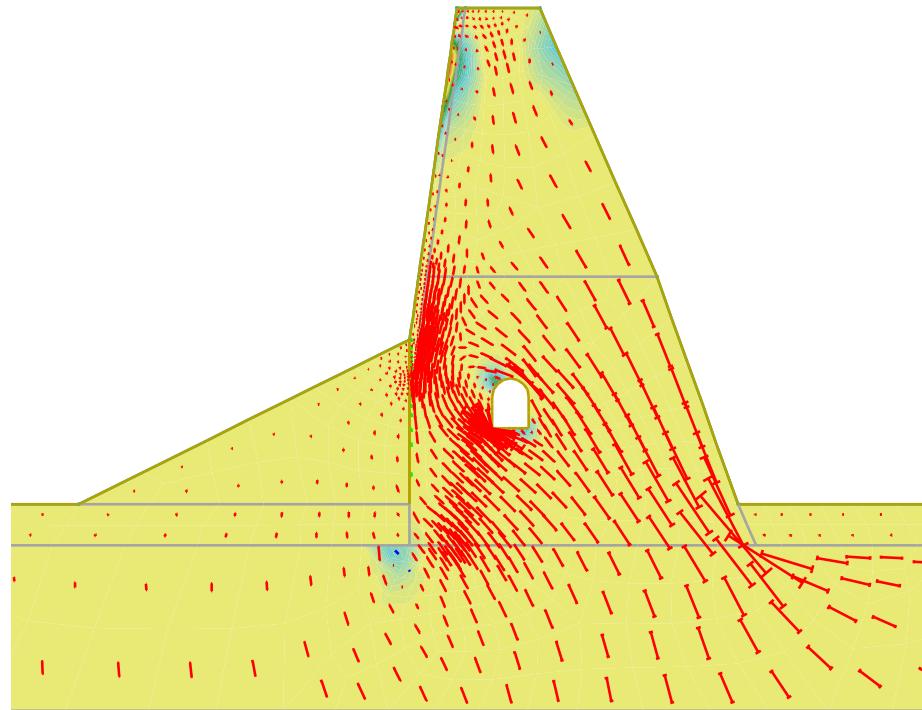


Winter

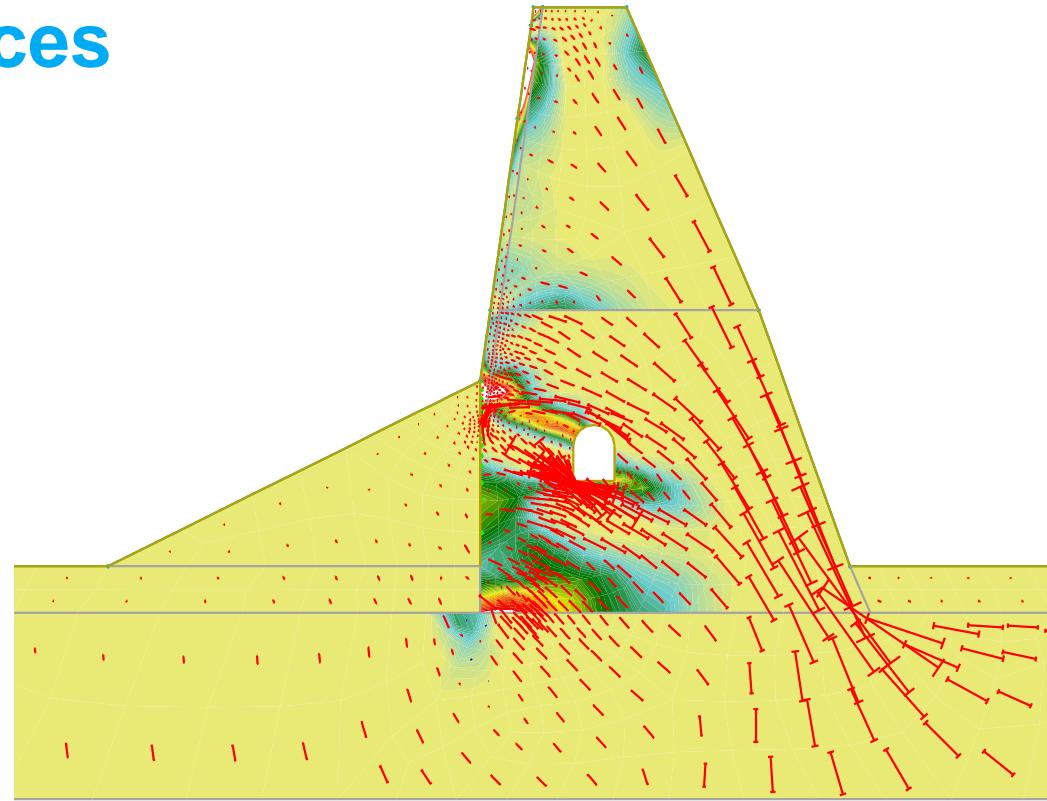
Earthquake

- There is a fluid structure interaction problem
- A simple approach just adds some mass to the barrage according to the distribution of Westergard
- You have to add a mass value depending on the distance of the nodes and their depth.
- How ?
 - Excel sheet (if you know the coordinates)
 - With a cubic load distribution (if you have good software)

Stresses including Earthquake



Near Collapse with 7 x Earthquake forces



Fluid-Elements - Eulerian Approach

- Differential equation connecting velocity of non viscous fluid to pressure:

$$\nabla p = -\rho_f \dot{u}$$

- The density of the fluid is constant
- Velocities are so small that higher order effects may be neglected, (Convection, Cavitation etc.).
- Shear stresses (viscous effects) are neglected



Fluid Structure Interaction



- Flow is described by velocities (Eulerian-Approach)
- Structure is described by displacements (Lagrangian Approach)
- Coupling quite complex

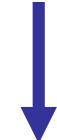
Lagrange Approach for Fluids

- Stresses with Compression and Shear modulus:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \sigma_z \end{bmatrix} = \frac{E}{(1+\mu)(1-2\mu)} \begin{bmatrix} 1-\mu & \mu & 0 & \mu \\ \mu & 1-\mu & 0 & \mu \\ 0 & 0 & (1-2\mu)/2 & 0 \\ \mu & \mu & 0 & 1-\mu \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \varepsilon_z \end{bmatrix}$$

→

$$K = \frac{E}{3(1-2\mu)}$$

$$G = \frac{E}{2(1+\mu)}$$


- Quite Common in Soil Mechanics
- Limit State $G \Rightarrow 0$, $\mu \Rightarrow 0.5$
- Some viscous effects possible

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \sigma_z \end{bmatrix} = \begin{bmatrix} K + \frac{4}{3}G & K - \frac{2}{3}G & 0 & K - \frac{2}{3}G \\ K - \frac{2}{3}G & K + \frac{4}{3}G & 0 & K - \frac{2}{3}G \\ 0 & 0 & G & 0 \\ K - \frac{2}{3}G & K - \frac{2}{3}G & 0 & K + \frac{4}{3}G \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \varepsilon_z \end{bmatrix}$$

Water

- Compression modulus

$$K = 2000 \text{ N/mm}^2$$

- dynamic Viscosity

$$\eta = 0.0000000165 \text{ Ns/mm}^2$$

- Shear Modulus very small

$$\begin{aligned}\tau &= G * \gamma = G * \frac{\dot{\gamma}}{\omega} = \eta * \dot{\gamma} \\ \Rightarrow G &= \eta * \omega\end{aligned}$$



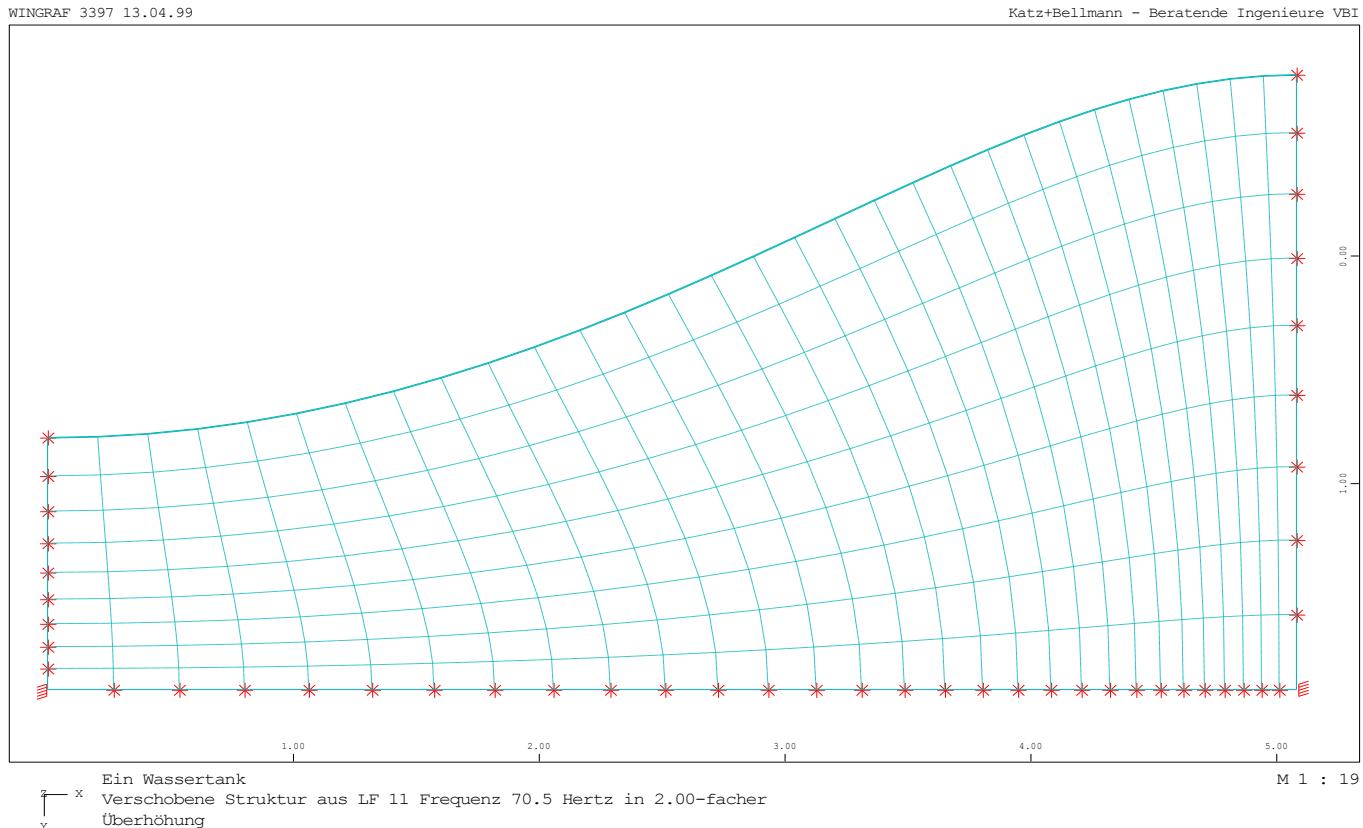
First Test

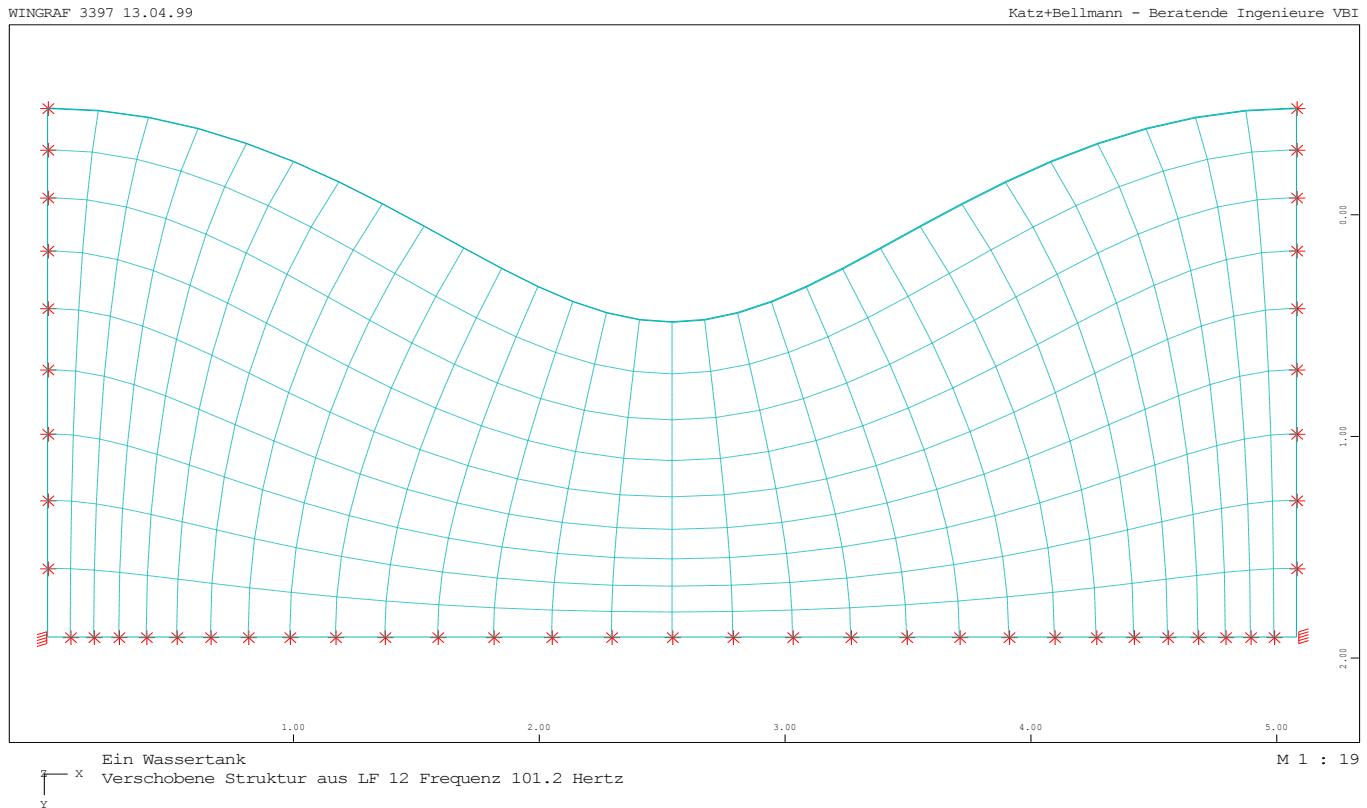


HEAD WATERTANK

NODE 1 0.00 0.0 ; 10 = 1.905
301 5.08 0.0 ; 310 = 1.905

MAT 1 K 2E6 G 2E3 GAM 10.0





1st Eigen frequency

| G-Modulus | Plane Strain | K+G plane strain | Plane Stress |
|-----------|--------------|---------------------|--------------|
| 200 000 | 70.61 | 70.45 | 69.18 |
| 2000 | 8.19 | 7.08 | 6.94 |
| 20 | 3.55 | 0.71 | 0.69 |
| 0.2 | 4.50 | 0.07 | 0.069 |
| Reference | 0.36 | 0.36 | 0.36 |



Flaws

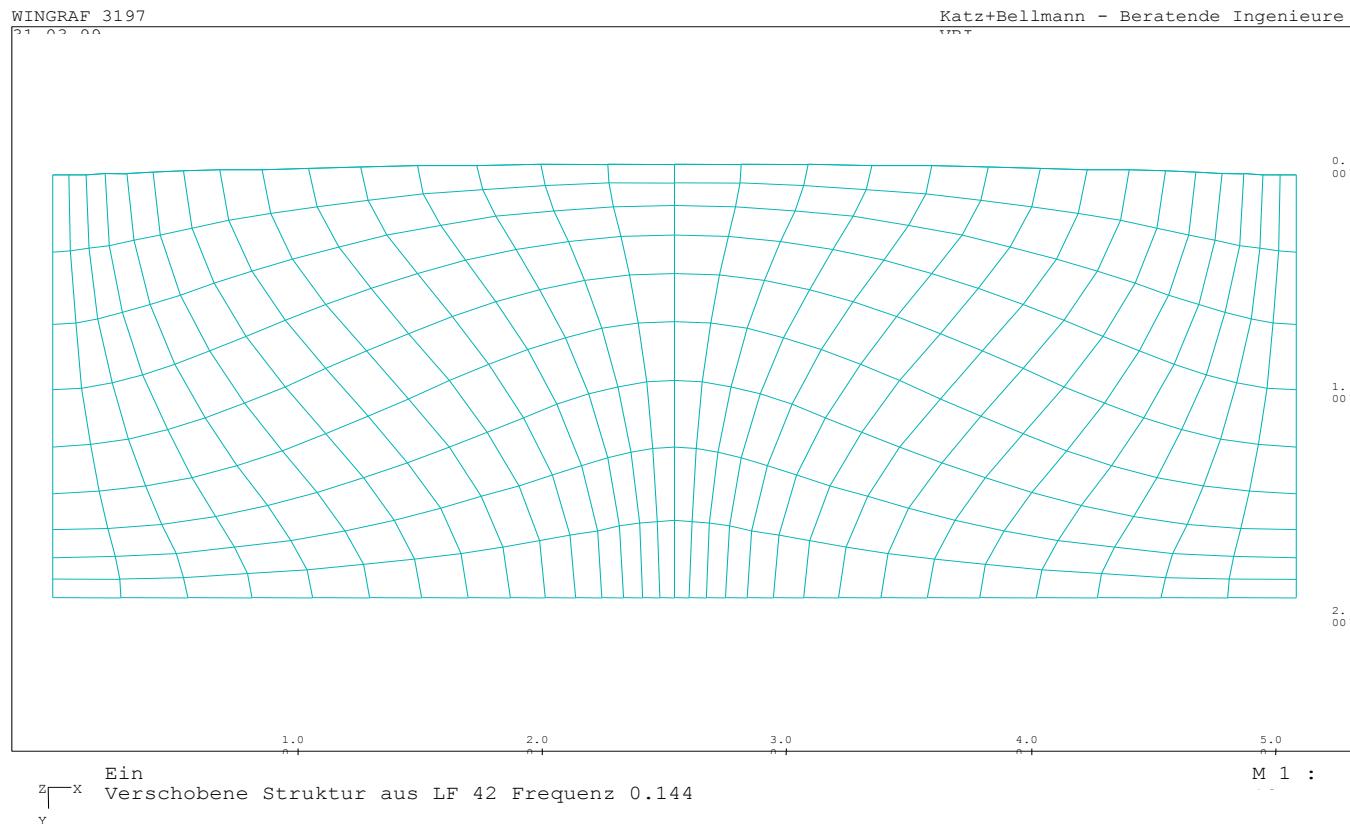


- Locking of incompressible media
=> Three field approximation
- Potential of free surface is missing

BOUN 1 TITLE 'Free Surface'

BOUN 1 301 10 CY 10.0

==> Frequencies become bounded





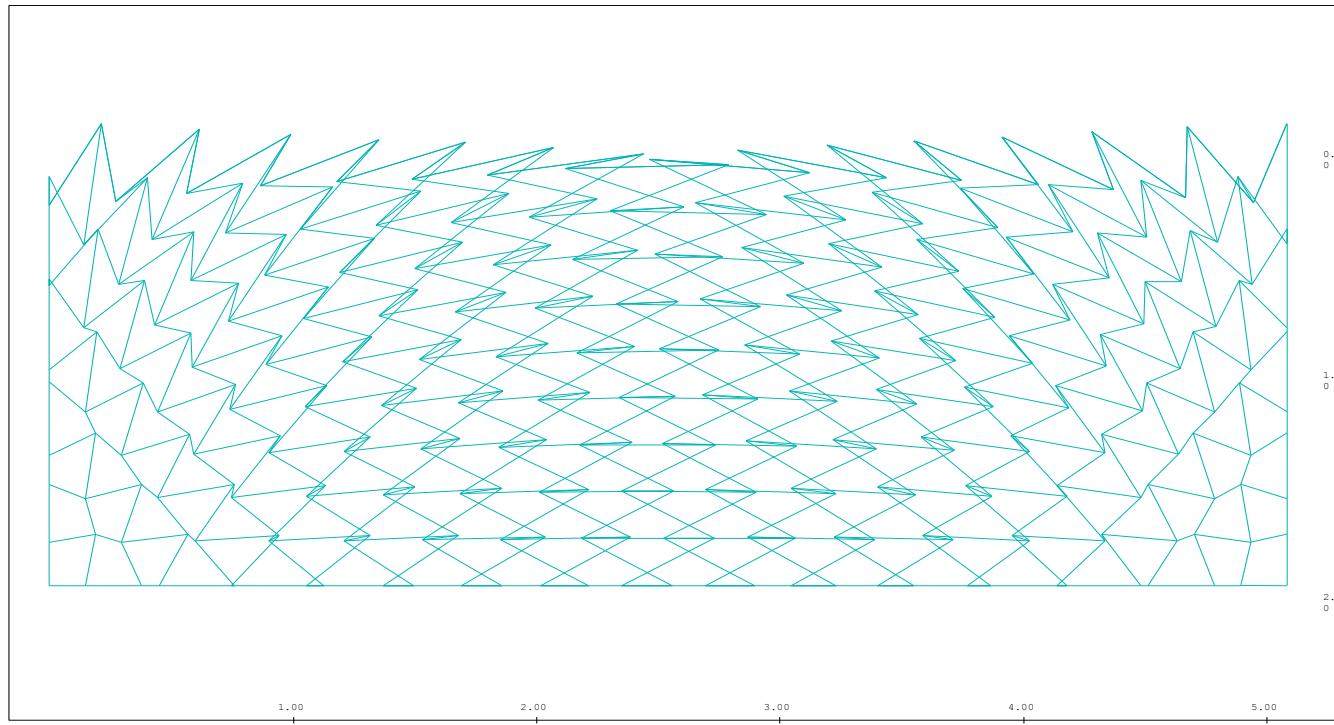
Flaws



- Stiffness of surface now larger then fluid stiffness
- Rotational deformations have no stiffness
 - => Introduce Penalty function

WINGRAF 3197 31.03.99

Katz+Bellmann - Beratende Ingenieure VBI



Ein Wassertank
Verschobene Struktur aus LF 43 Frequenz 0.697 Hertz in 0.500-facher
Überhöhung



Flaws



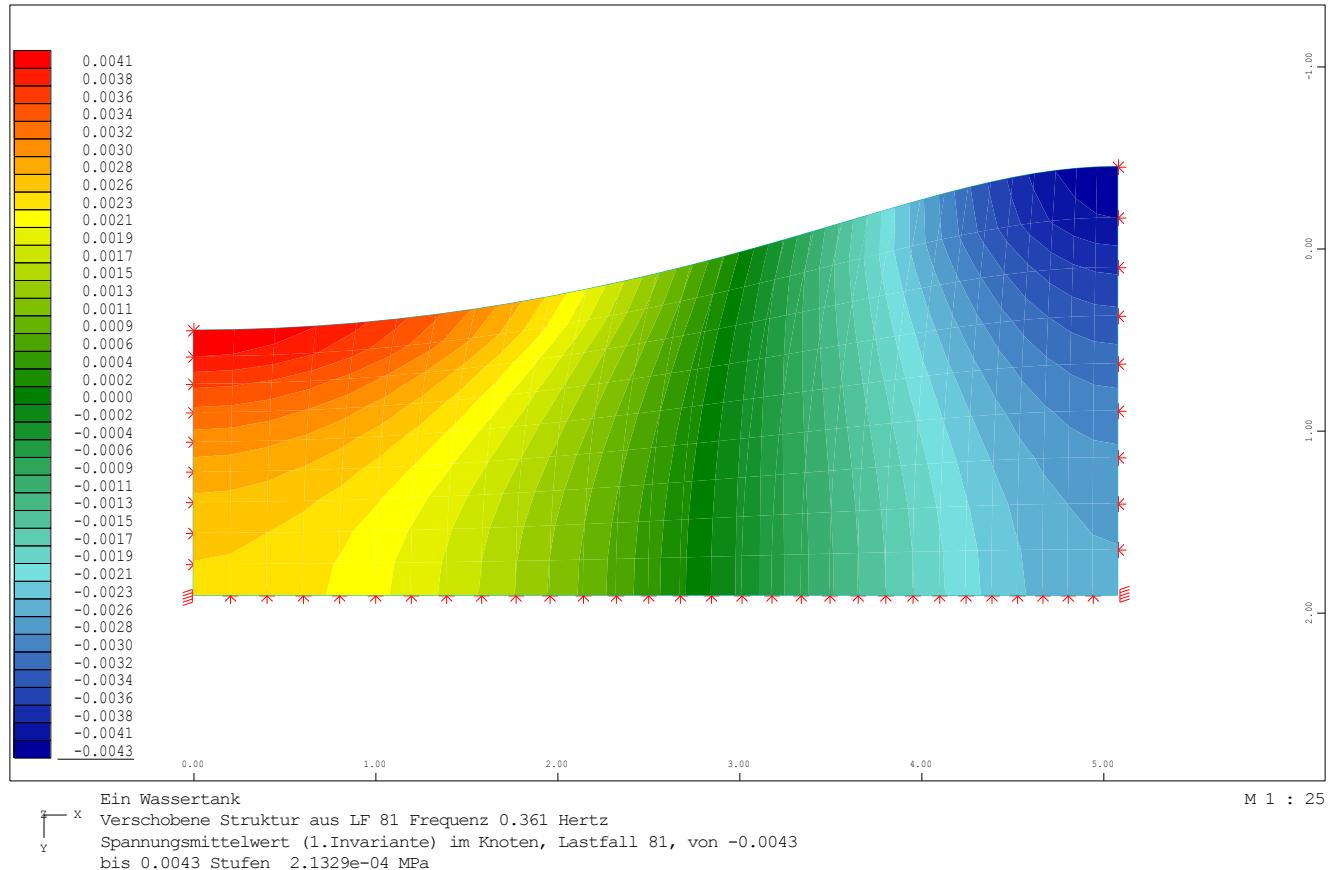
- „spurious modes“
- Mass matrix has still modes which are suppressed in the stiffness matrix
=> Projection by Kim/Yong

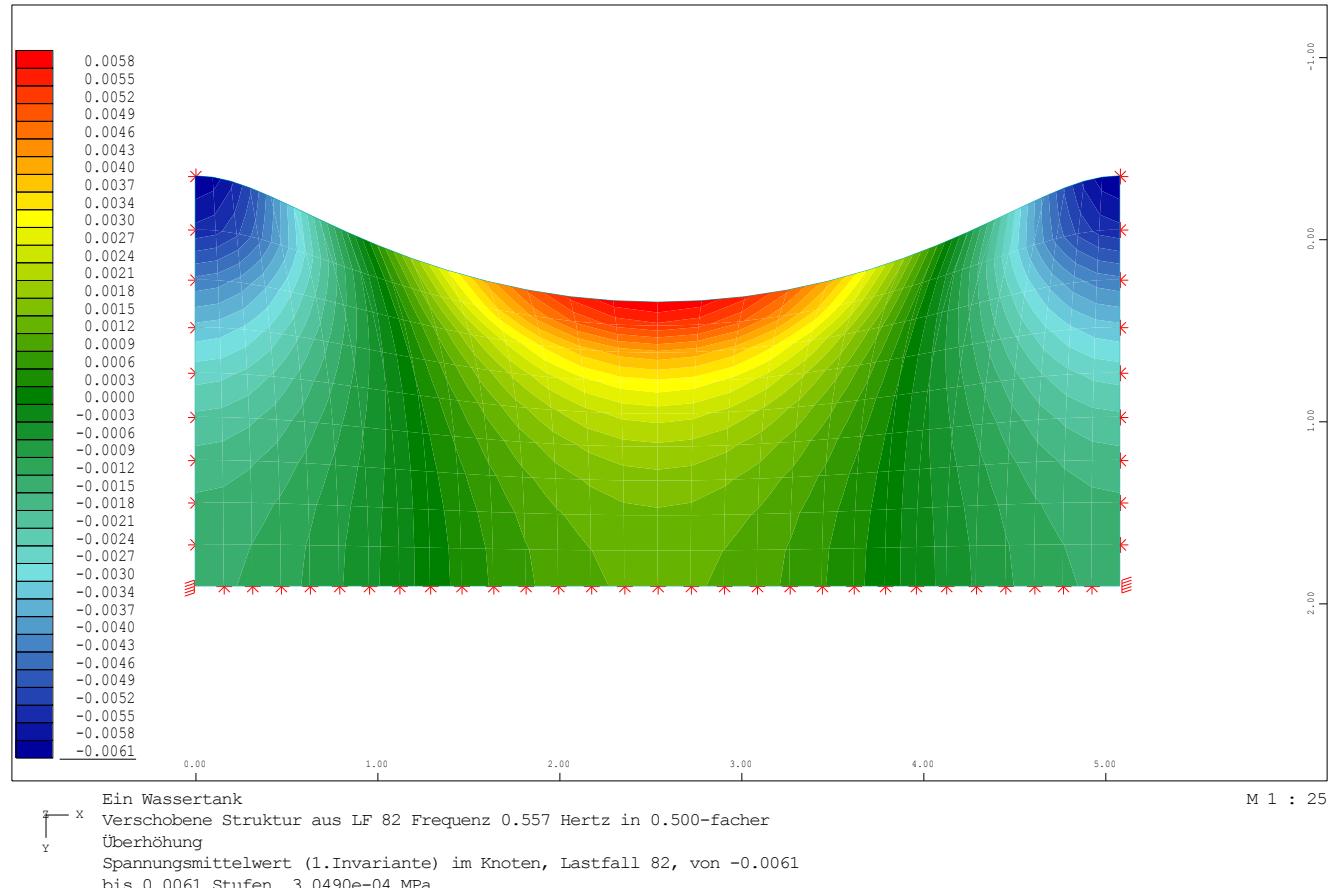


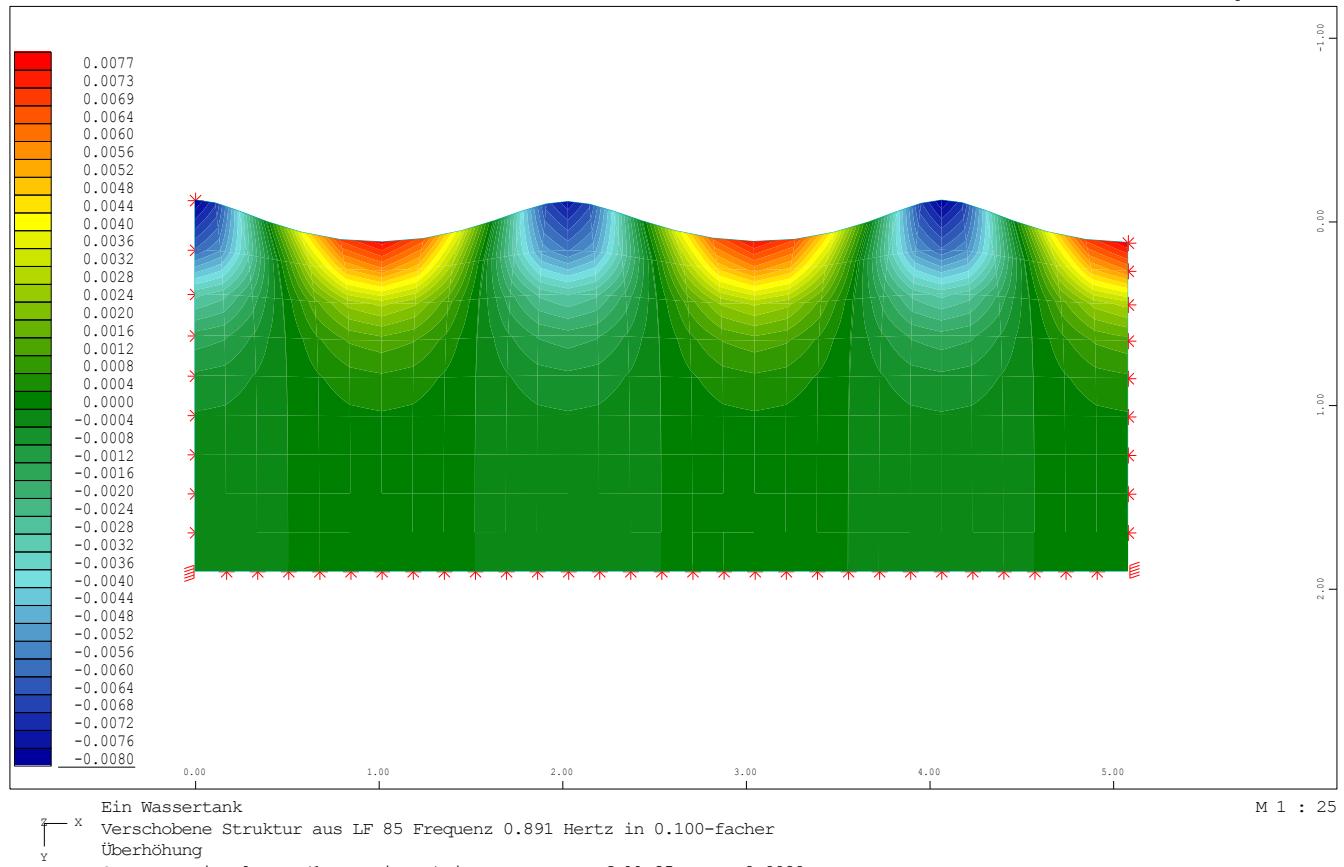
Eigenfrequencies



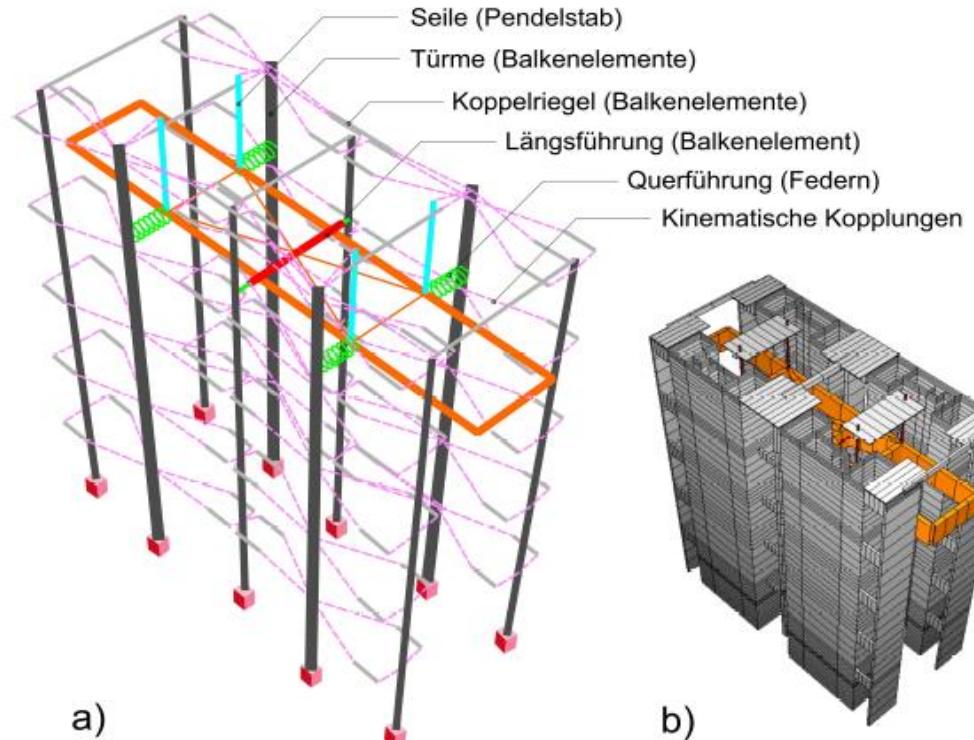
| G-Modulus | f1 | f2 | f3 |
|-----------|--------|---------|--------|
| 200 000 | 70.455 | 101.210 | |
| 2000 | 7.101 | 10.246 | 13.254 |
| 20 | 0.951 | 1.846 | 2.730 |
| 0.2 | 0.371 | 0.582 | 0.730 |
| 0.02 | 0.361 | 0.557 | 0.689 |
| Reference | 0.357 | 0.555 | |





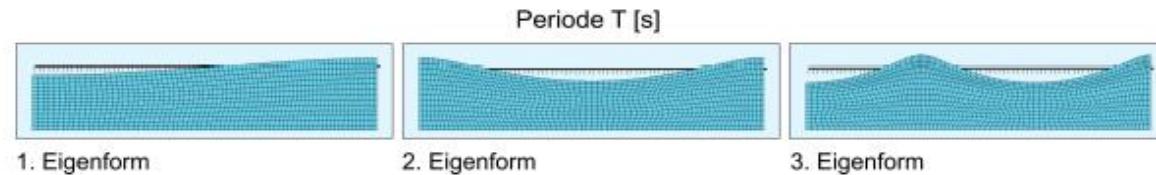


Three Gorges Dam China



Problem of Water in trough

- Ship elevator investigated by Krebs & Kiefer Karlsruhe with SOFiSTiK
- Seismic response of sloshing water in trough
- CFD Model with ca 1 Million cells
- 3D model of water with ca 1000 lagrange elements



Comparison CFD / FEM

