Industrial Applications of Computational Mechanics Plates and Shells – Mesh generation – static SSI

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FEM - Reminder

- A mathematical method
- The real (continuous) world is mapped on to a discrete (finite) one.
- We restrict the space of solutions
- We calculate the optimal solution within that space on a global minimum principle

Plates (Slabs and shear walls)

- Classical solution for shear walls (Airy stress function F)

$$\Delta\Delta F = 0$$

- Classical Plate bending solution (Kirchhoff)

$$\Delta\Delta w = p$$

- FE / Variational approach for shear walls

$$\Pi = \frac{1}{2} \int \varepsilon D \varepsilon \, dV = \text{Minimum}$$

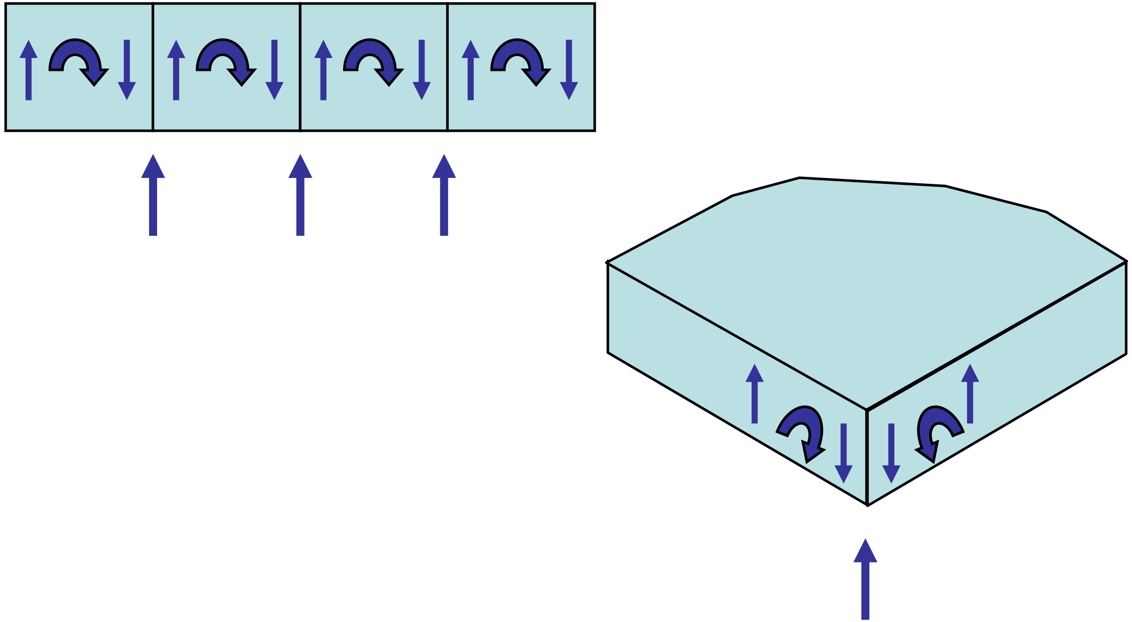
- FE / Variational approach for bending plates

$$\Pi = \frac{1}{2} \int \kappa D \kappa \, dV = \text{Minimum}$$

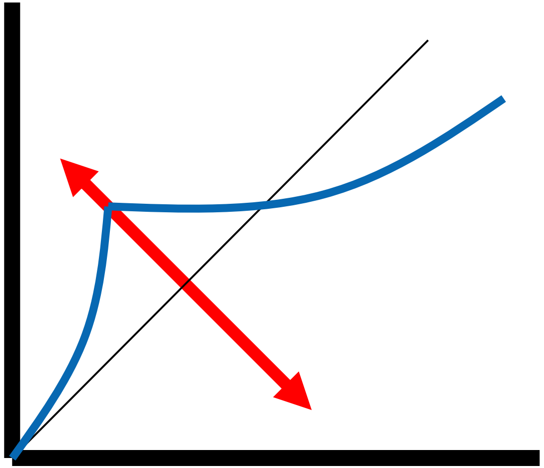
Plate elements

- Kirchhoff Theory
  - Introducing equivalent shearing force
  - Shear force is calculated from 3rd derivative
    Precision of those values are not acceptable
  - Better elements quite complex
  - Hybrid elements
    mixed functional of strains and stresses
    = quite good but rather complex
    = difficult for non linear effects

Equivalent shear force



Condition at the corner



μ= 0 / plate without torsion

$$\begin{bmatrix} m_{xx} \\ m_{yy} \\ m_{xy} \end{bmatrix} = \frac{E \cdot t^3}{12\left(1-\mu^2\right)} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & 1-\mu \end{bmatrix} \cdot \begin{bmatrix} k_{xx} \\ k_{yy} \\ k_{xy} \end{bmatrix}$$

- For simpler analysis set $\mu = 0$
  => Minimum transverse reinforcement of a plate 20 % (DIN)
- Torsion-free-Plate sets the 3rd diagonal term = 0
  - More reinforcement in the mid span
  - Less reinforcements in the corners
- General Rule
  - It is difficult to save reinforcements by a nonlinear analysis

Kinematic of plates without shear deformations

- Problem of skewed supported edges

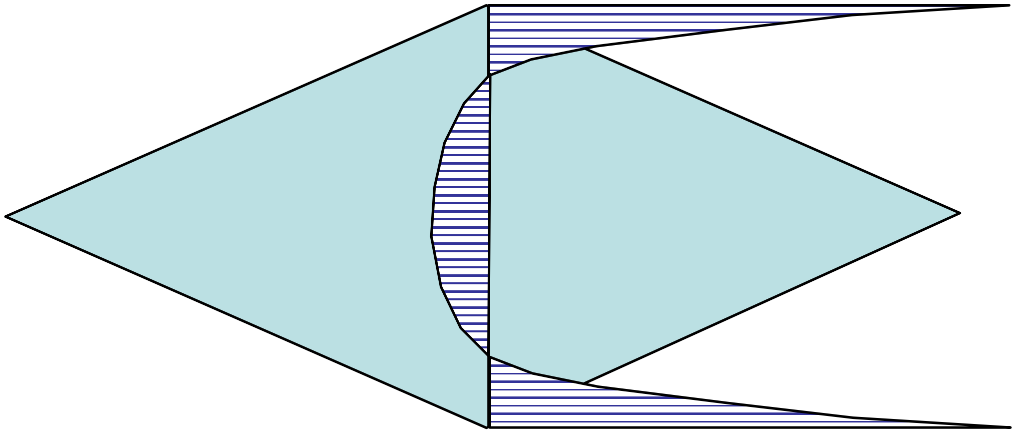


Plate elements

- Mindlin/Reissner Theory
  - Introducing shear deformations
  - Two coupled differential equations
  - Shear force is calculated from the 1st derivative !
  - Elements very simple
  - Problem for thin plates
    (shear locking)
  - Problem with spurious modes
    (under integrated Elements)

Mindlin/Reissner Theory

$$\Theta_x = \varphi_x - \frac{\partial w}{\partial x}; \quad \Theta_y = \varphi_y - \frac{\partial w}{\partial y}$$

$$k_x = \frac{\partial \varphi_x}{\partial x}; \quad k_y = \frac{\partial \varphi_y}{\partial y}; \quad k_{xy} = \frac{\partial \varphi_x}{\partial y} + \frac{\partial \varphi_y}{\partial x}$$

$$\begin{bmatrix} m_{xx} \\ m_{yy} \\ m_{xy} \end{bmatrix} = \frac{E \cdot t^3}{12\left(1-\mu^2\right)} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & 1-\mu \end{bmatrix} \cdot \begin{bmatrix} k_{xx} \\ k_{yy} \\ k_{xy} \end{bmatrix}$$

$$\begin{bmatrix} q_x \\ q_y \end{bmatrix} = \frac{G \cdot t}{1.2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \Theta_x \\ \Theta_y \end{bmatrix}$$

Kinematic of plates including shear deformations

- build in

  $w = 0 \quad ; \quad \varphi_n = 0 \; ; \quad \varphi_t = 0$

- hard support

  $w = 0 \quad ; \quad \varphi_t = 0$

- soft support

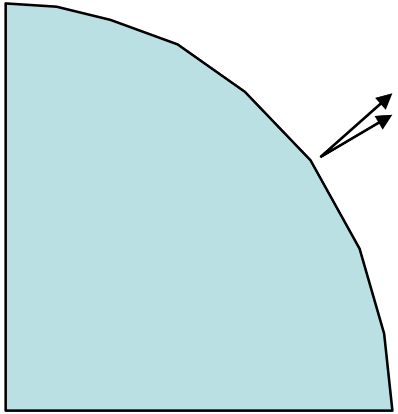
  $w = 0$

- sliding edge

  $w = 0 \quad ; \quad \varphi_n = 0$

Circular plates

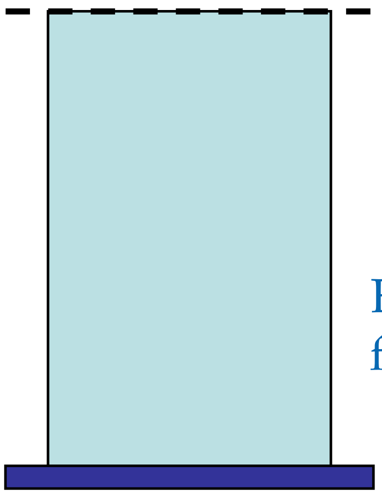
- Be careful when modelling support AND geometry !



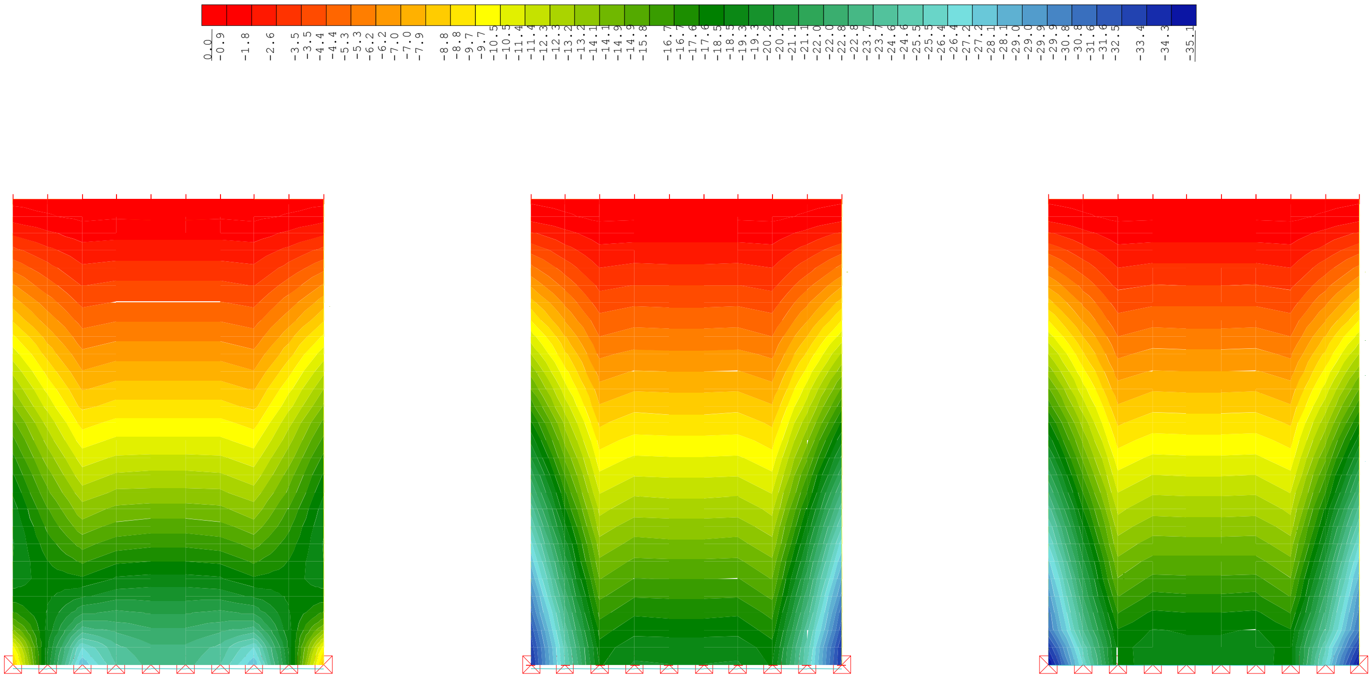
Smallest errors in the geometry may create a „build in“ effect

Boundary Layer

- Boundary region is critical for shear force

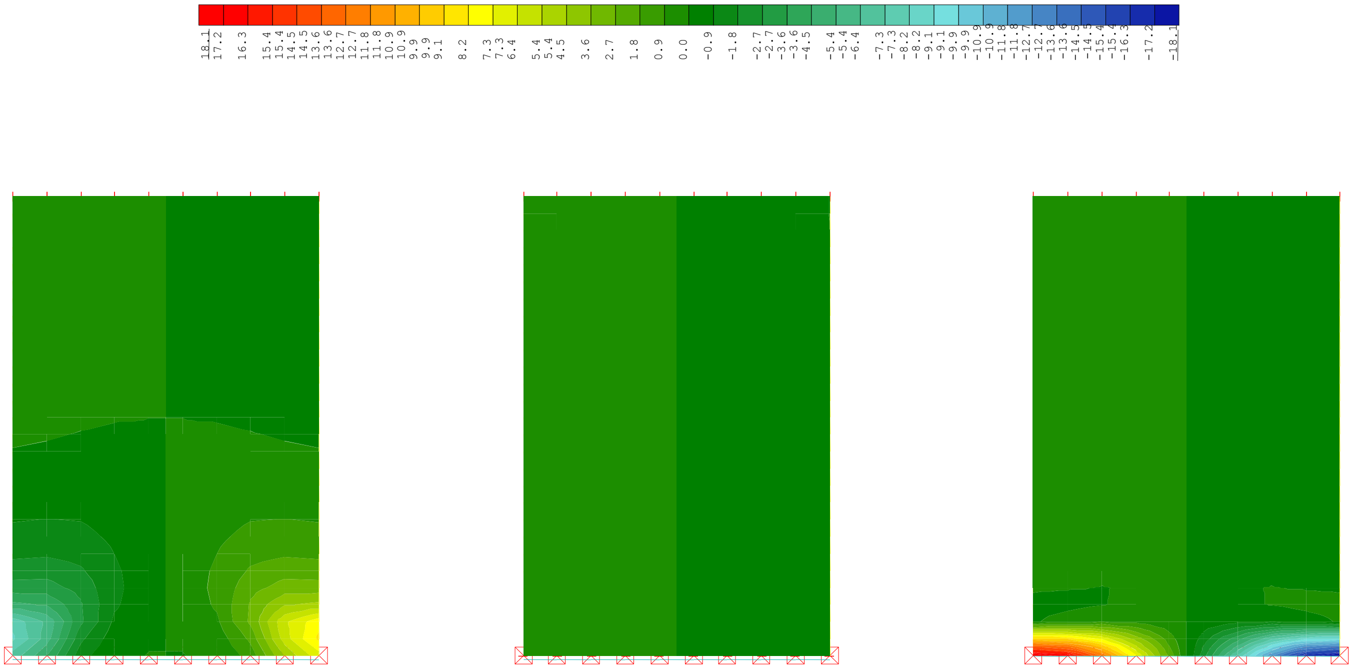


Shear force in longitudinal direction



Build in Hard support Soft support

Shear force in transverse direction



Build in Hard support Soft support

SOFiSTiK-elements

- Based on Hughes / Bathe-Dvorkin
  (discrete Kirchhoff-Modes enforce dM/dx=V)
- Quadrilateral enhanced with
  non conforming modes
- Properties:
  - Shear deformations without “locking”
  - Linear moment distribution
  - Constant shear force

No Problem:

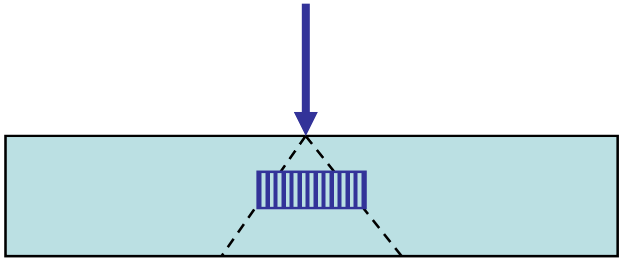
- Locking
- spurious modes
- Thick Plates
- Shear forces
- Skewed meshes

Problems:

- Loading
- Support Condition
- Design

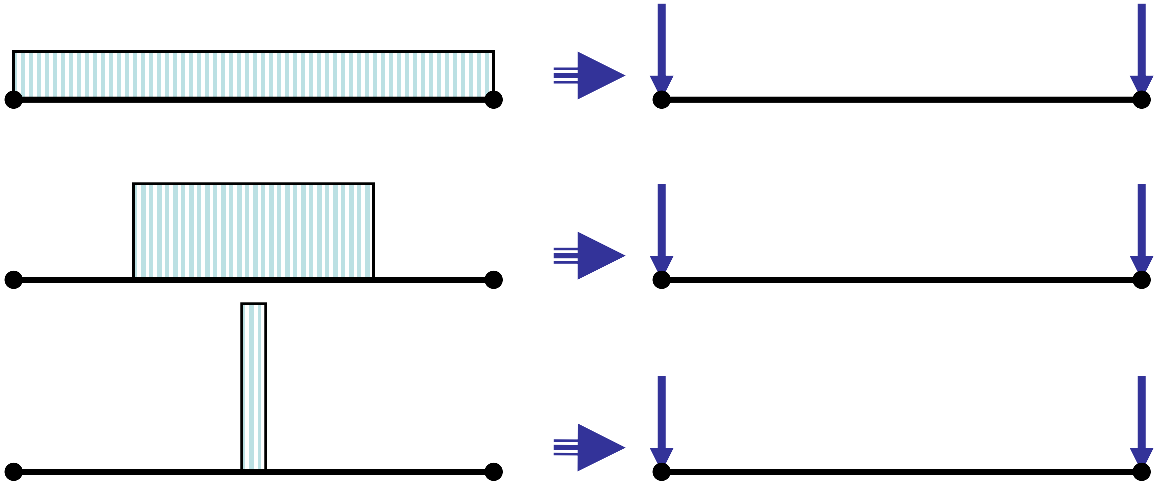
Loading

- There are no point loads !

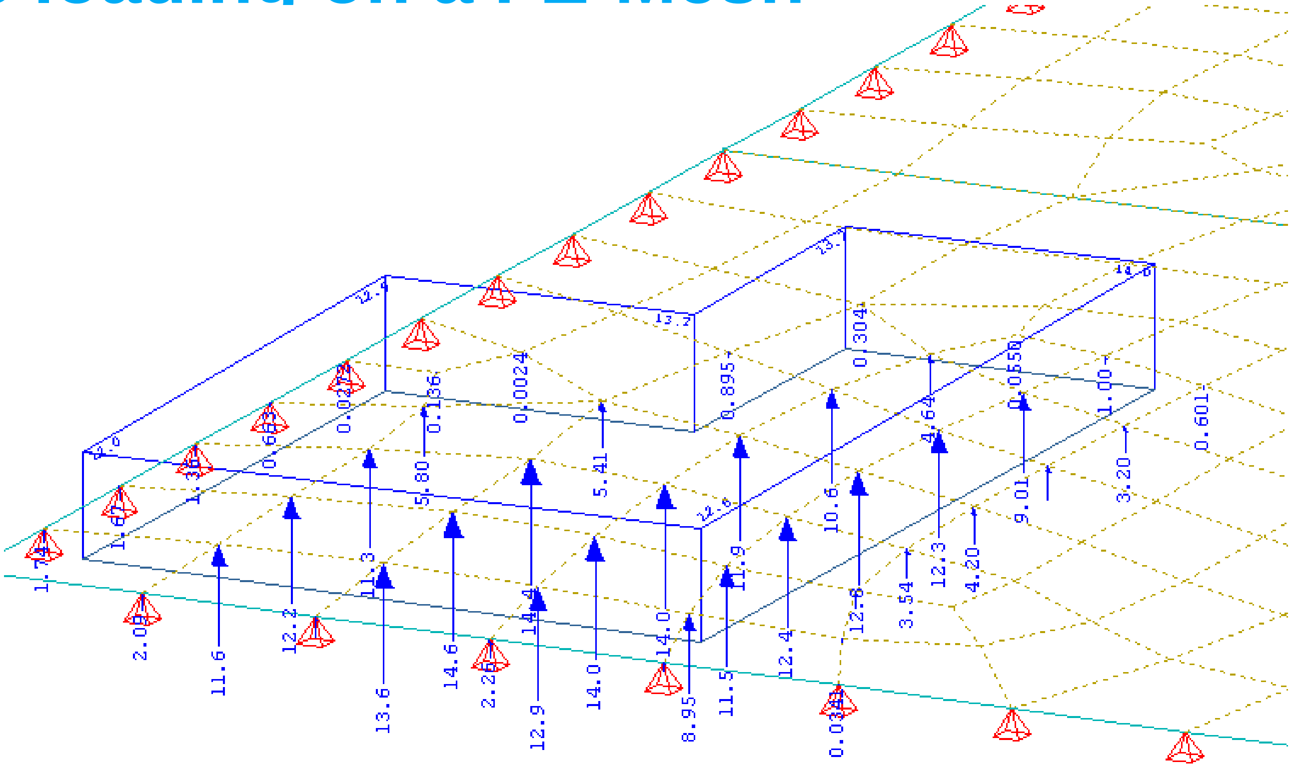


Nodal Loads

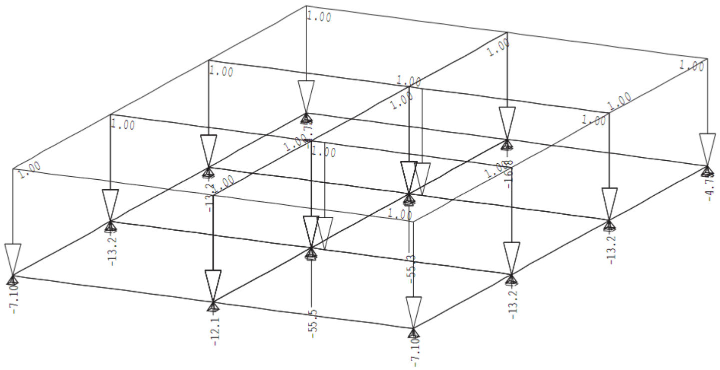
- Nodal loads are no point loads
- There are no nodal moments for the Mindlin-Plate

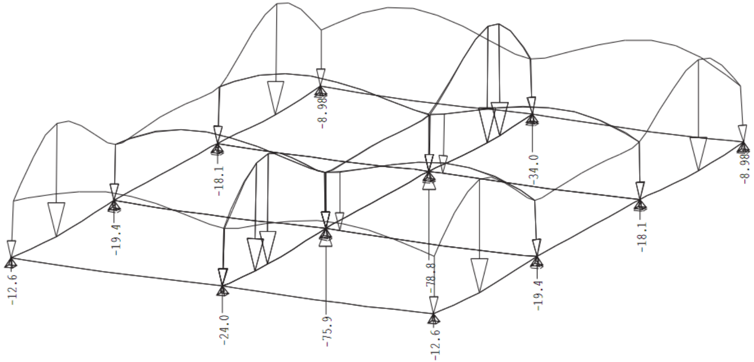


Free loading on a FE-Mesh



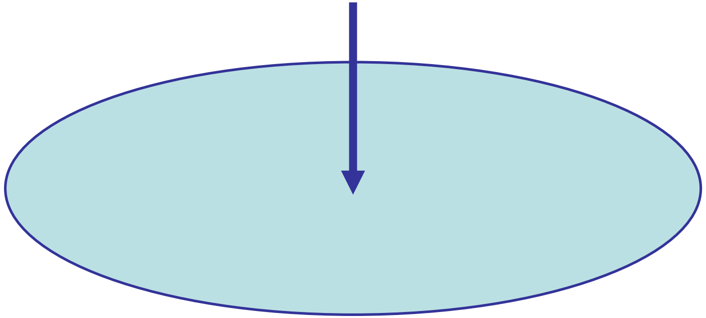
Non conservative loading (water ponds)





Building of ponds by deflection of roof

Verification Example



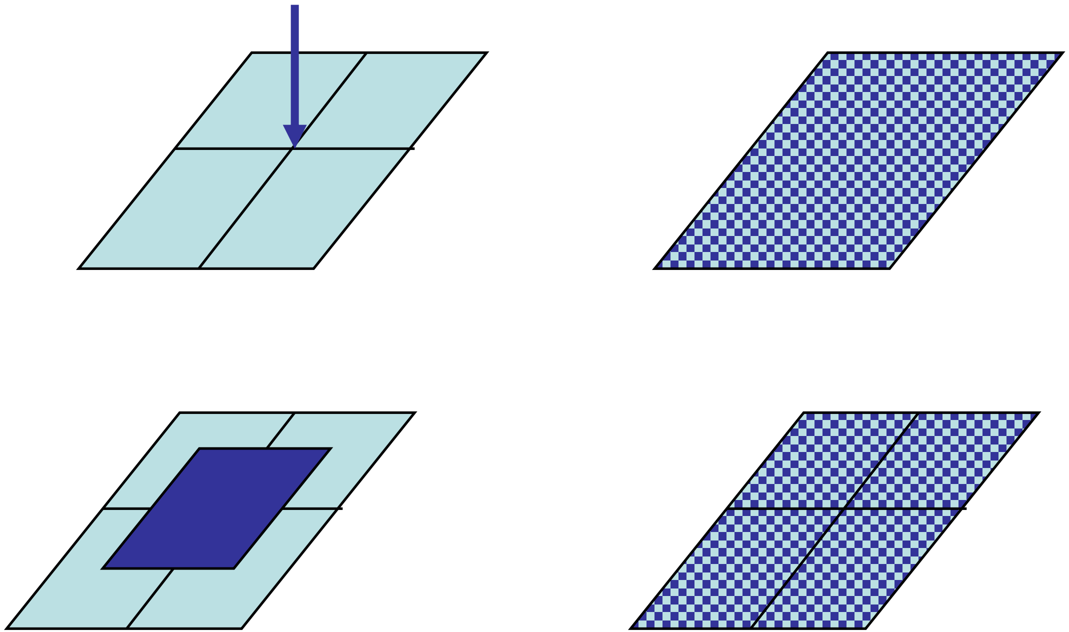
Circular plate with point load

$$w = r^2 \cdot \ln(r)$$

$$\textit{Moment} \qquad m = \ln(r)$$

$$\textit{Shear} \qquad v = \frac{1}{r}$$

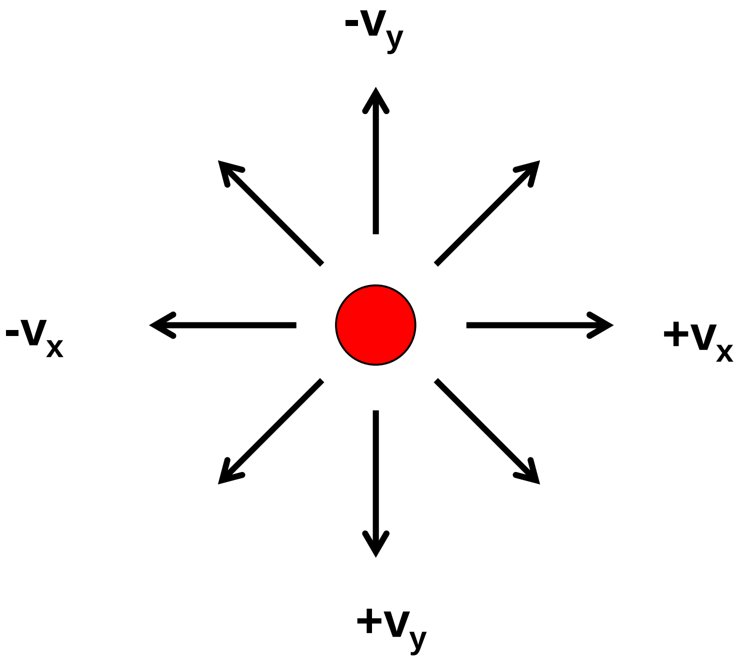
Load definitions



Results for the centre

- Point load
  - Deformations are finite for Kirchhoff but infinite for Mindlin/Reissner
  - Moments singular of logarithmic order
  - Shear is singular of order 1/r
- Area loading
  - Deformation always finite
  - Moments always finite !
  - Shear is 0.0 at center !

Sign of the shear in a plate $v_r > 0$

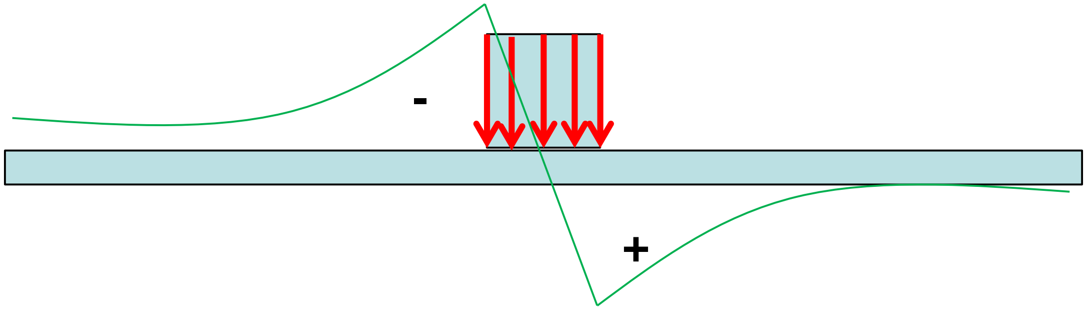


Sign of the shear

- Resultant shear stress is always positive

$$\sigma_v = \sqrt{\sigma_x^2 + 3\tau^2}$$

- To allow superposition of results we have to work on the components with the correct sign



Deflections at the centre

	Point Load		Area loading	
h/t	Theoret.	FE	Theoret.	FE
0.00	3.318			
0.44		3.298	3.256	3.281
0.88		3.307	3.248	3.275
1.76		3.307	3.222	3.226

h = mesh size

t = element thickness

Shear at centre

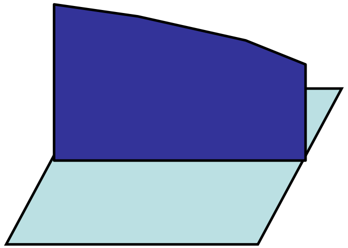
	Point Load		Area loading	
h/t	Theoret.	FE	Theoret.	FE
0.00	∞		0.0	
0.44	289.37	247.2	72.34	74.5
0.88	144.69	120.9	36.17	36.2
1.76	72.34	57.7	18.09	18.0

(Element has constant shear)

Moment at centre

	Point Load		Area loading	
h/t	Theoret.	FE	Theoret.	FE
0.00	∞			
0.44		56.7	44.40	43.3
0.88		49.7	37.78	36.7
1.76		43.4	31.15	30.6

Moment for design



Integral of theoretical forces along the element / length

compared to values in centre of element

	Point Load		Area Loading	
h/t	Theoret.	FE	Theoret.	FE
0.00	∞			
0.44	43.17	43.1	42.08	39.2
0.88	36.55	36.6	35.33	32.9
1.76	29.93	30.5	28.61	26.6

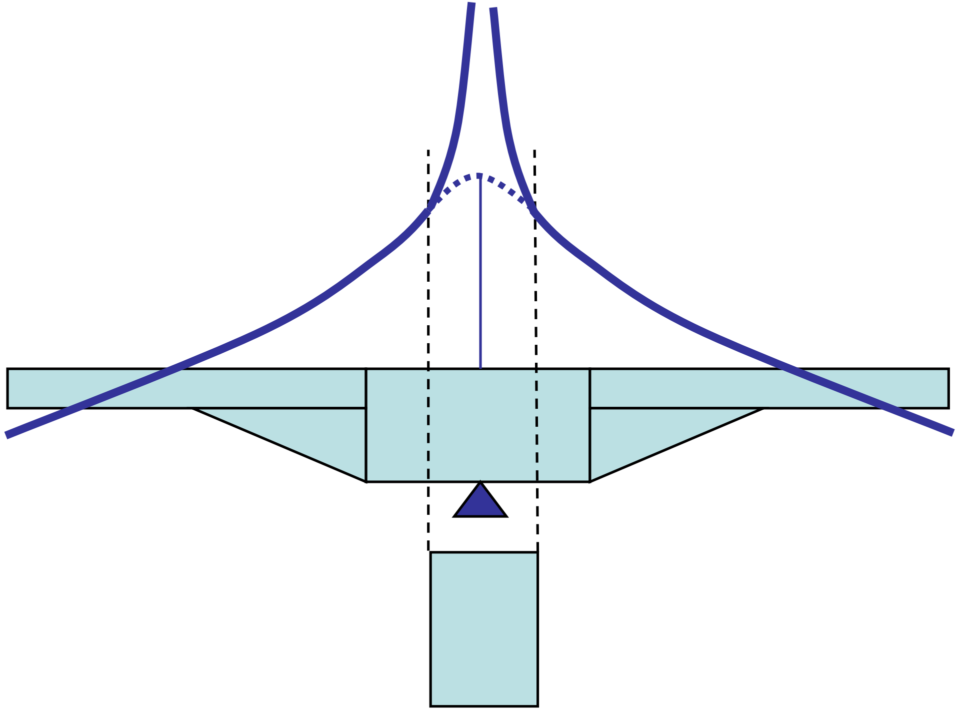
Recommendations

- A reasonable mesh size is not smaller than
  the thickness of the plate,
- but we need at least 3 to 5 elements for every span.
- Point loads on meshes finer than that limit have to be avoided.
- Distributed loadings will not cope with the full value of the moments if only one single element is loaded.
- So there is a best fit of the loadings for any given mesh size !
- Design should be based on integral values (centre)

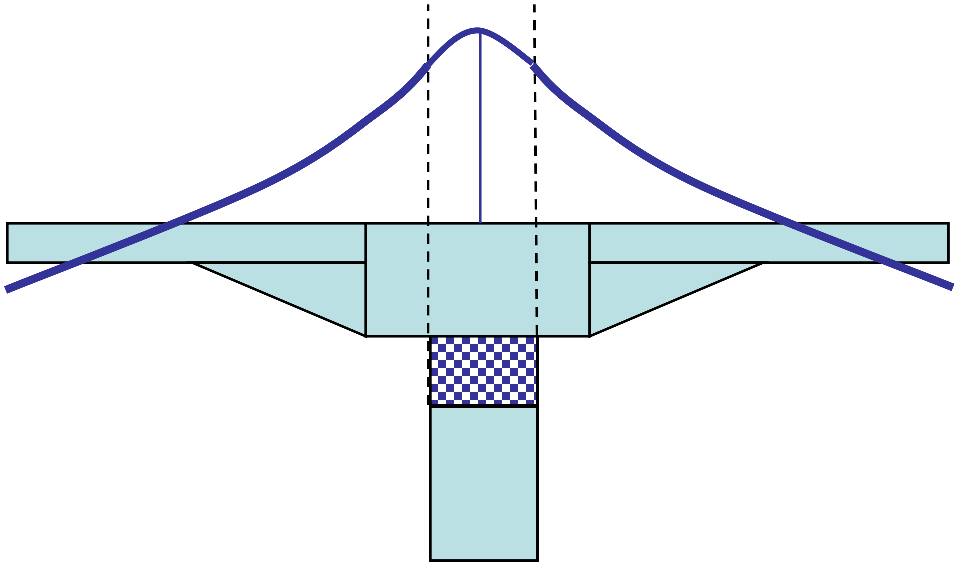
Supports

- Similar to the load problem
- Point Support, Build in effects
- Elastic Bedding (Winkler Assumption)
  - Problematic, if other supports are rigid
  - Unwanted build in effects are possible
- Kinematic Constrained Support
  - Simple, not so easy for automatic mesh generation
  - EST (equivalent stresses) as a general method

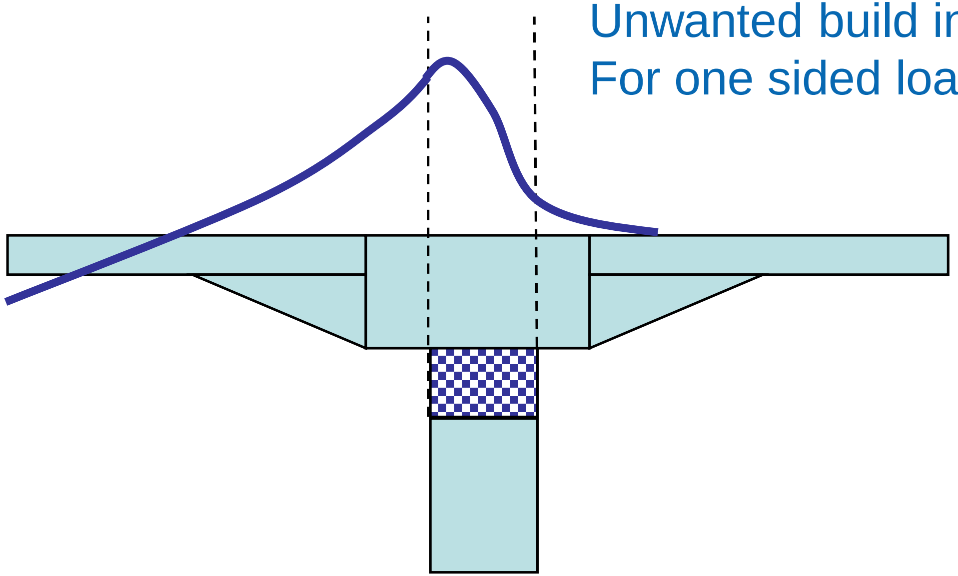
Point Support



Elastic support (Winkler)

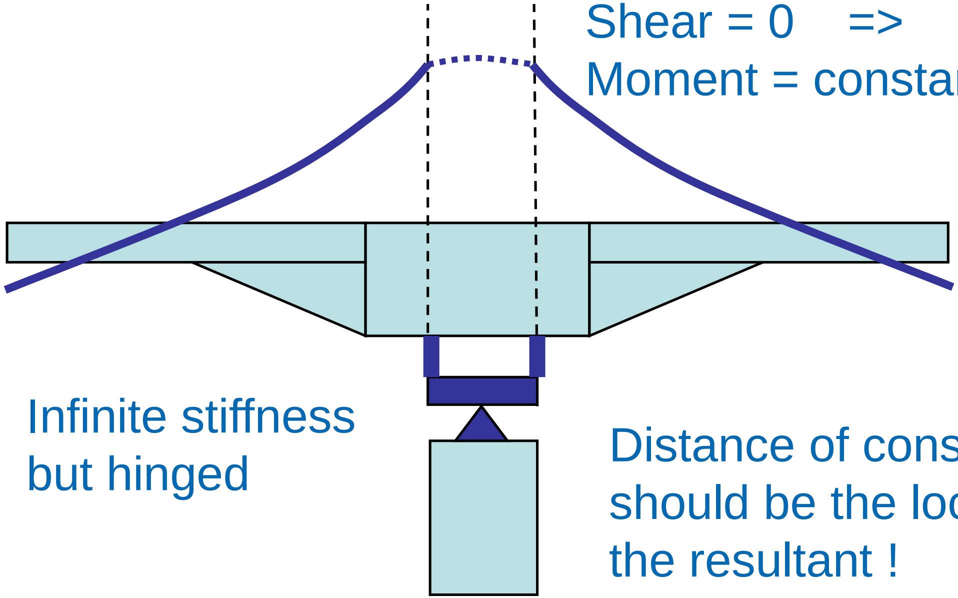


Elastic support (Winkler)



Unwanted build in effect
For one sided loading

Kinematic Constraint

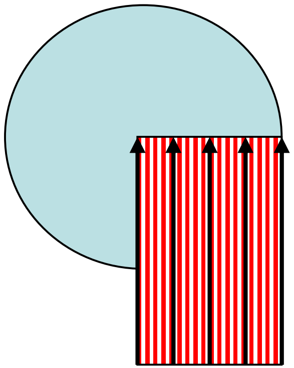


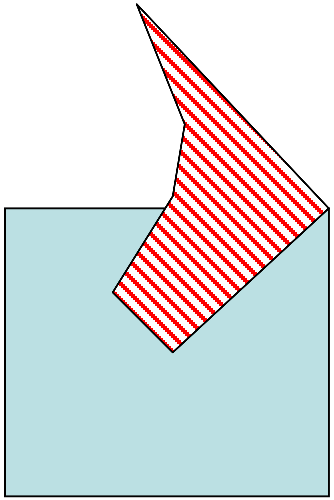
Shear = 0 =>
Moment = constant

Infinite stiffness
but hinged

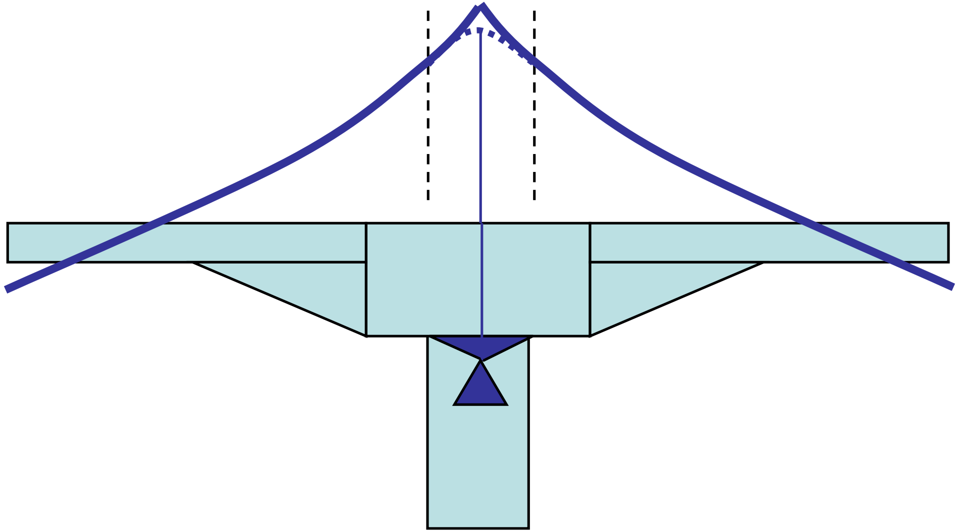
Distance of constraint
should be the location of
the resultant !

Variations of Support





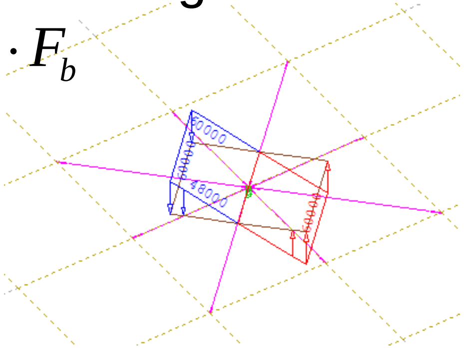
Slab-Designer Support



- Select mesh size based on dimension of column
  Use 4 elements to model the column region
- Point Support with optional elastic rotational support (springs)
- Enhance the central thickness for the design (haunch 1:3)

EST – Equivalent Stress Transformation (Werkle, 2002, 2004)

- Original name was equivalent stiffness transformation
- If the support is done by a beam section, the stresses in the beam caused by normal force and moments are always linear
- If we integrate this stress with the shape functions we get nodal forces for the finite element mesh: $F_{pl} = T^T \cdot F_b$

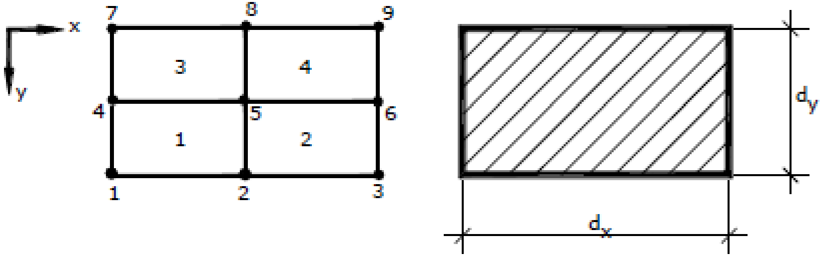


- We may us this distribution equation as a kinematic constraint

$$u_b = u_{pl} \cdot T$$

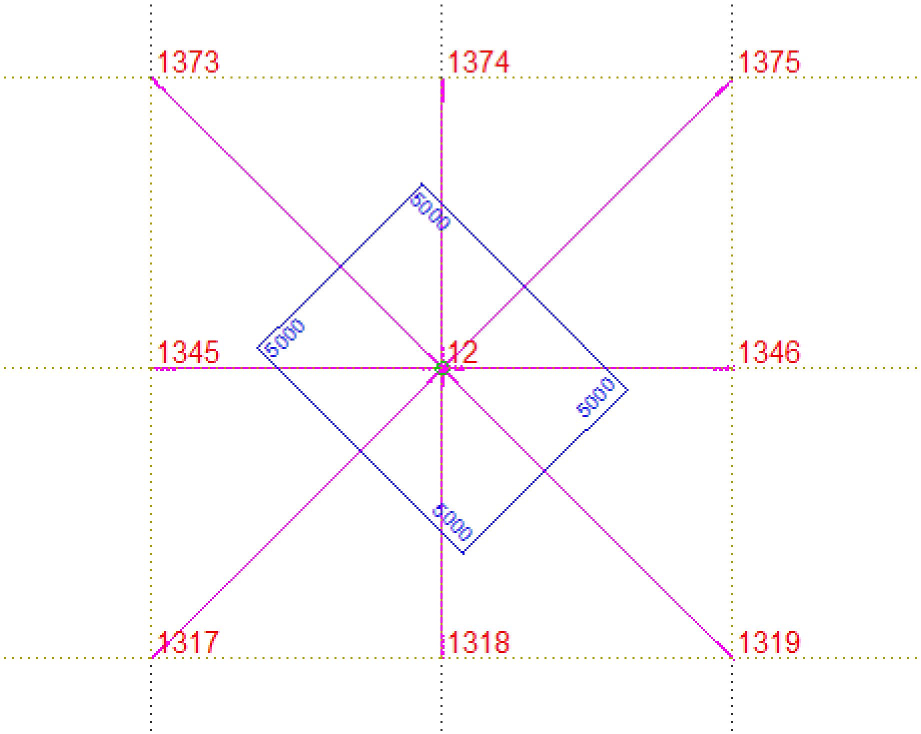
- Works for any mesh topology and any shape of the section!

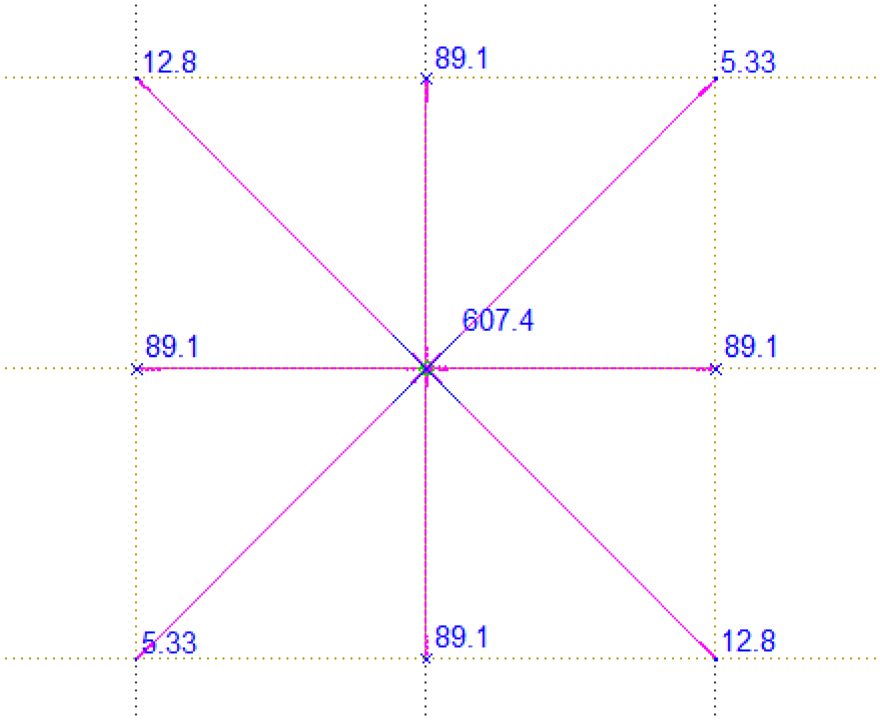
Example from Werkle



$$\underline{T} = \begin{bmatrix} \frac{1}{16} & \frac{1}{8} & \frac{1}{16} & \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{16} & \frac{1}{8} & \frac{1}{16} \\ \frac{1}{4 \cdot d_x} & 0 & \frac{-1}{4 \cdot d_x} & \frac{1}{2 \cdot d_x} & 0 & \frac{-1}{2 \cdot d_x} & \frac{1}{4 \cdot d_x} & 0 & \frac{-1}{4 \cdot d_x} \\ \frac{1}{4 \cdot d_y} & \frac{1}{2 \cdot d_y} & \frac{1}{4 \cdot d_y} & 0 & 0 & 0 & \frac{-1}{4 \cdot d_y} & \frac{-1}{2 \cdot d_y} & \frac{-1}{4 \cdot d_y} \end{bmatrix} \quad \text{with } \underline{w}_{St} = \begin{bmatrix} w_z \\ \phi_{yy} \\ \phi_{xx} \end{bmatrix}.$$

A more general example





Resolving equations

- The T-Matrix

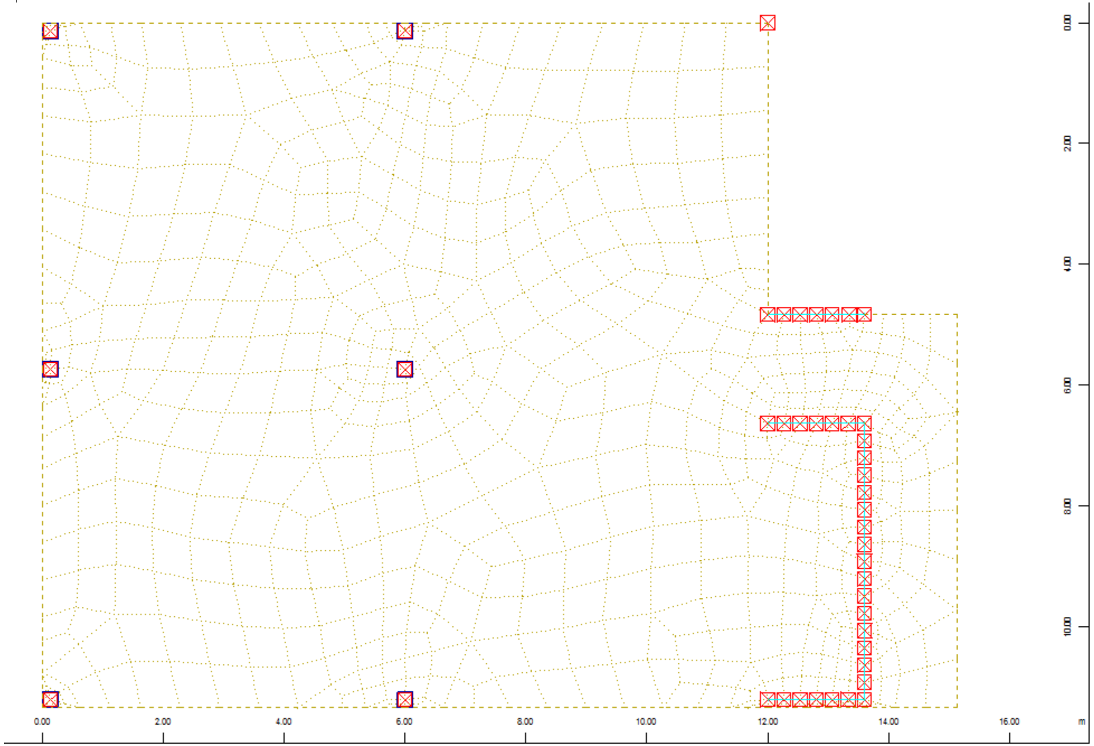
$$u_{12b} = 0.607 \cdot u_{12} + 0.089 \cdot (u_{18} + u_{45} + u_{46} + u_{74})$$
$$+ 0.013 \cdot (u_{19} + u_{73}) + 0.0053 \cdot (u_{17} + u_{75})$$

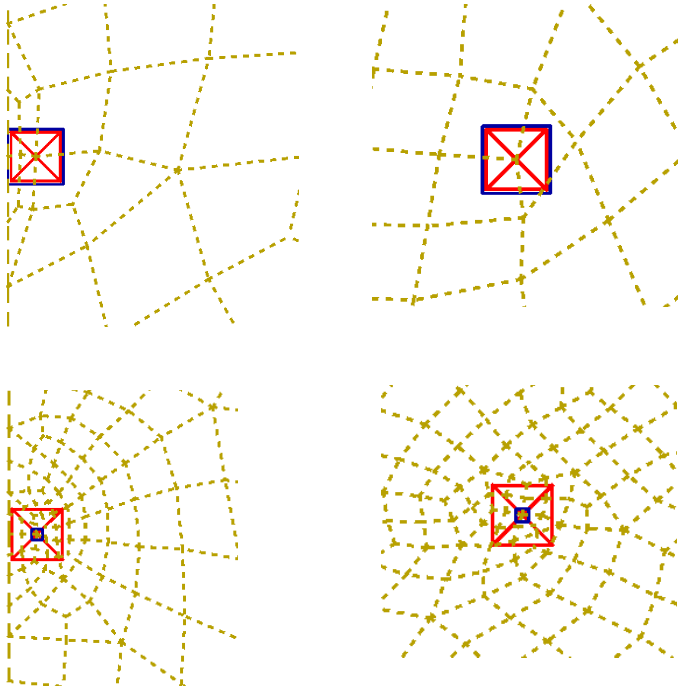
- If nodes 12b and 12 are identical:

$$0.393 \cdot u_{12b} = 0.089 \cdot (u_{18} + u_{45} + u_{46} + u_{74})$$
$$+ 0.013 \cdot (u_{19} + u_{73}) + 0.0053 \cdot (u_{17} + u_{75})$$

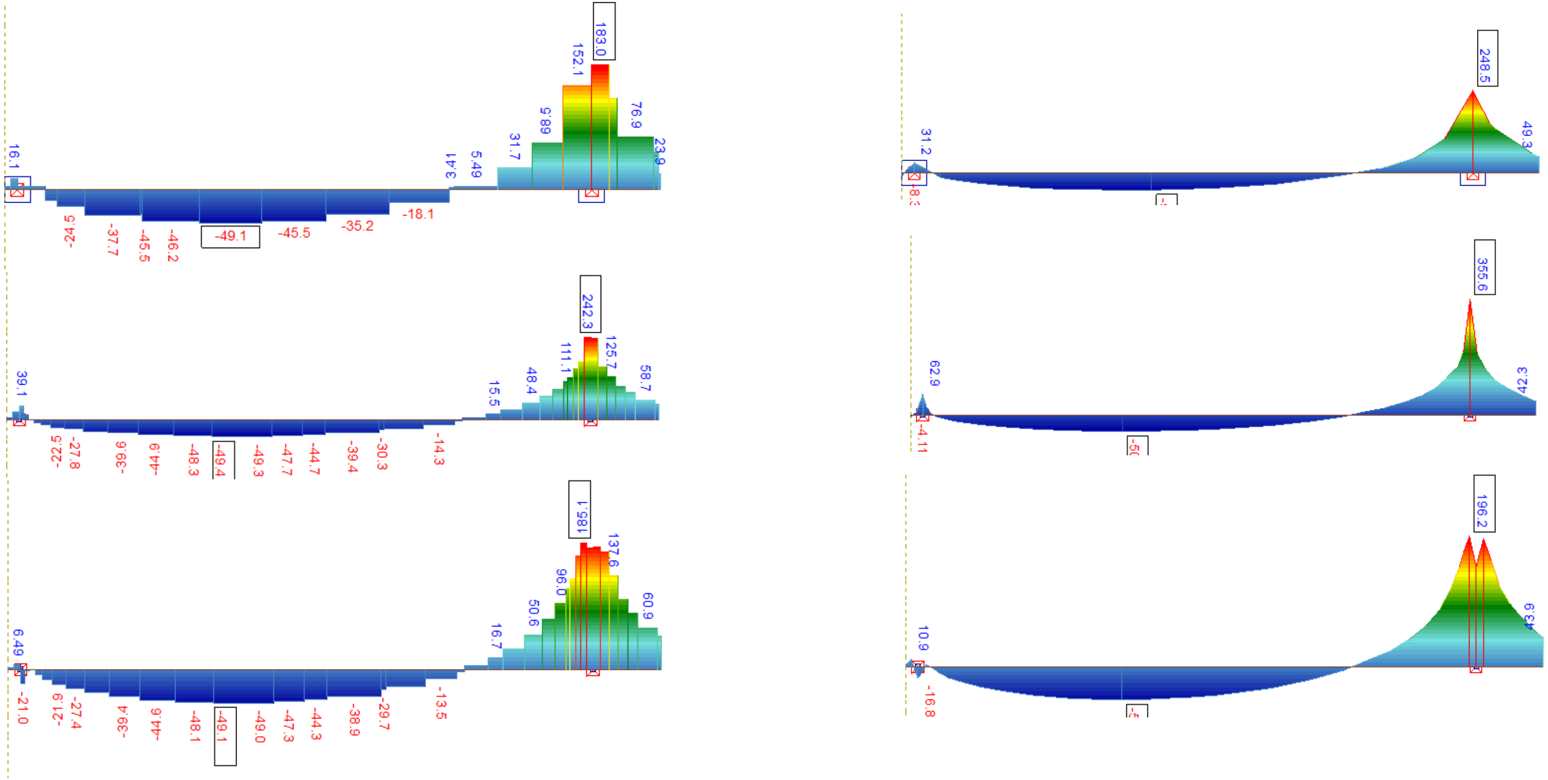
$$u_{12} = 0.227 \cdot (u_{18} + u_{45} + u_{46} + u_{74}) + 0.0326 \cdot (u_{19} + u_{73}) + 0.0136 \cdot (u_{17} + u_{75})$$

Slab Example with different Meshing





Slab Example: Moment m-xx



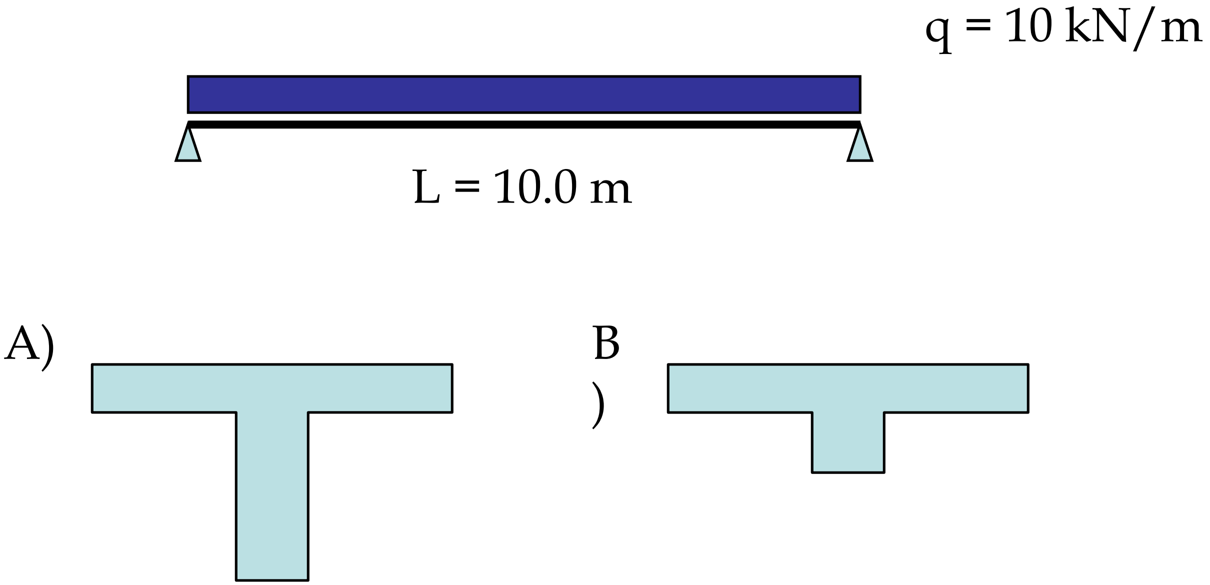
Remarks to the EST

- The EST technique is a general tool to solve nearly all connecting problems.
- It may be used to combine shear walls with beam elements
- It could be used to describe a shear distribution as well
- The shape of the column has always an effect, but if the size of the column is smaller than the mesh size, the missing resolution will generate rather similar results.

General Recommendations

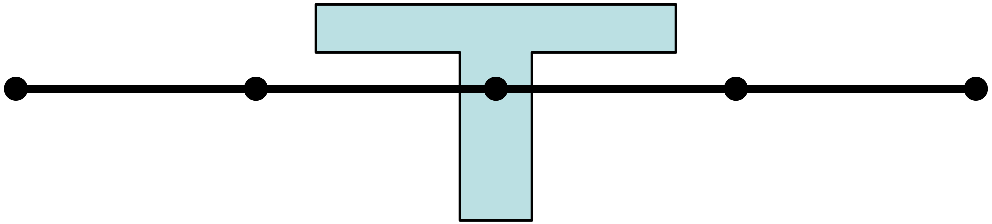
- Use EST technique whenever possible.
- Columns with a width less than the plate thickness may be modelled as point loads, as long as the element mesh size is selected sufficiently large.
- Elastic supports will smooth singularities introduced by rigid supports (especially useful for walls)
- Extreme care is required if elastic and rigid supports are used within the same system!

T-Beams



Possible Models

- Shell elements (SH)
- Shell elements and eccentric beam (SEB)
- Plate and eccentric Beam (PEB)
- Plate and assigned T-Beam (PB)



Assigned T-Beam

- Bending Stiffness of beam adjusted on total system

  $I_{yy}(beam) = I_{yy}\ (P+B) - A_{yy}\ (plate)$

- Transformation of forces during post processing (ΔN is calculated based on the stiffness difference)

  F (P+B) := F (beam) + F(plate)

  F (plate) := F (plate) - ΔN(P+B)

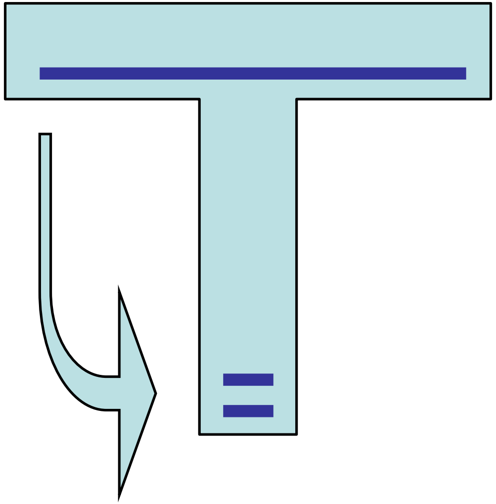
High Web

	Ref.	SH	SEB	PEB	PB
Deflection	0.841	0.899	0.860	0.588	0.843
m – Plate	3.23	3.06	2.98	1.94	2.88
n – Plate	-181.6	-170.3	-179.3	(-201)	(-162)
M – Beam	30.99	(44.50)	32.00	22.10	122.11
N – Beam	+181.6	+170.3	+179.3	201.5	(162)
As – Beam	4.69	6.56	6.28	7.05	4.58
As – Plate	0	0	0	0.43	0.59
As – Links	0.65	2.04	0.84	0.85	0.63

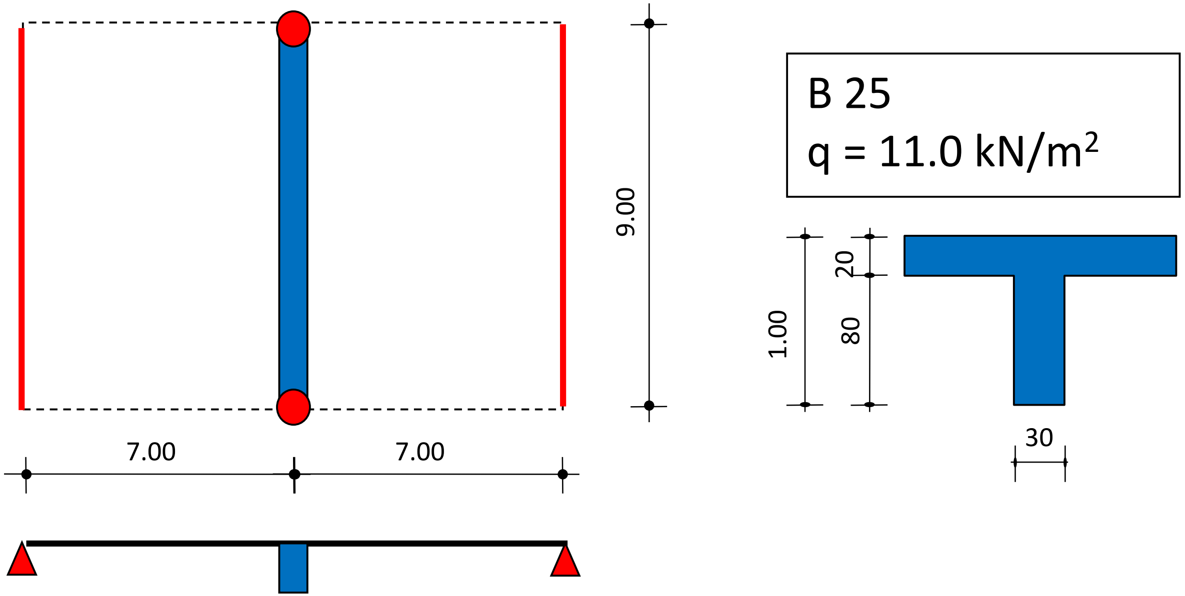
Low Web

	Ref.	SH	SEB	PEB	PB
Deflection	11.989	11.426	12.145	11.122	12.103
m – Plate	46.04	43.70	46.73	42.86	46.38
n – Plate	-360.3	-353.8	-356.9	(379.4)	(363)
M – Beam	6.91	(21.98)	7.14	6.57	79.69
N – Beam	+360.3	+353.8	+356.9	+379.4	(363)
As – Beam	13.17	12.44	12.50	13.28	8.30
As – Plate	0	3.53	4.14	9.46	9.58
As – Links	1.90	4.16	8.38	8.78	1.17

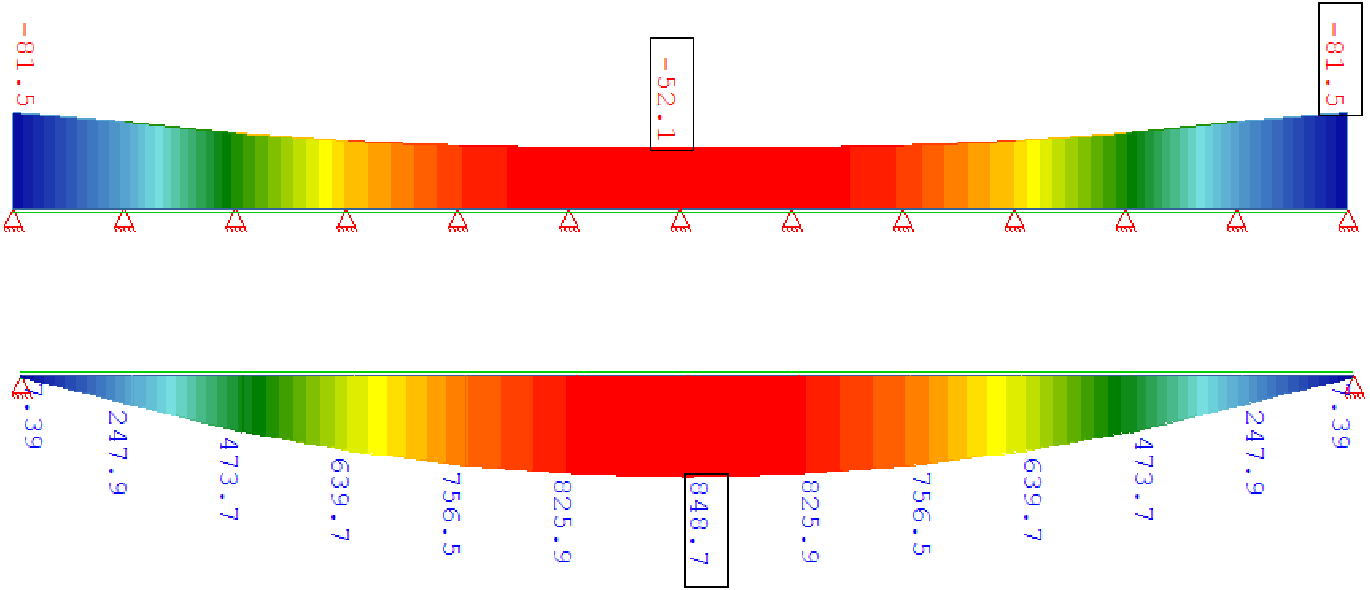
Rearrangement of the plate reinforcements



A small benchmark



Hogging transverse moments of plate / Moments of beam:



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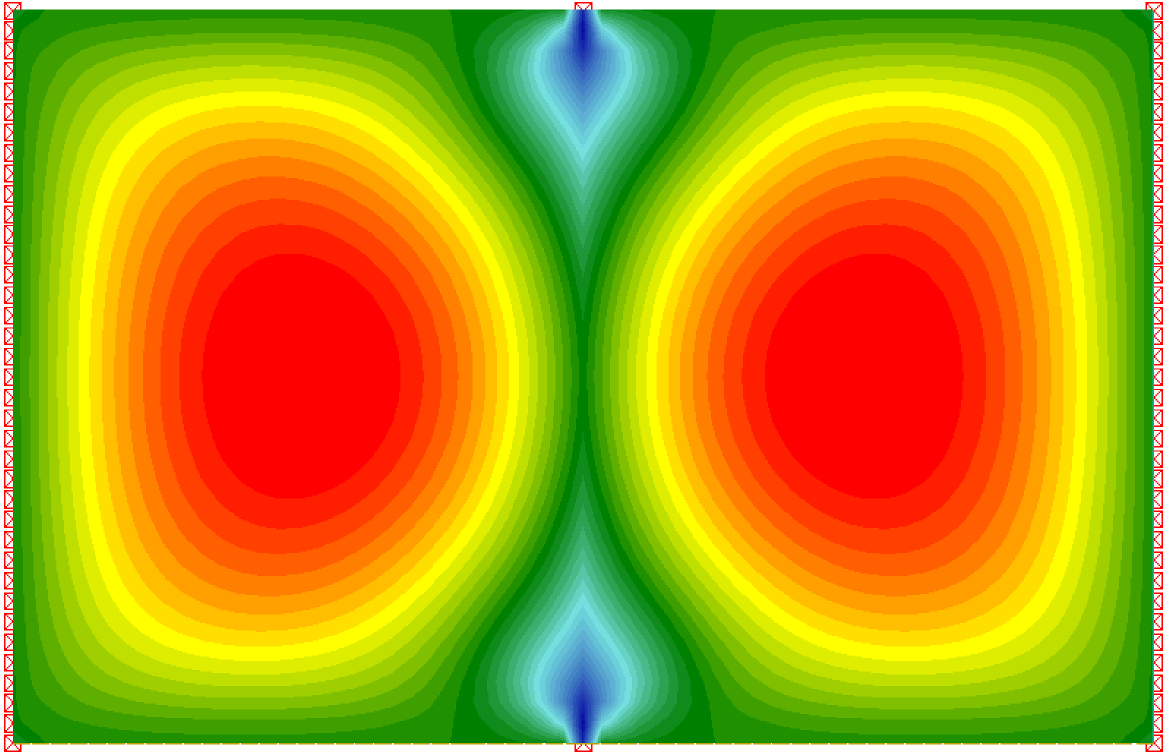
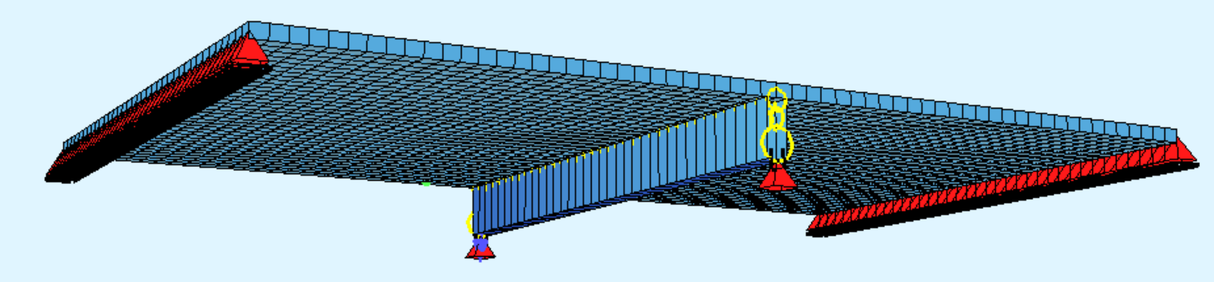
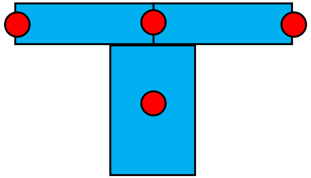
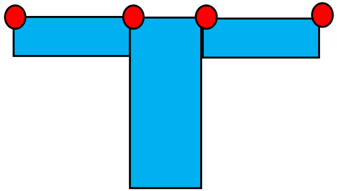
Moments m-xx of the plate



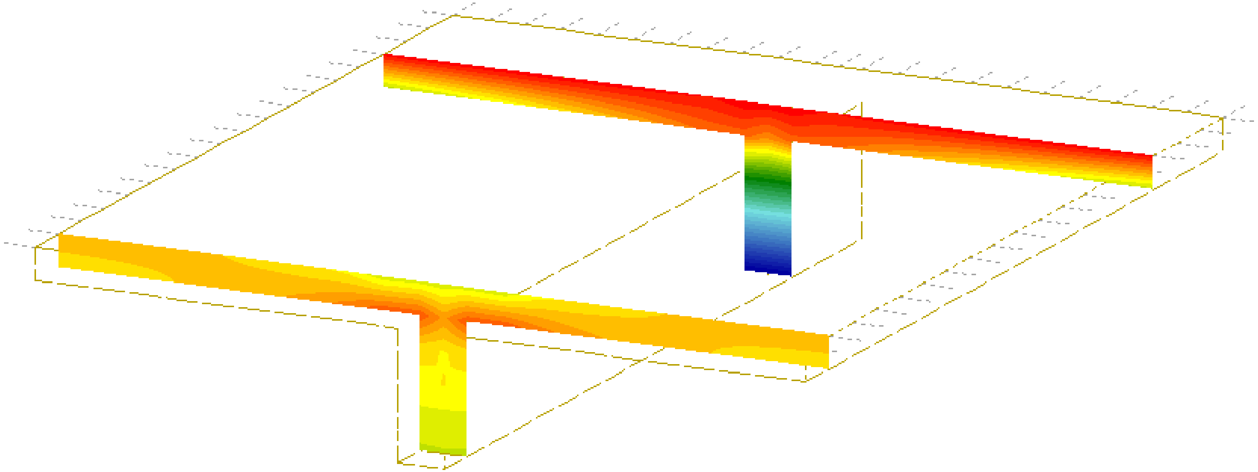
Modelling in 3D with shells and beams

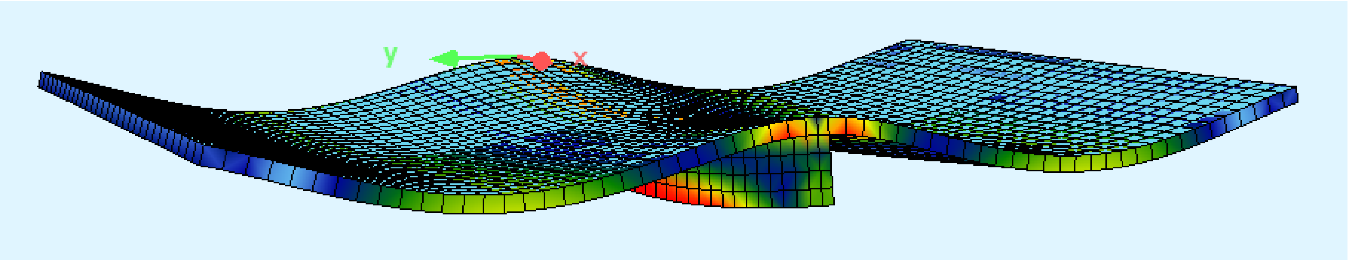




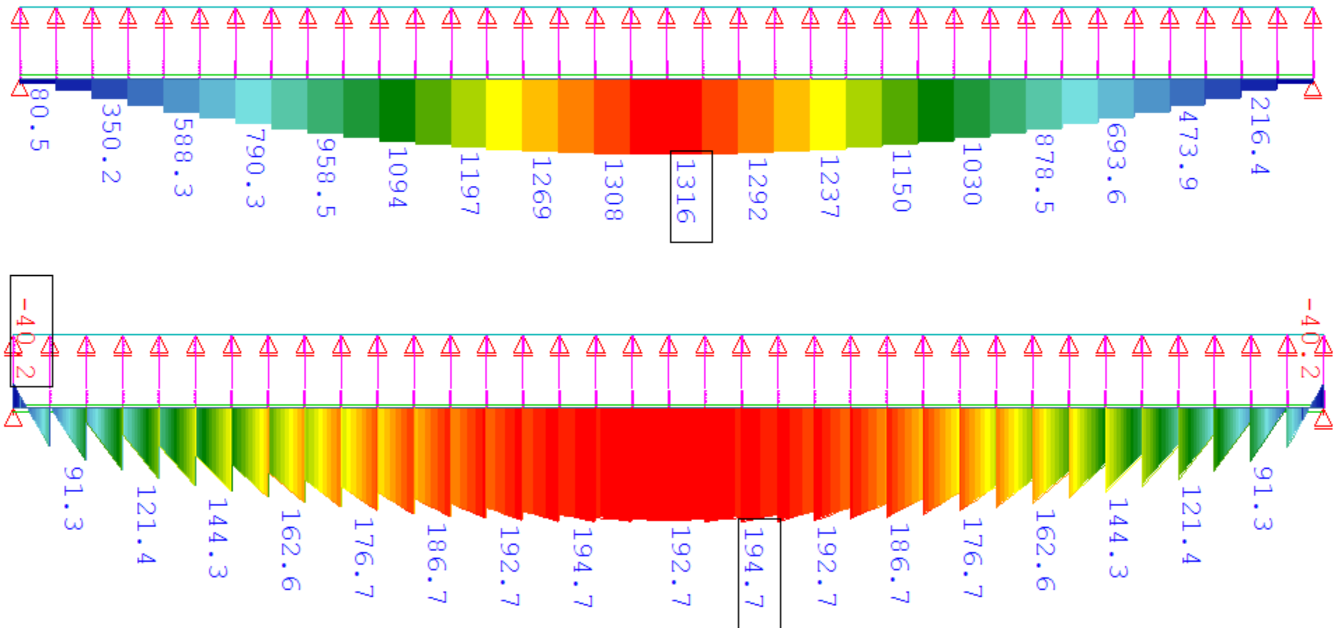


Modelling as 3D Continua

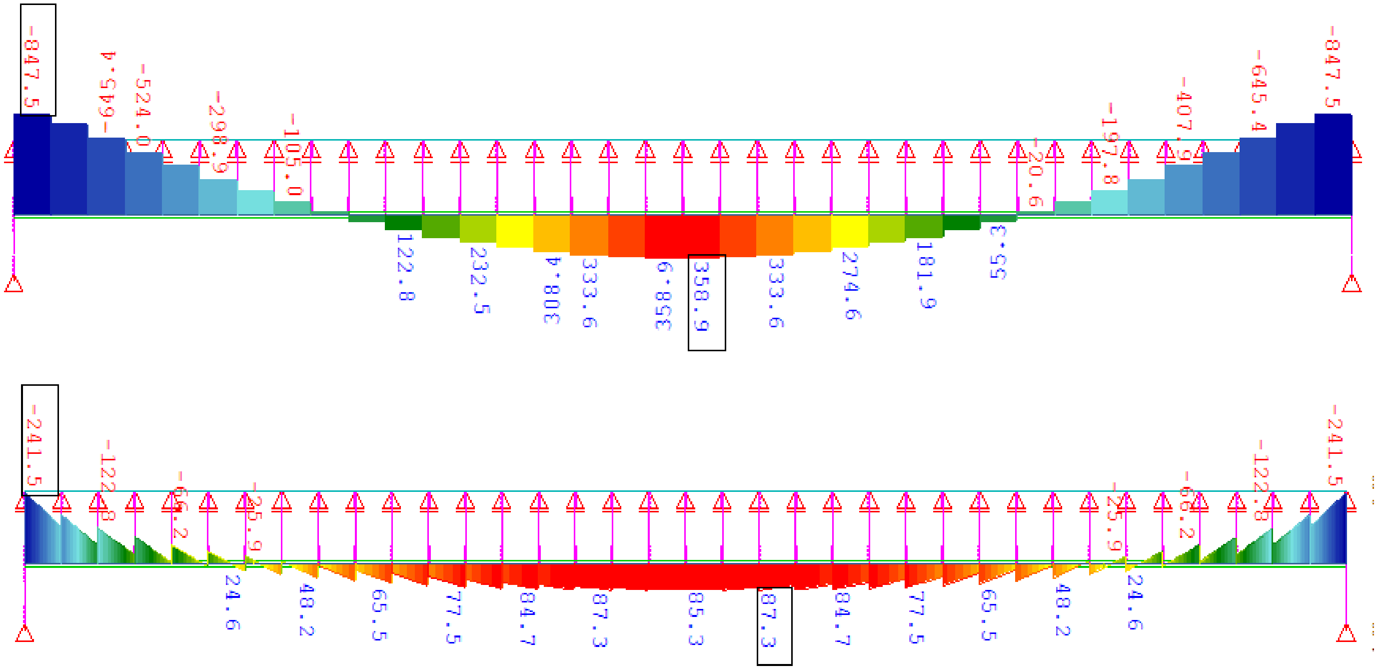




Influence of horizontal support
a) N / M for free supports



Influence of horizontal support
b) N / M for fixed supports



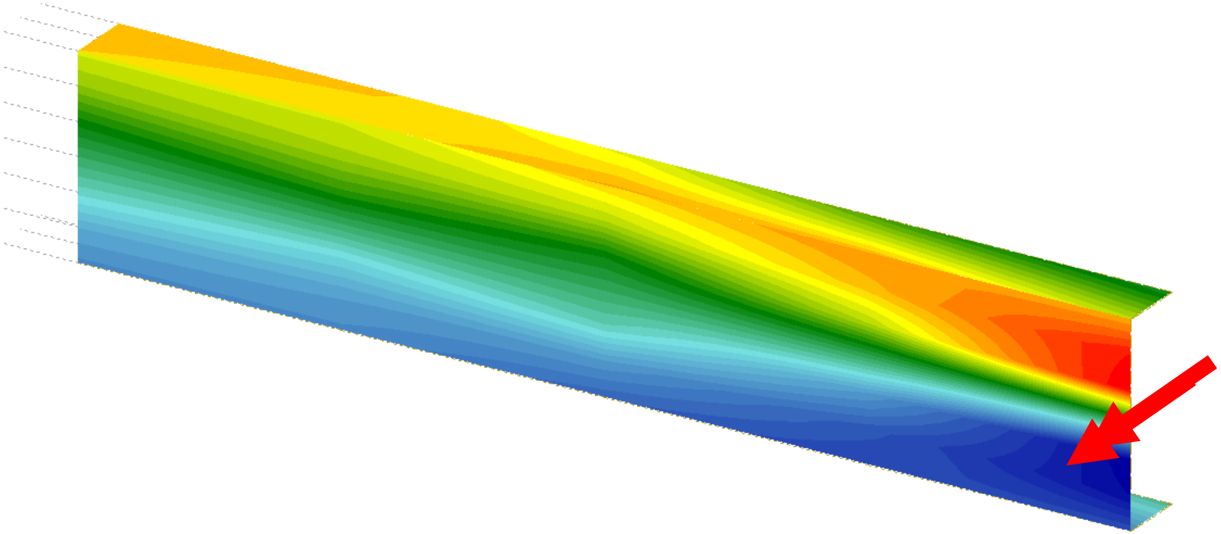
Recommendations

- It is possible to deal with the T-Beam-Problem with a simple plate bending program
- If the height of the beam is small compared to the plate, results may differ to what you expect for classical analysis methods.
- Special considerations are required for the design process

Shell elements

- Combination planar plate and membrane elements
  - 6^{th} „Drilling Degree of Freedom“
  - Twist of elements
- Degenerated 3D-Continua elements
- Special curved shell elements
- Rotational symmetric elements
- textile membranes, Form finding

Example cantilever with single moment



- Vertical displacements are precise within 2.4 o/oo
- Local rotation is higher by a factor of 3.7

Channel Shape Cantilever with self weight

Modelling	u-z [mm]	u-yy[mrad]	u-xx[mrad]
Classical beam theory	74.483	-11.814	-62.025
Beam theory & warping torsion	74.071	-11.814	-54.296
FE-Model conform	59.711	-9.629*	-43.935
FE-Model with assumed strains	74.119	-11.835*	-63.151
FE-Model with drilling degrees	74.825	-11.877	-63.796

The FE-System is too soft!

Drilling Stiffness

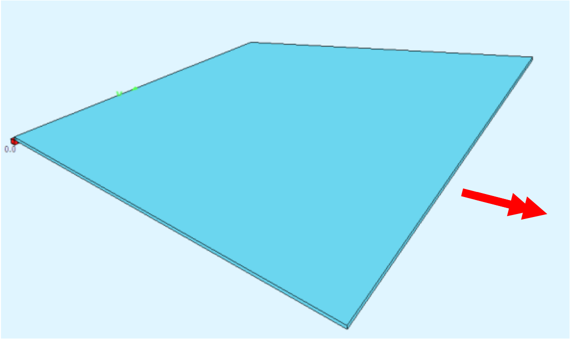
- Factor 2, or do not forget the edge terms!

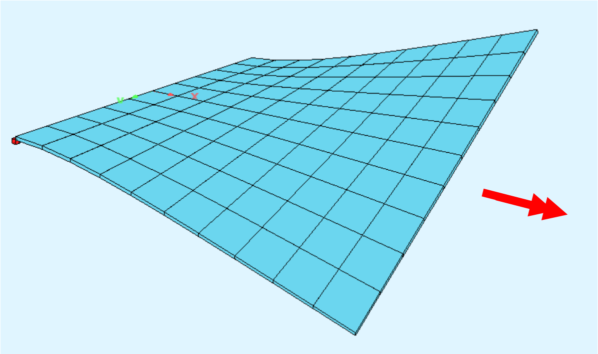
$$beam \quad M_t = G \cdot I_t \cdot \theta' = \frac{G \cdot b \cdot t^3}{3} \cdot \theta' \;\; ; \;\; \tau_{\max} = \frac{M_t}{I_t} \cdot t = \frac{3M_t}{b \cdot t^2}$$

$$plate \quad m_t = K \cdot (1-\mu) \cdot \frac{\partial^2 w}{\partial x \partial y} = \frac{E \cdot t^3}{12(1+\mu)} \cdot \frac{\partial^2 w}{\partial x \partial y} = \frac{G \cdot t^3}{12} \cdot \left[\frac{\partial \phi_x}{\partial y} - \frac{\partial \phi_y}{\partial x} \right] = \frac{G \cdot t^3}{6} \cdot \theta'$$

$$M_t = \int m_t ds + \frac{b}{2} \cdot [m_t(0) + m_t(b)] = 2b \cdot m_t$$

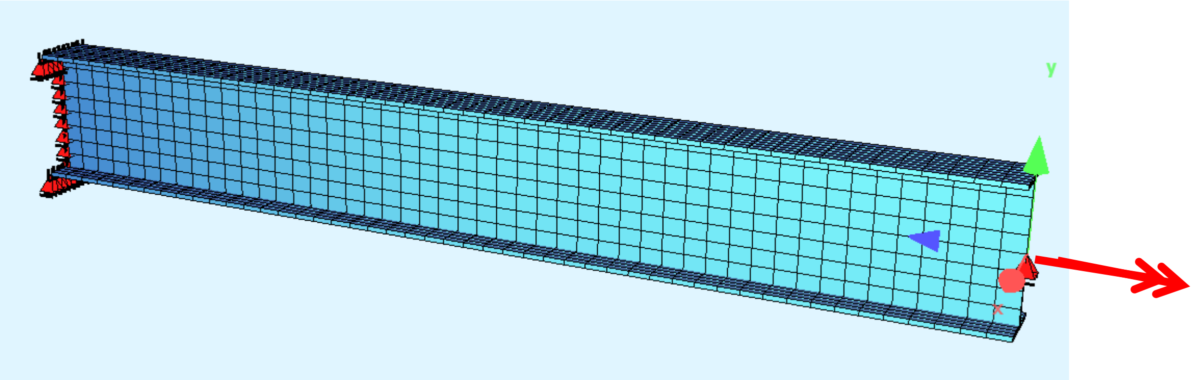
Not everything looking like torsion is torsion





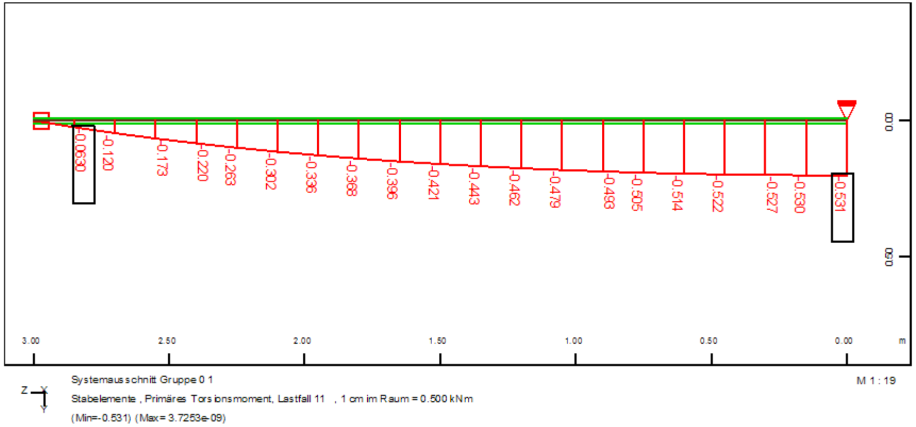
- Analytic solution: $w = a \cdot x \cdot y \Rightarrow m_t = a \cdot K \cdot (1-\mu)$

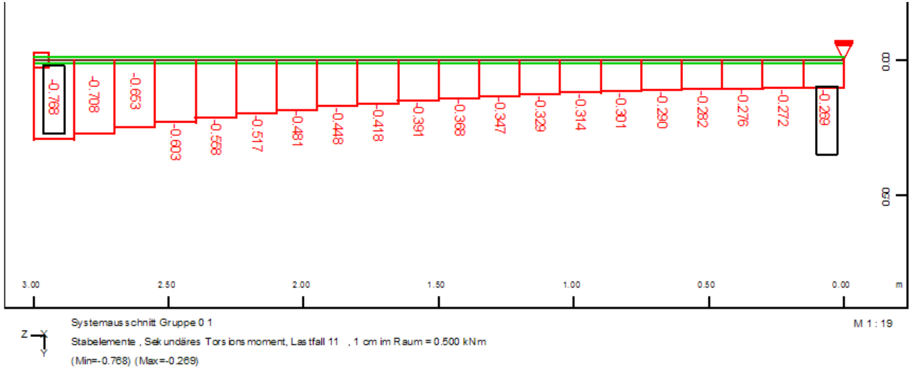
Pure Torsion for a cantilever



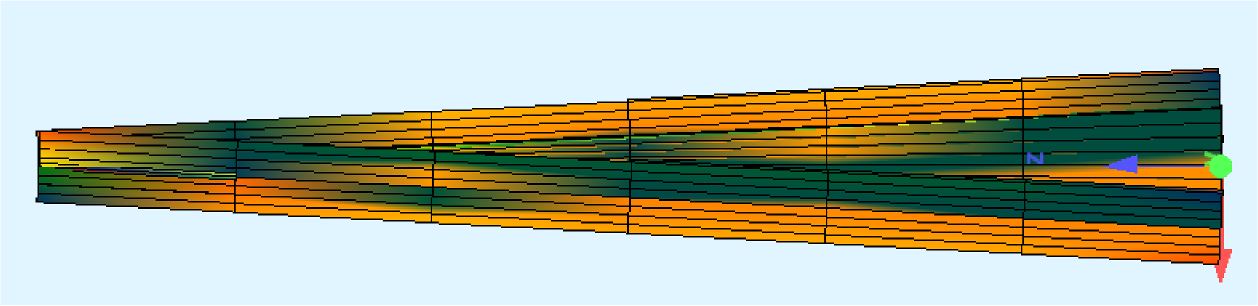
- Rotation beam system: 72.4 mrad
- Rotation FE-System: 37.2 mrad
- Beam system with warping torsion 33.6 mrad
- FE system with free warping support 75.7 mrad

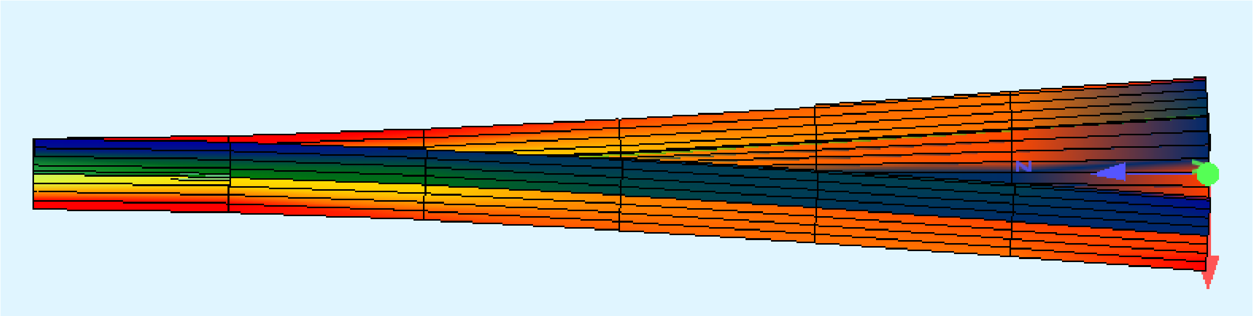
Primary & Secondary Torsional Moment





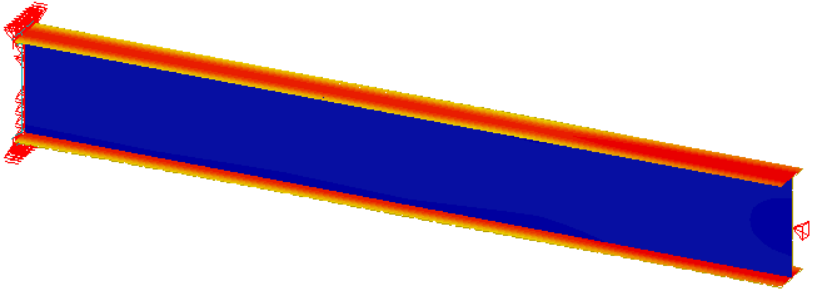
Build-In Support conditions for FE

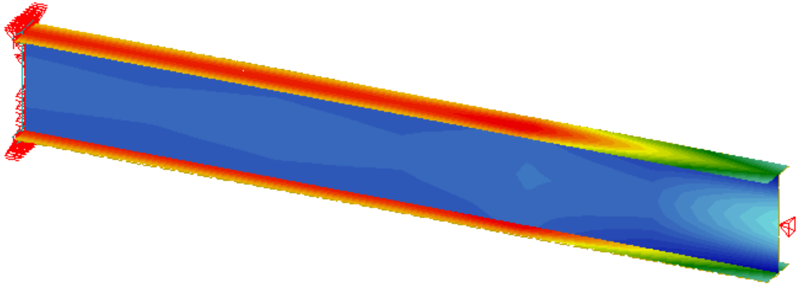




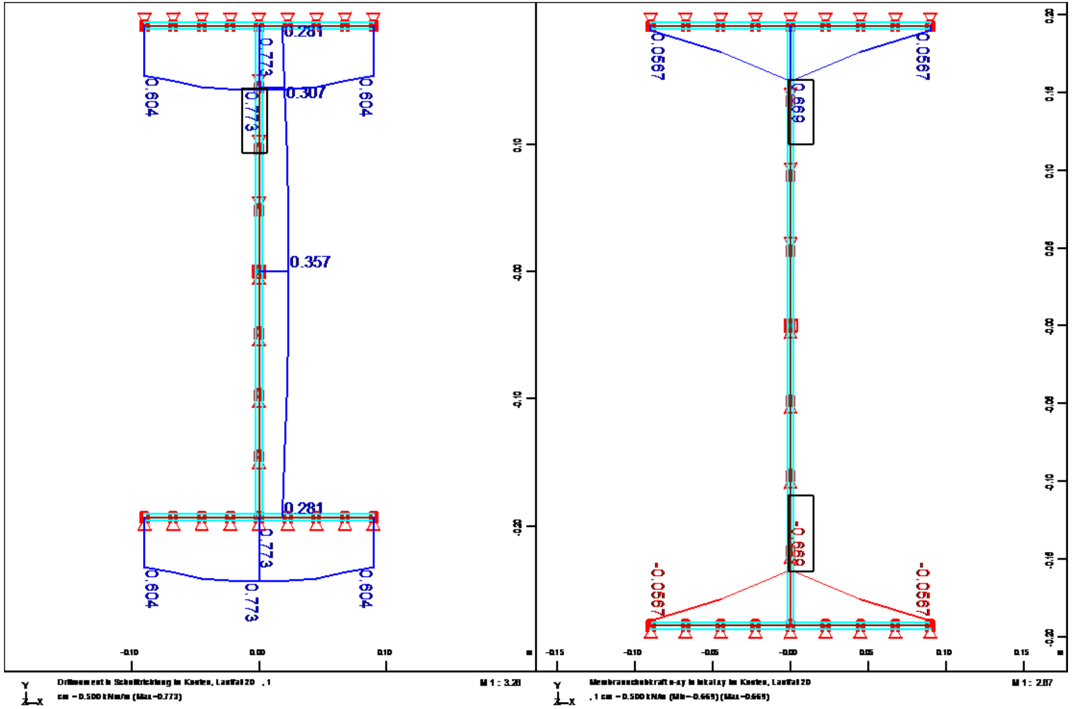
Loaddefinition

- Distributed Drilling moments (Saint-Venant)
- Opposite directed warping shear in the flanges

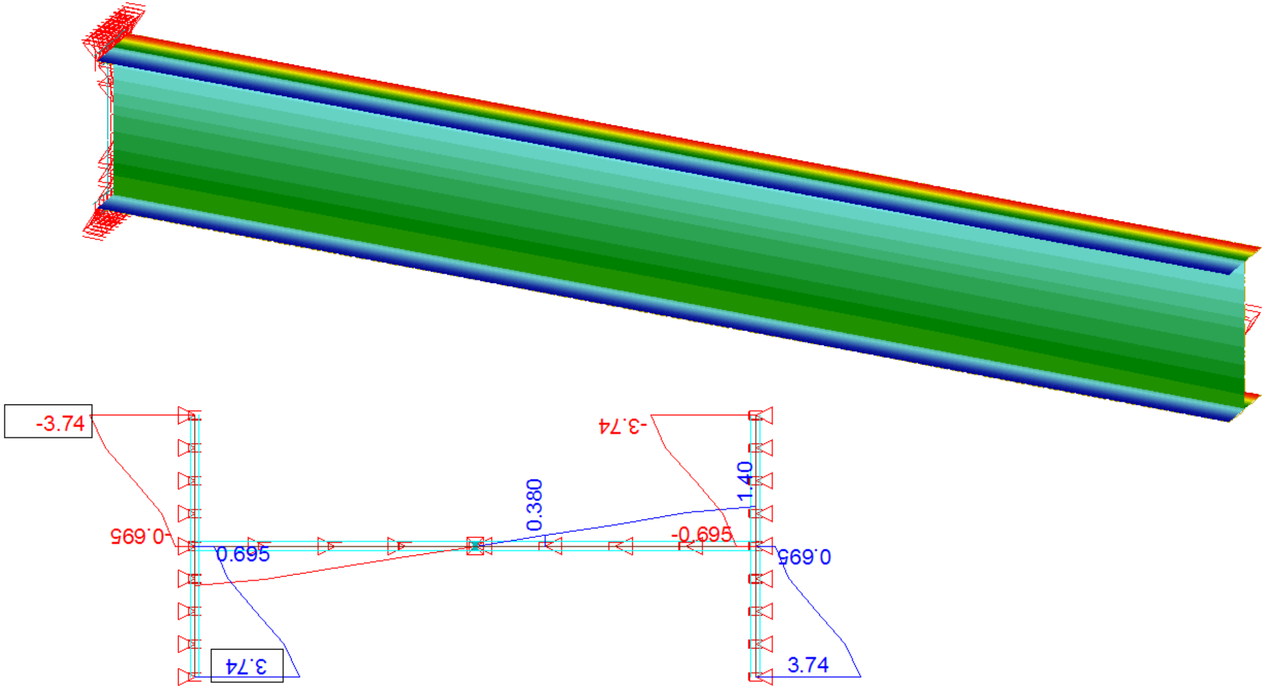




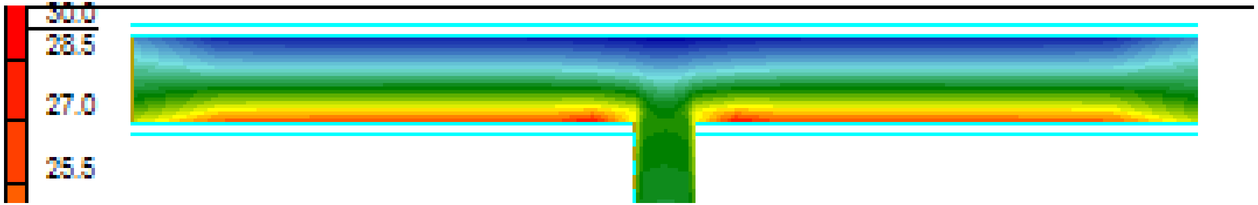
Stresses within section (m_t & τ_s)

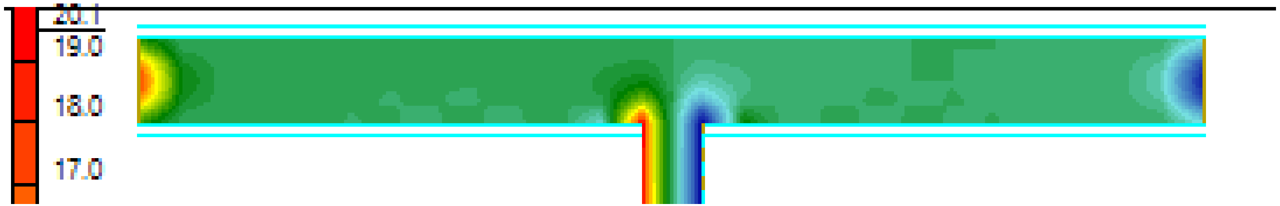


Longitudinal stress and plate shear

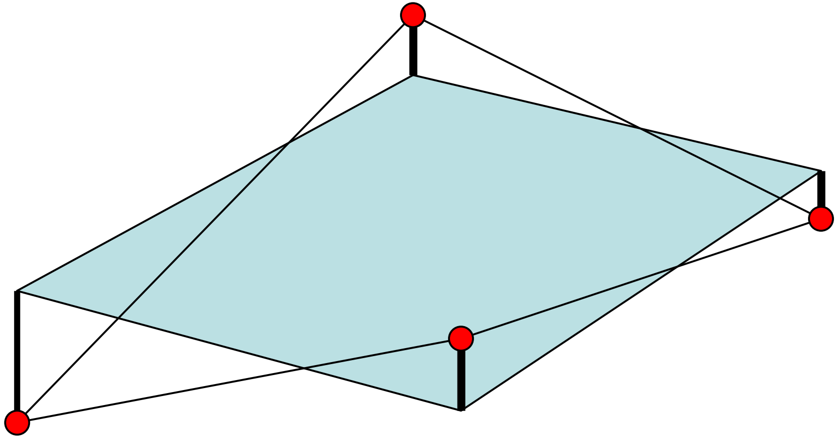


Shear in 3D Volume model (τ_{xy} / τ_{xz})

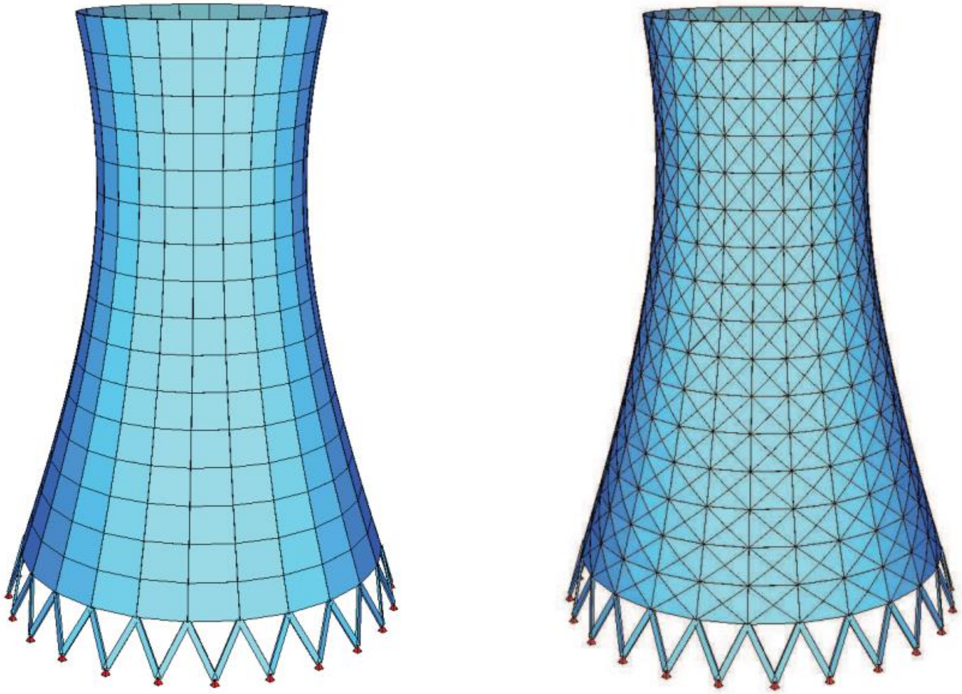




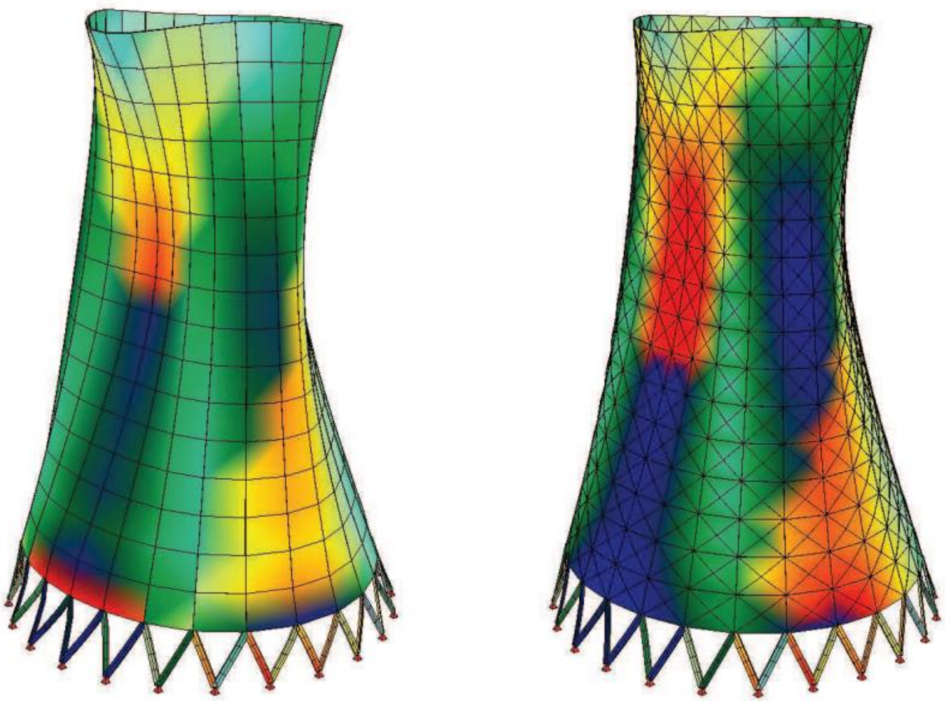
Twist = out of plane effects



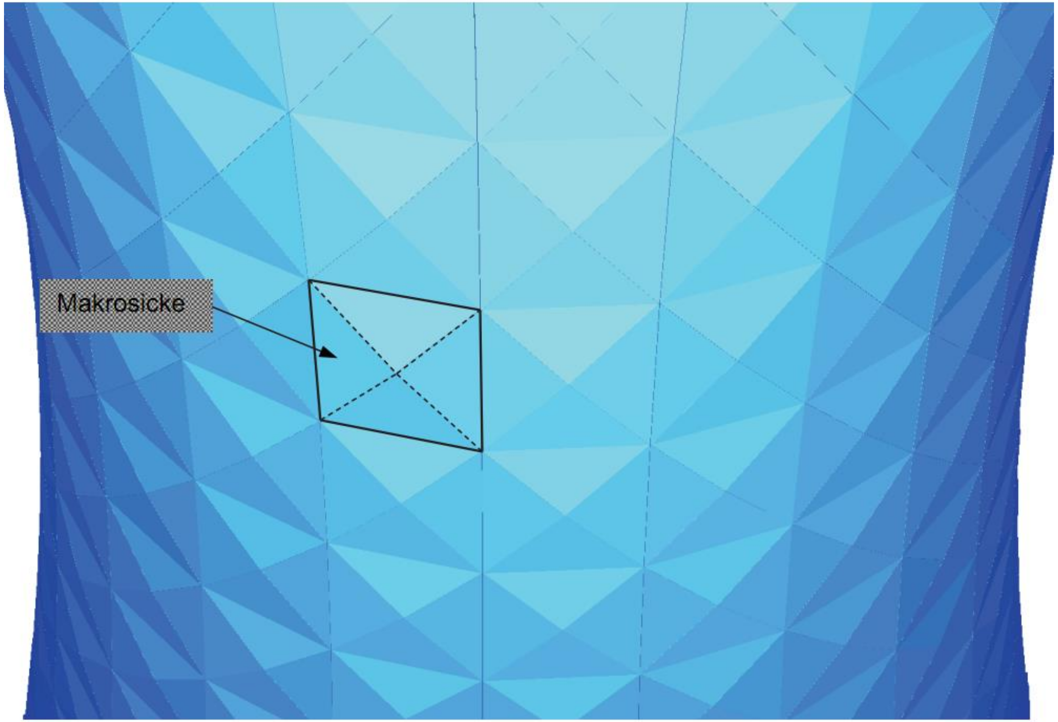
FEM-Meshes for a cooling tower



Deformations

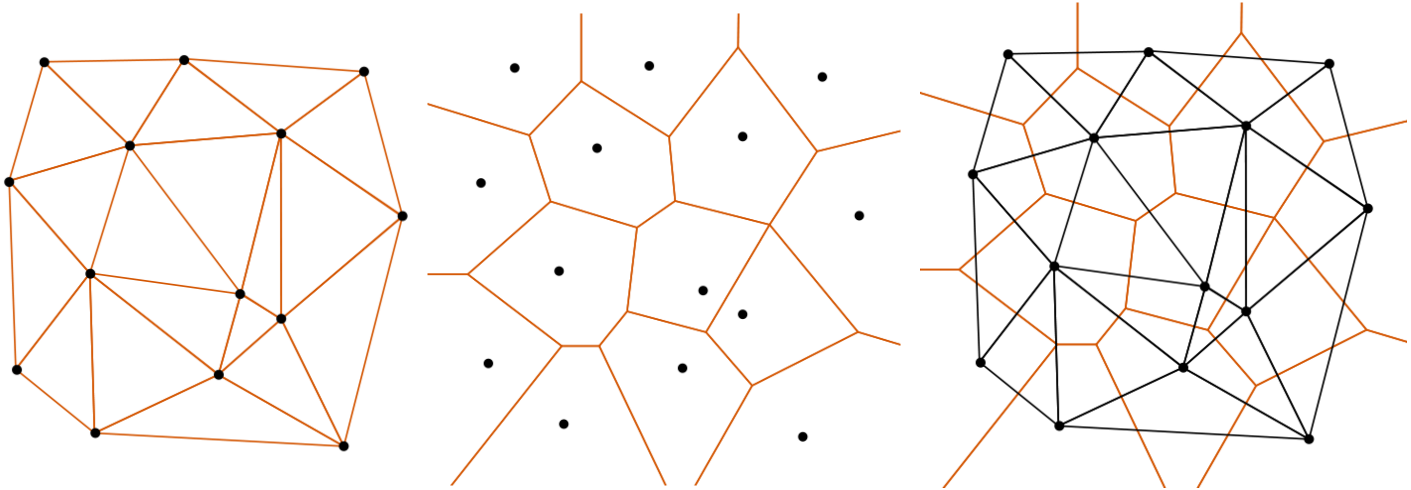


And the reason is:



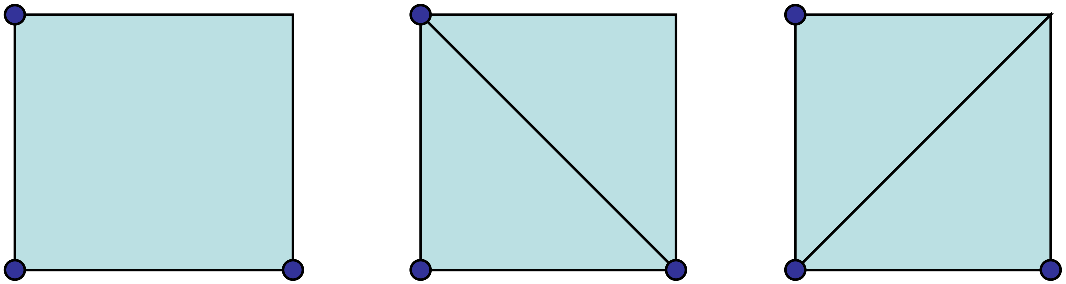
Triangular mesh is always possible

- Delauney triangularisation / Voronoi Diagrams



Mesh Quality

- Quadrilateral is better than two triangles



$$w = a \cdot x \cdot y$$

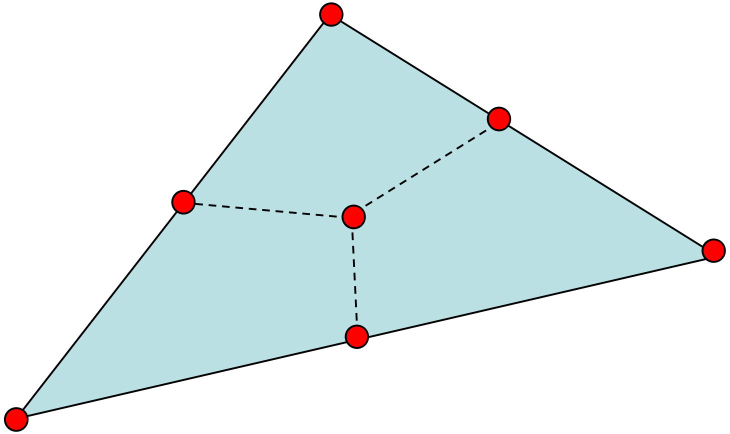
- Even if distorted

Mesh quality

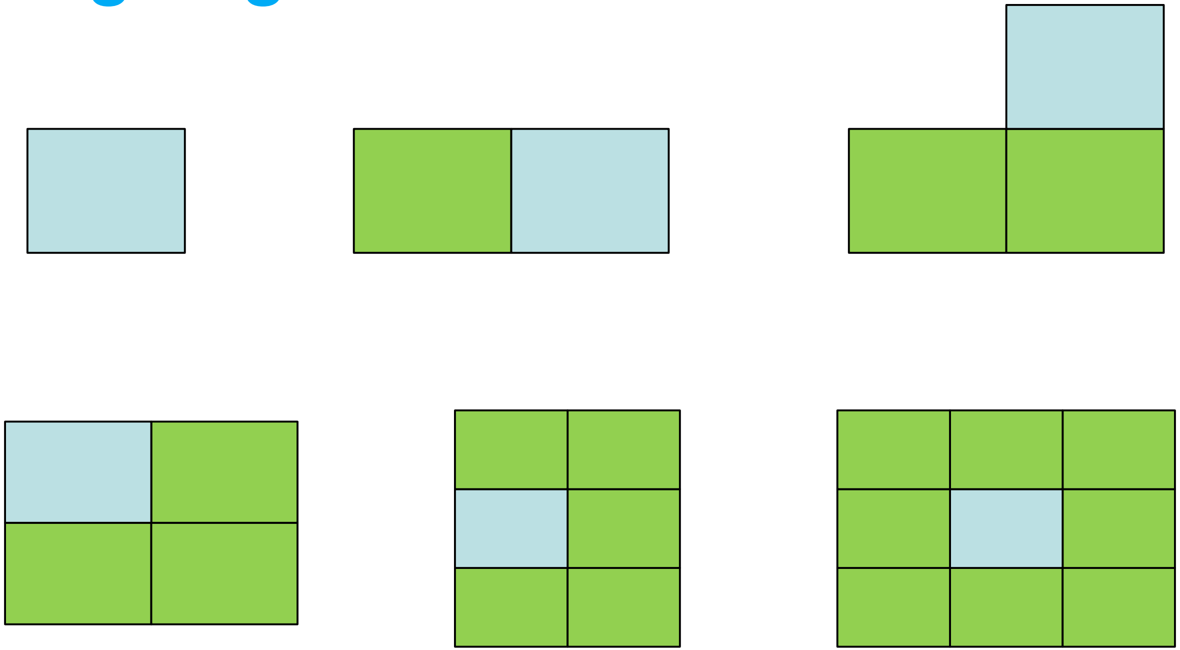
- Ratio of sides
  - Optimal 1:1
  - Tolerable 1:5, Special cases (1:100)
- Interior Angle
  - Triangle 60 degree
  - Quadrilateral 90 degree
  - Error increases for smaller angles
  - Angles > 180 degrees are impossible

QUAD Meshes are better

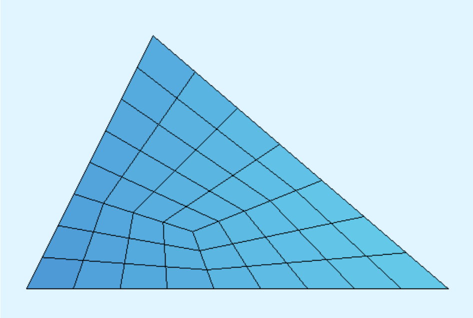
- But not always possible ?
- Every Triangular mesh may be converted to a QUAD mesh:



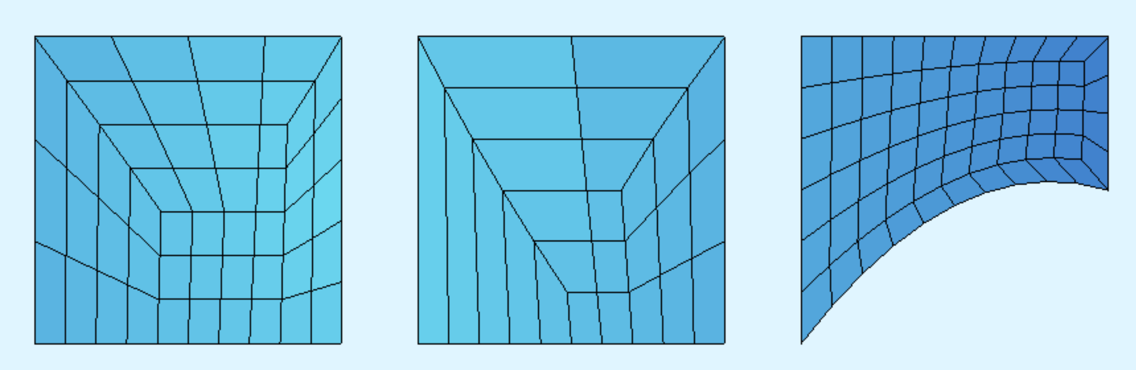
Every QUAD mesh has an even number of bounding edges



If the number of edges is even, a QUAD mesh is nearly always possible



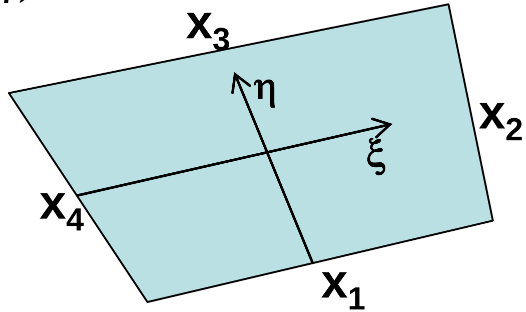
For triangular regions, every stepping has to be less than the sum of the other two

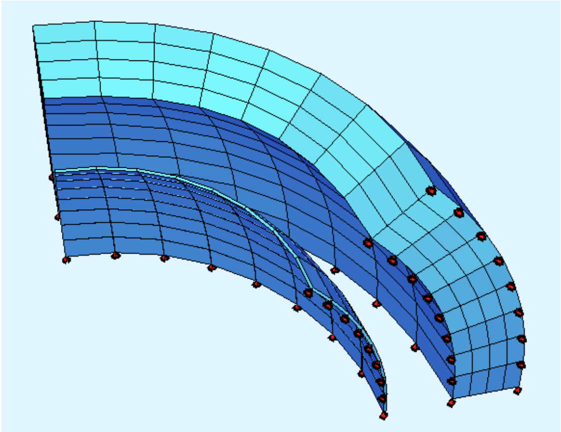


Coons-Patches 2D and 3D

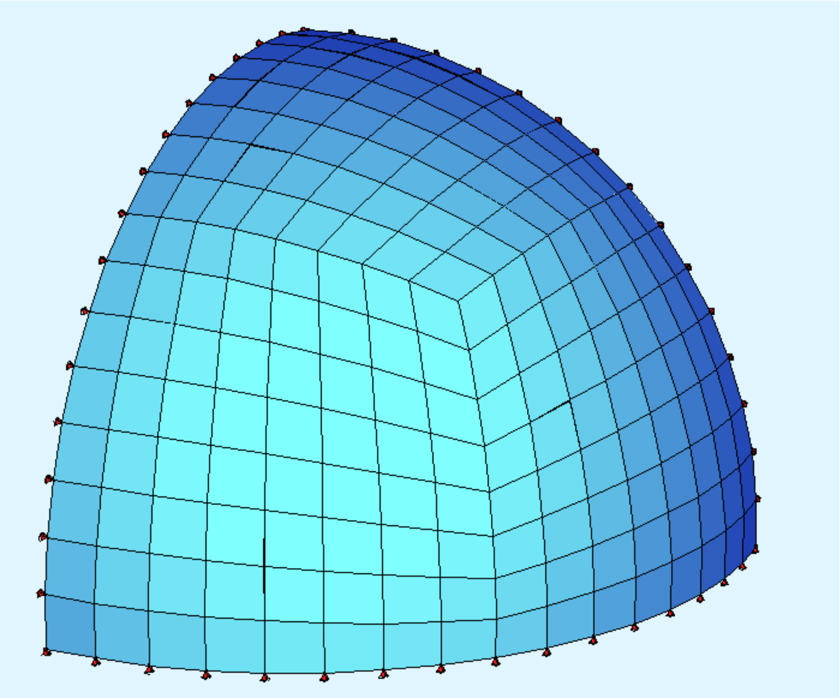
- Idea: Interpolation between opposite edges
- Quadrilateral topology: Two interpolations, thus a bilinear interpolation is subtracted:

$$\vec{x}(\xi,\eta) = \vec{x}_1(\xi)\cdot(1-\eta) + \vec{x}_3(\xi)\cdot\eta + \vec{x}_4(\eta)\cdot(1-\xi) + \vec{x}_2(\eta)\cdot\xi$$
$$-\sum_{i=1}^{4} \vec{x}_{ci}\cdot N_i(\xi,\eta)$$

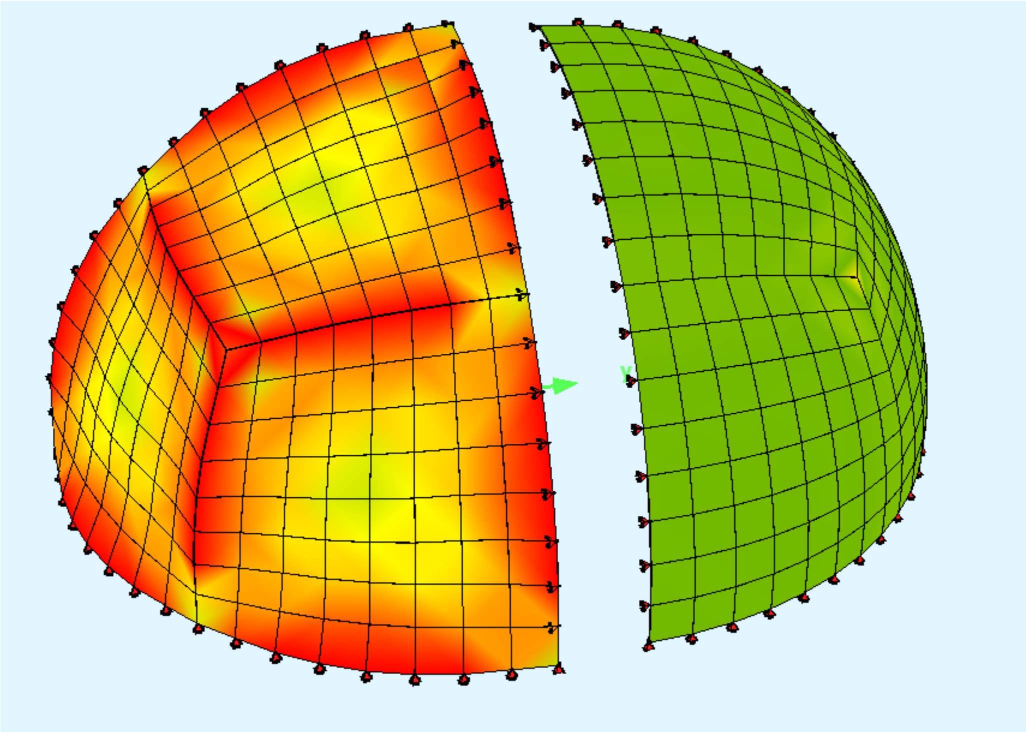




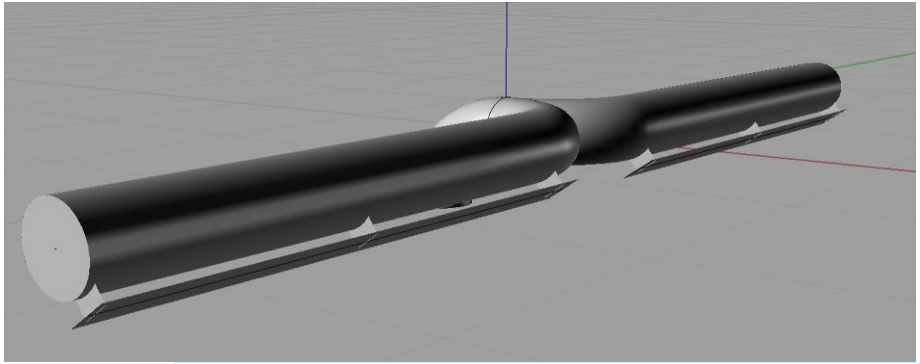
Mesh division of a sphere ?



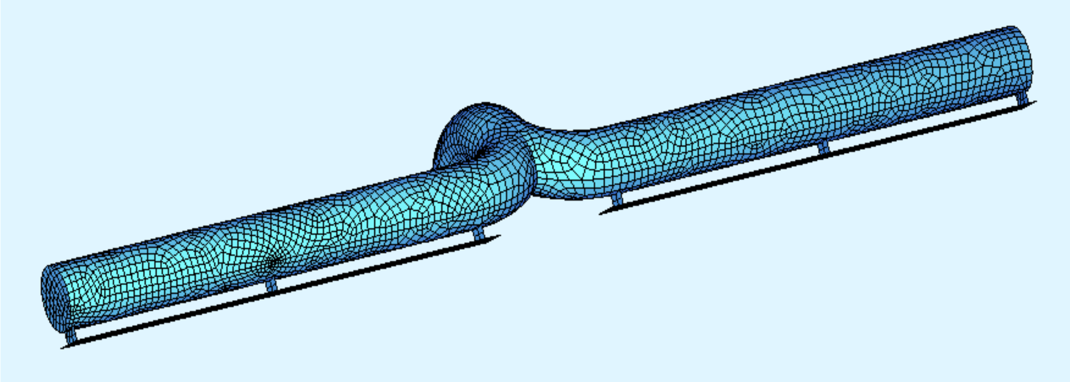
Stresses for Coons Patch / Exact Geometry



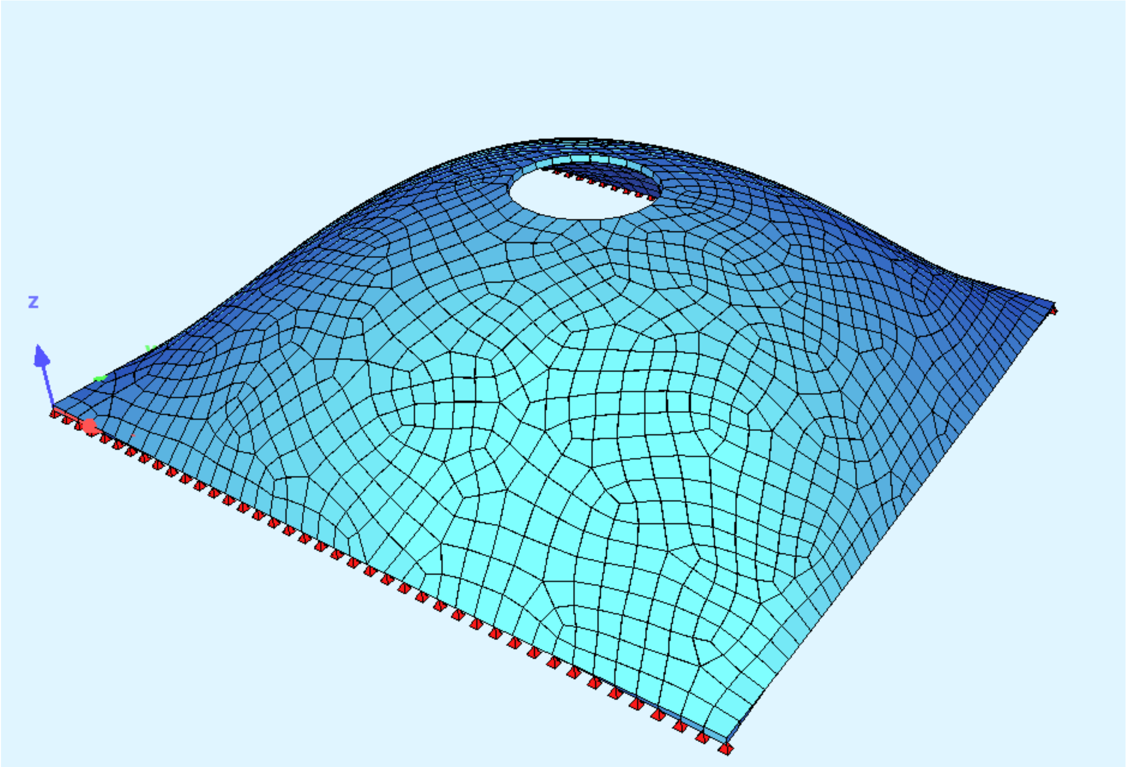
Intersection of shells



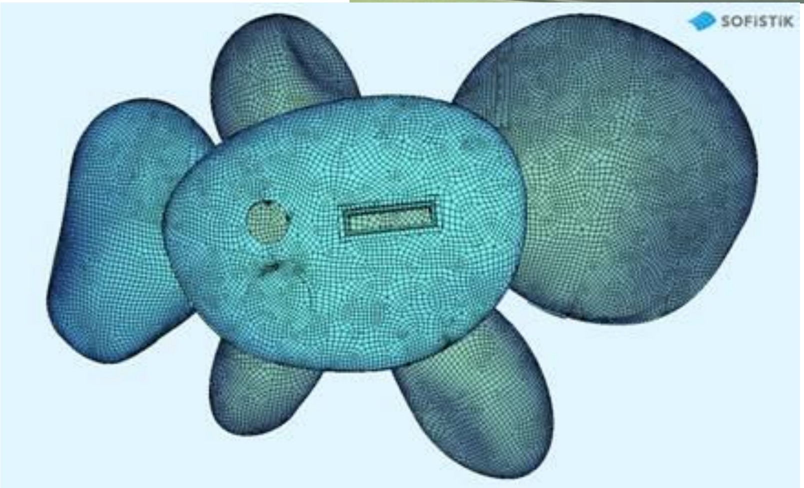
NURBS-modelling
with Rhinoceros®



Mapped mesh with a hole



Example

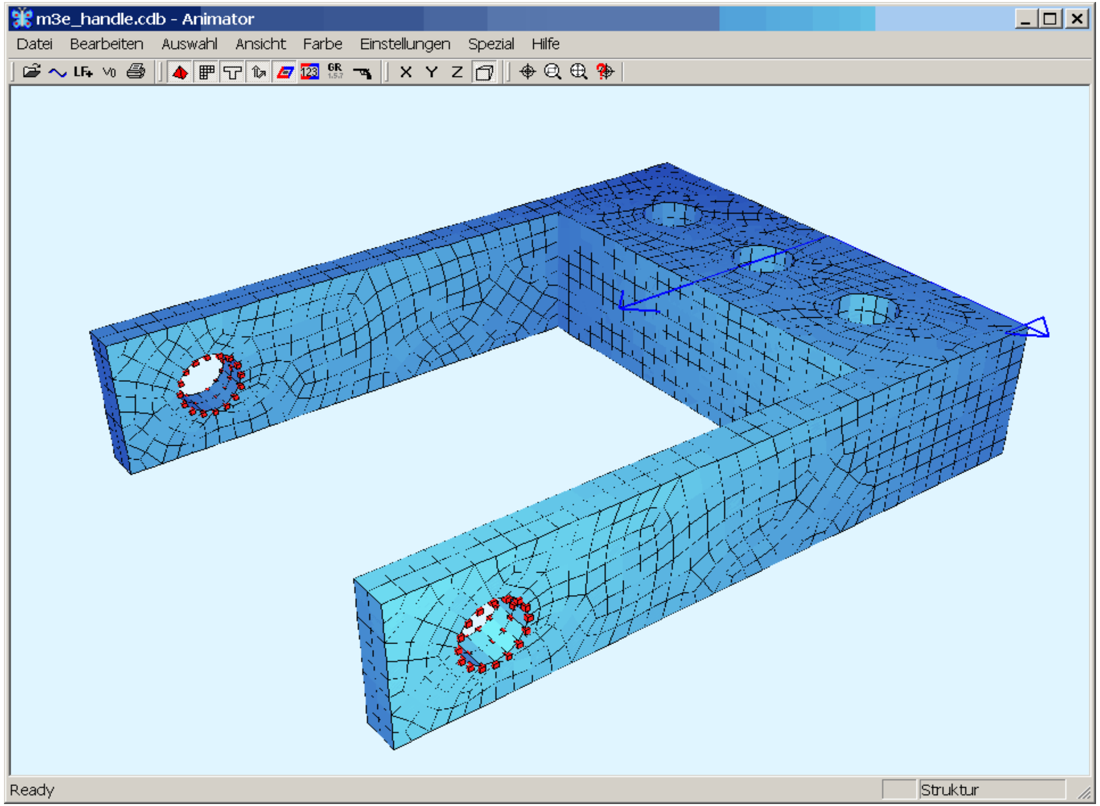


NURBS-modelling with Rhinoceros®

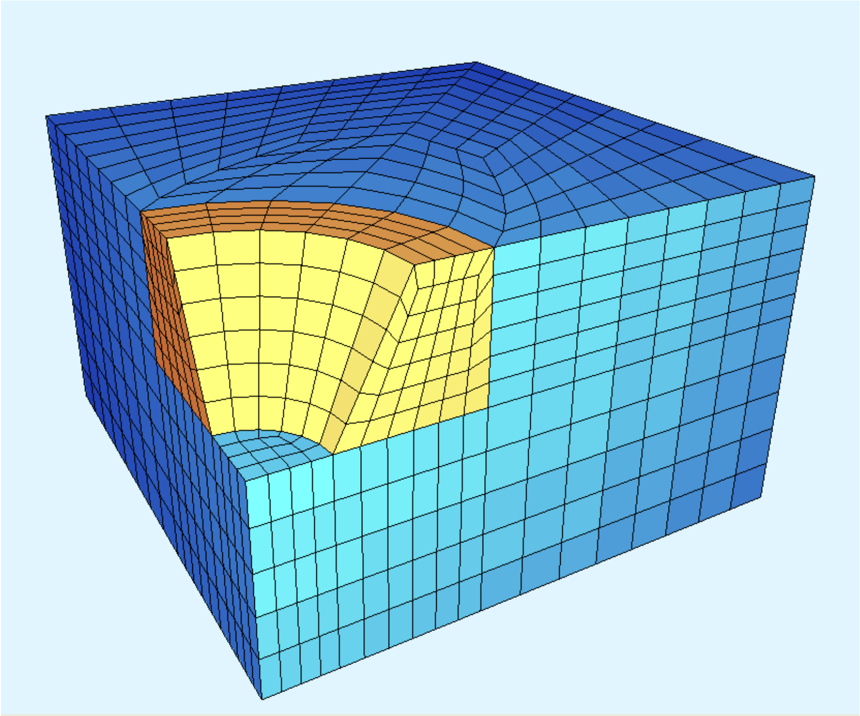
Problems

- The description of the surface allows meshes with singular geometry (spheres)
- For the FE-Mesh this is a very bad idea!
  A mapping of the Jacobian is then required.
- Be careful about approximating geometries!
- Water-Tightness of meshes
  (purify the CAD meshes)
- Ignore tiny details of a CAD structure irrelevant for the analysis.

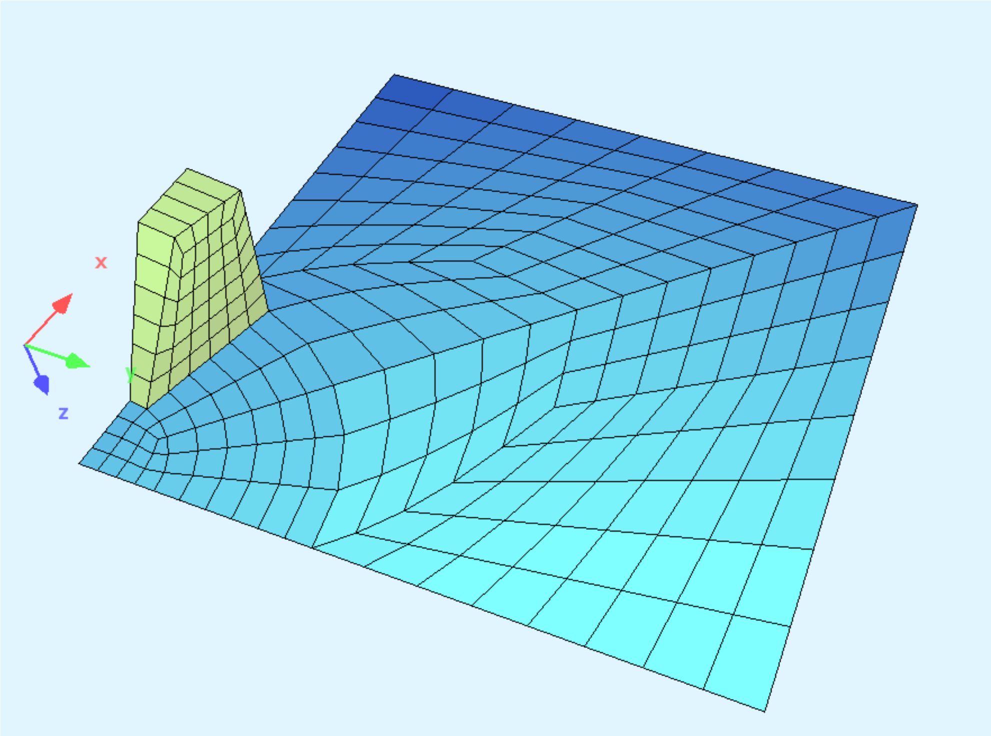
3D Extrusions-Mesh Generator



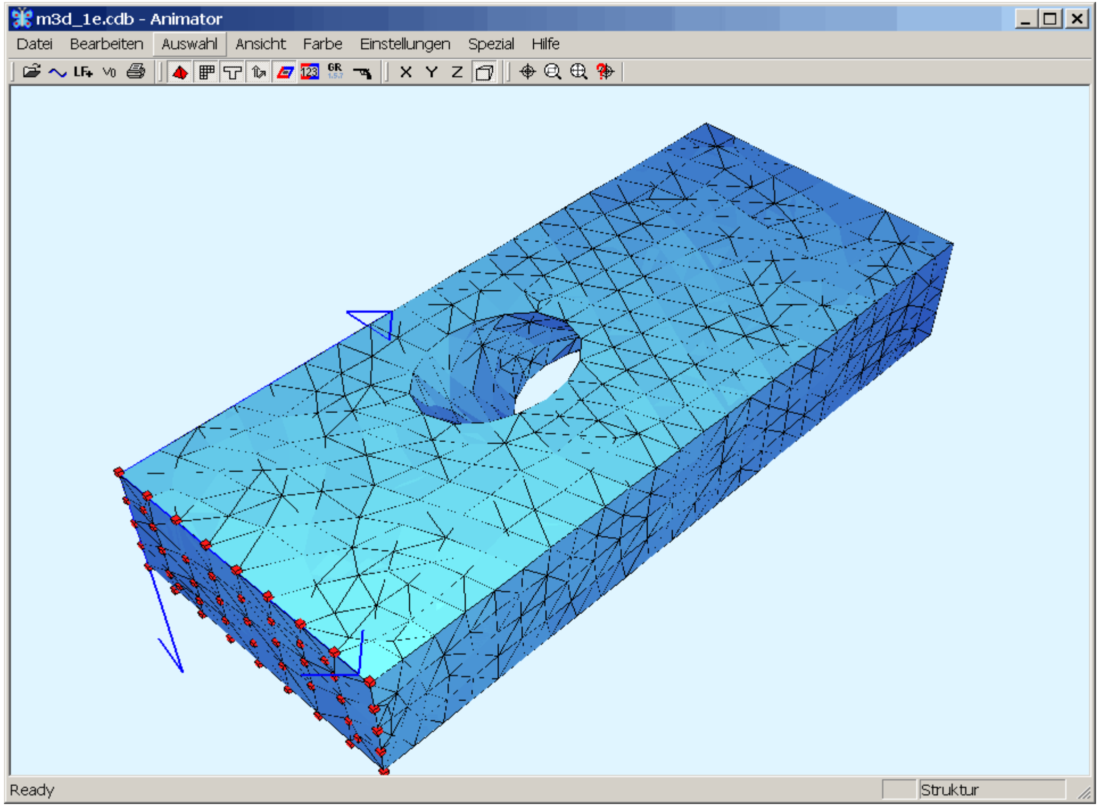
3D extrusion / sweep along circle



The 2D Start Faces

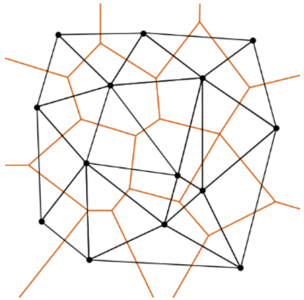


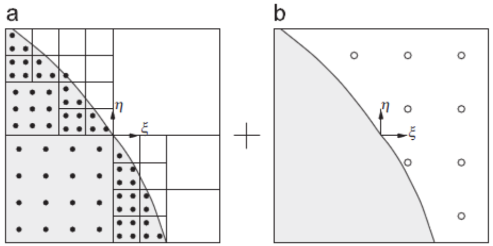
3D Tetraeder Mesh Generation



Possibilities for 3D FEM

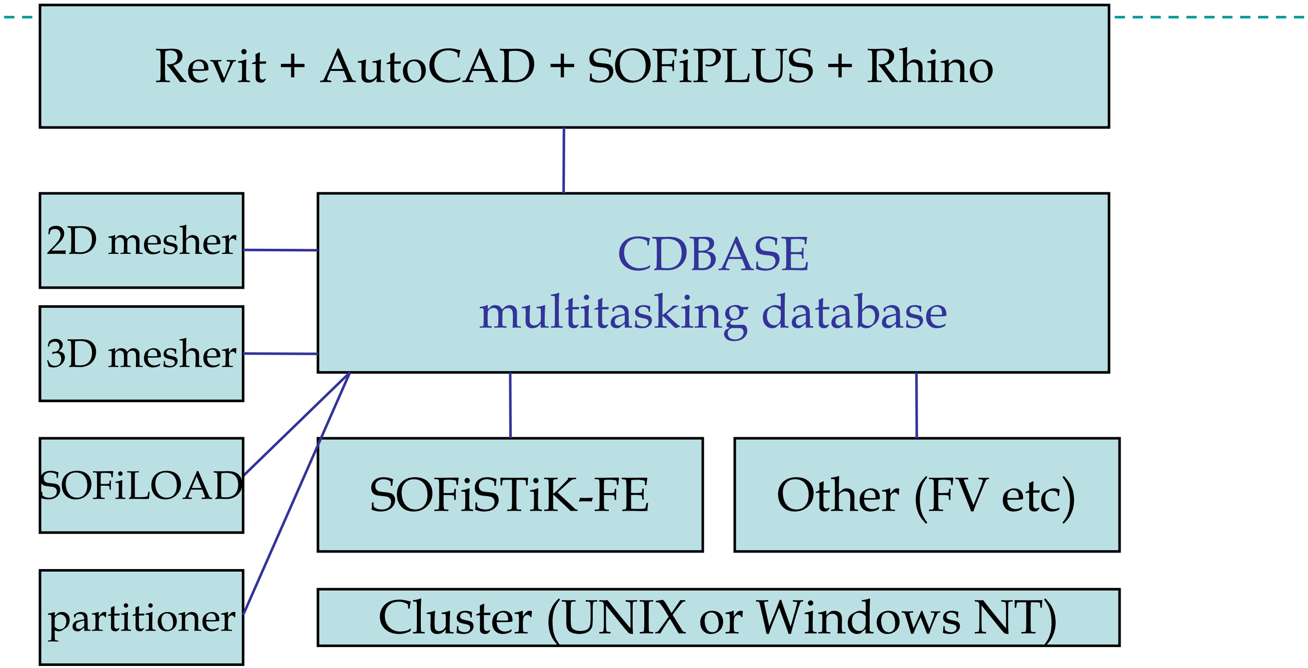
- Hexahedral Elements by Extrusion etc.
- Tetrahedral Mesh
  - Constant strain elements not acceptable
  - High order Elements need high order interfaces
  - Virtual polyhedral elements





- Finite Cell Approach

The SOFiSTiK System

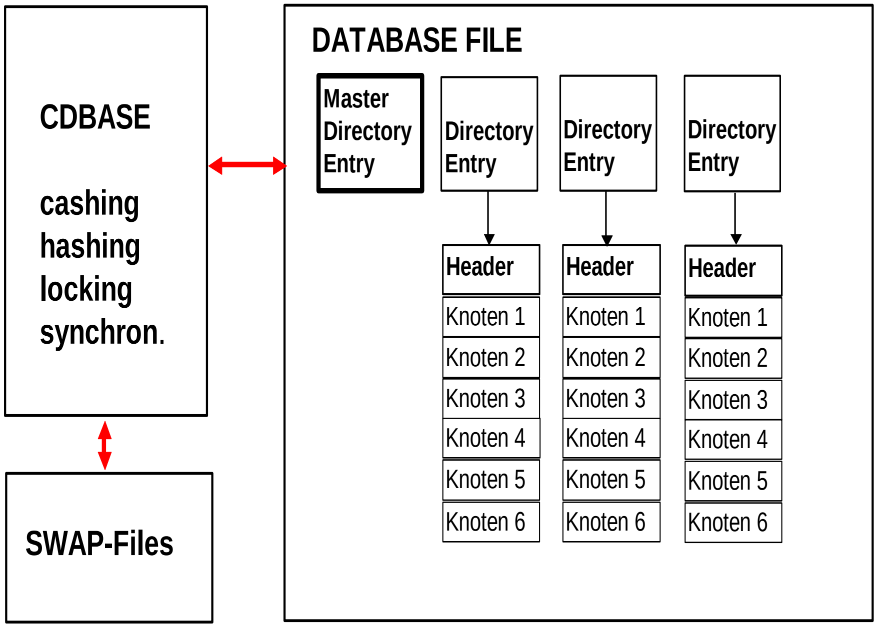


Geometric / Structural Elements

- Points (Supports, column heads, Monitorpoints)
- Lines and curves
  - Lines, Arcs, Klothoids, Splines, Nurbs
  - Assigned properties:
    Sections, elastic or rigid supports, Interface-conditions
- Surfaces
  - Planar, Rotation, Extrusion / Sweep / Lofts
    Coons-Patches, B-Splines, Nurbs
- Automatic Intersections of all elements
  = geometric definition independent
  to inherited structural elements

Effective Communication of data

- CDBASE - Database
  - clear Interface
  - Data structures
  - Performance
  - locking
  - merging
  - system-independent



Database

- Contains all data which might become important
- Example: Sections and Materials
    - Constants not directly bound to elements
    - Element has a pointer to the section / material
    - Section / material have tables with other data
- Material is not just a name or a constant
    - Elasticity constants
    - Strength
    - Weight / weight class / prices
    - Thermal properties etc.

Soil-Structure-Interaction

- Method 0
  - Foundations are rigid for the analysis of the structure
  - Loadings on foundations are compared against admissible stresses
- Method 1
  - Foundations are rigid for the analysis of the structure
  - Loadings on foundations are compared against a soil rupture analysis and a settlement analysis

Soil-Structure-Interaction

- Method 2
  - Foundations are rigid for the analysis of the structure
  - Loadings on foundations are compared against a soil rupture analysis and a settlement analysis
  - Settlements are applied as inforced deformations on the structure

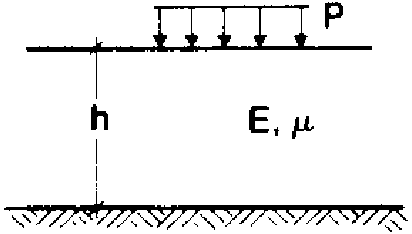
Soil-Structure-Interaction

- Method 3 = real Interaction
  - Winkler Assumption (3a) (Bettungsmodulverfahren)
  - Elastic Half-Space (3b) (Steifemodulverfahren)
  - Soil as a non linear Continua (3c)
- Extend of Model
  - Only the foundation itself (e.g. plate)
  - Total structure
  - All construction stages

Winkler Assumption

- Bedding modulus C [kN/m^3]
  = soil pressure / settlement
- Neglecting shear stresses
- Depending on the load pattern / load level
- Depending on the size of the structure
- Depending on the material, but NOT a material constant
- Constant loading creates constant settlements

Values of the Coefficient



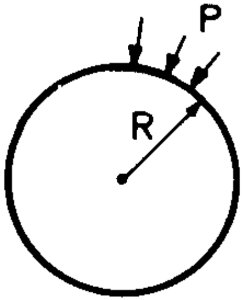
$$C_n = \frac{E}{h}\frac{(1-\mu)}{(1+\mu)(1-2\mu)} \text{ (plane strain)}$$

$$C_n = \frac{E}{h}\frac{1}{(1-\mu)(1+\mu)} \text{ (plane stress)}$$

$$C_t = \frac{E}{h}\frac{1}{(1+\mu)}$$

Values of the Coefficient

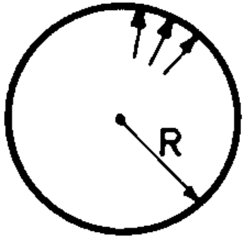
For a circular disc we get



$$C_n = \frac{E}{R}\frac{1}{(1-\mu)} \text{ (plane stress)}$$

$$C_n = \frac{E}{R}\frac{1}{(1+\mu)(1-2\mu)} \text{ (plane strain).}$$

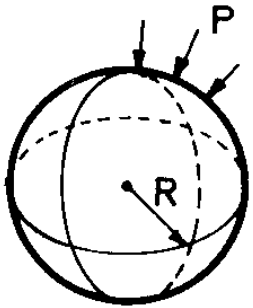
For a circular hole in an infinite disc holds



$$C_n = C_t = \frac{E}{R}\frac{1}{(1+\mu)} \text{ (plane strain and stress).}$$

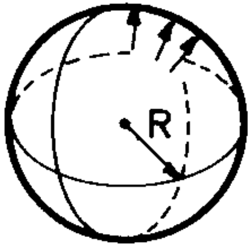
Values of the Coefficient

In three dimensions we get for a sphere



$$C_n = \frac{E}{R}\frac{1}{(1-2\mu)}.$$

and for a spherical cavity (internal pressure)



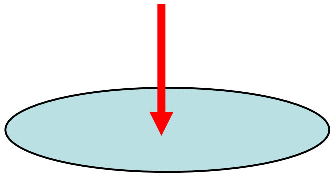
$$C_n = \frac{E}{R}\frac{2}{(1+\mu)}$$

Values of the Coefficient

If the loaded area is restricted, better values can be calculated using a uniform pressure or displacement only for that area. This is especially important if we have a semi-infinite body. In this case it is possible to use the displacements under a rigid circular die for example (Timoshenko[1]) to get

$$C_n = \frac{E}{R}\frac{2}{(1-\mu)(1+\mu)\pi}$$

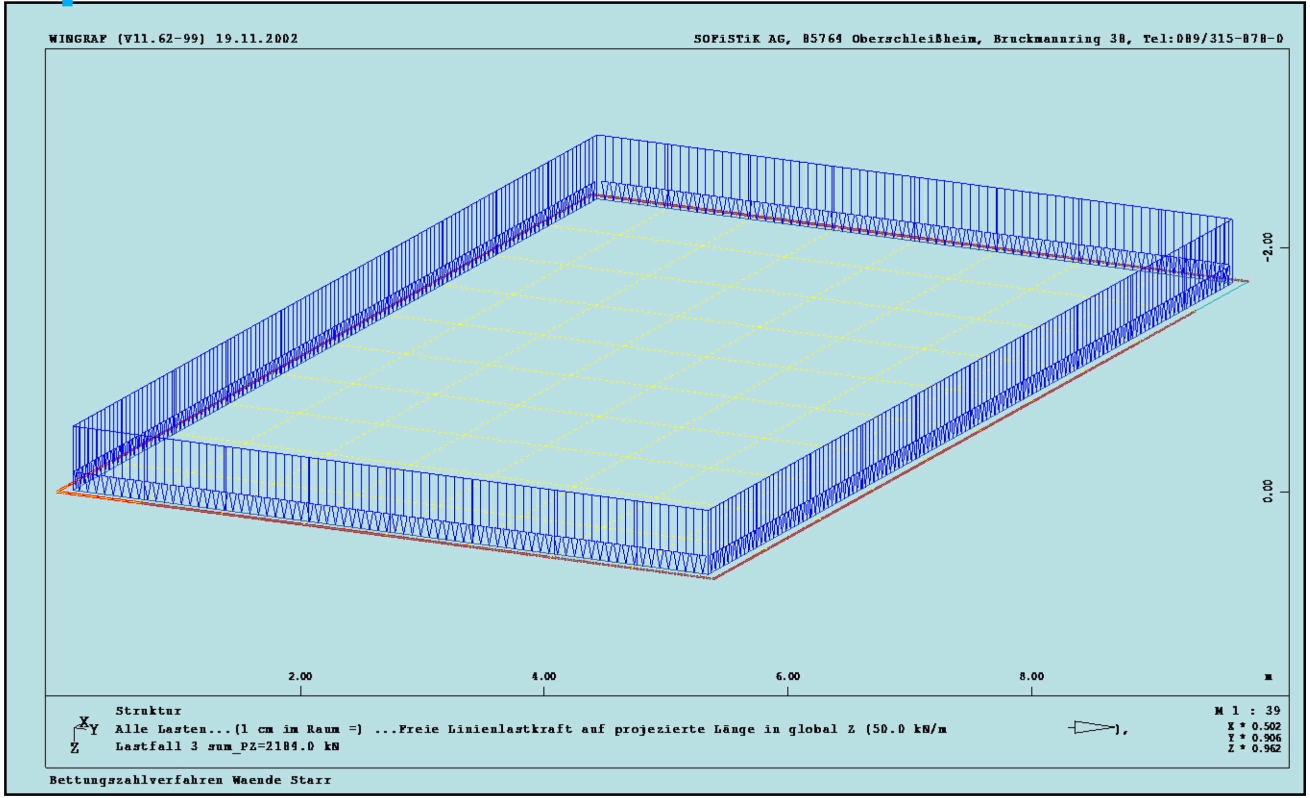
where R is the radius of an approximate circular area which equals the loaded area.



Stiffness Approach

- All methods where the shape of the settlements is accounted for
- Analytic Description of Half Space
  - Stress distribution based on elastic model
  - Deformations are calculated based on non linear properties of soil
  - Inversion of the flexibility matrix
- Modelling Half Space with Finite Elements
- Modelling Half Space with Boundary Elements
- Modelling Half Space with connected springs

Example



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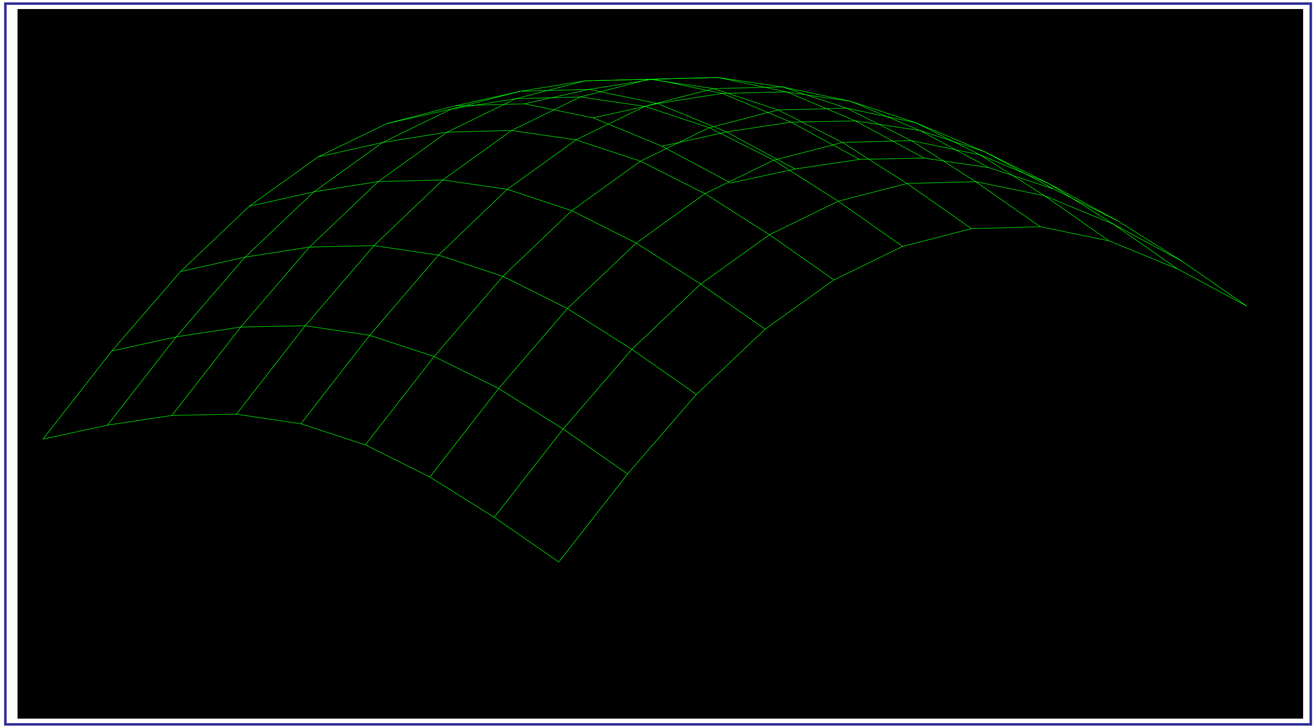
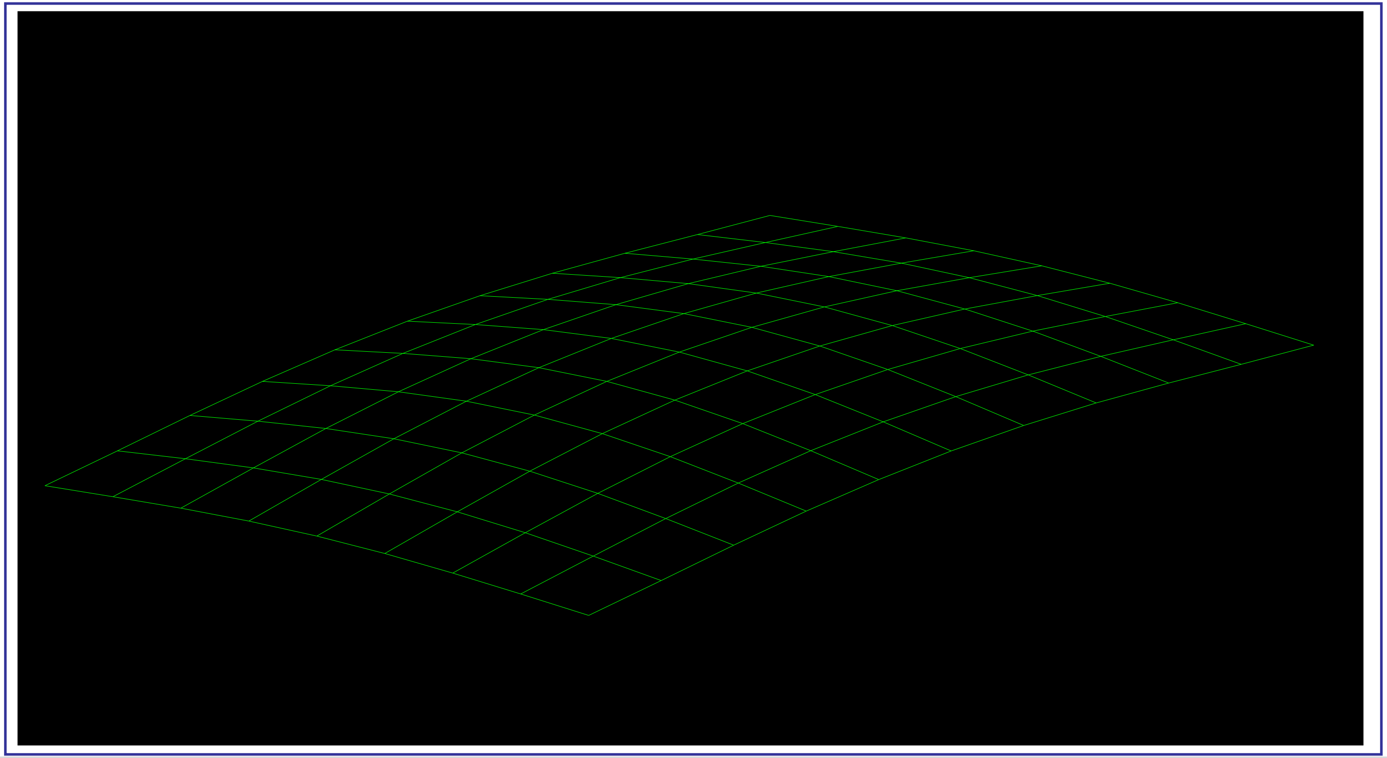
m
Struktur
Alle Lasten...(1 cm im Raum =) ...Freie Linienlastkraft auf projezierte Länge in global Z (50.0 kN/m
Lastfall 3 sum_PZ=2184.0 kN
M 1 : 39
X * 0.502
Y * 0.906
Z * 0.962
Bettungszahlverfahren Waende Starr

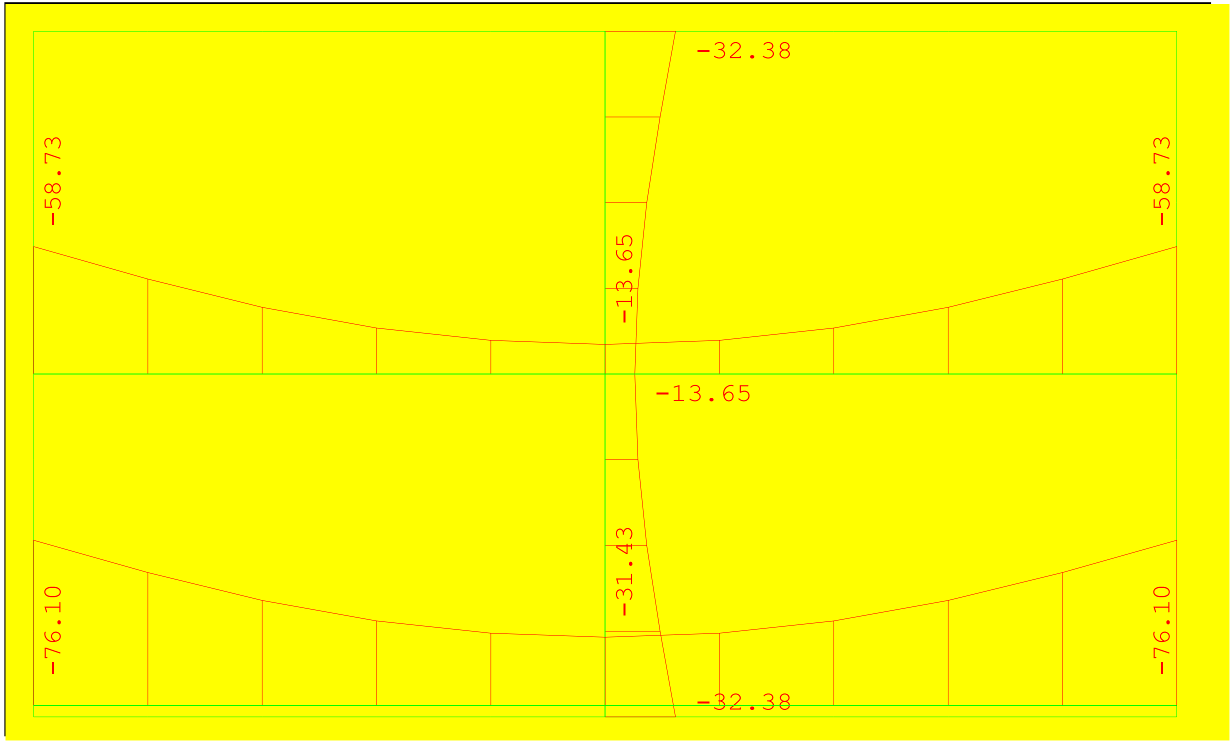
Winkler Assumption



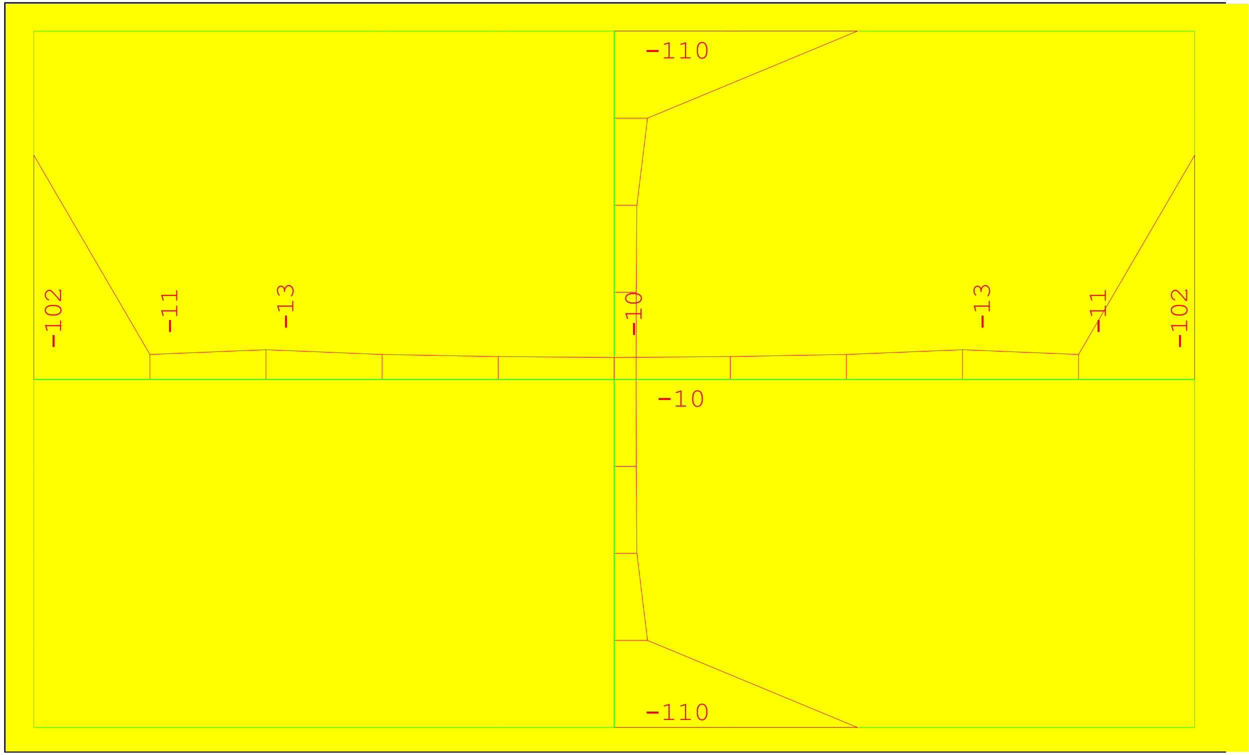
Stiffness Approach



Winkler Assumption - pressure



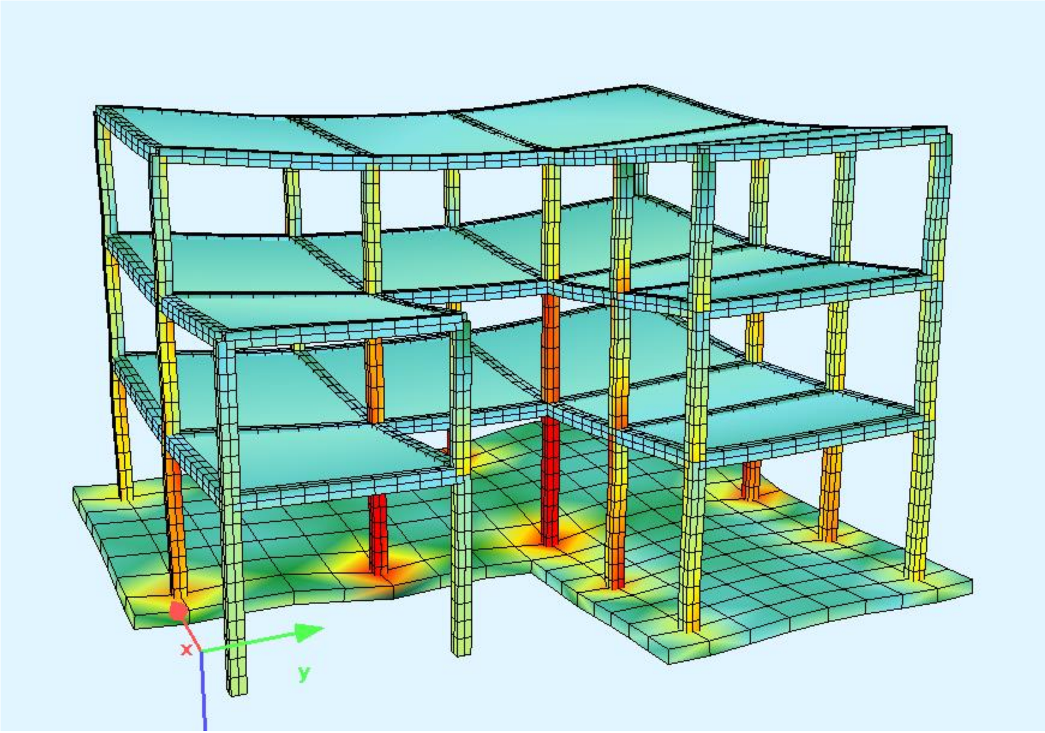
Stiffness Approach



Comparison

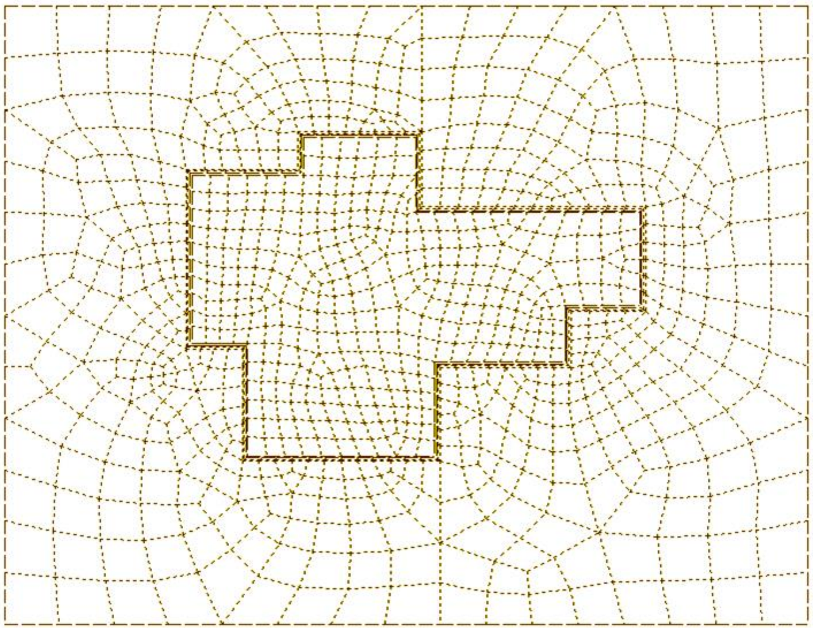
- The Winkler assumption yields more negative moments in the foundation plate
- The stiffness approach yields more positive moments in the foundation plate
- Effort for stiffness based methods considerably higher
- Simple enhancement for the Winkler assumption with increased coefficients at the edges

Combined Frame / Slab / Soil



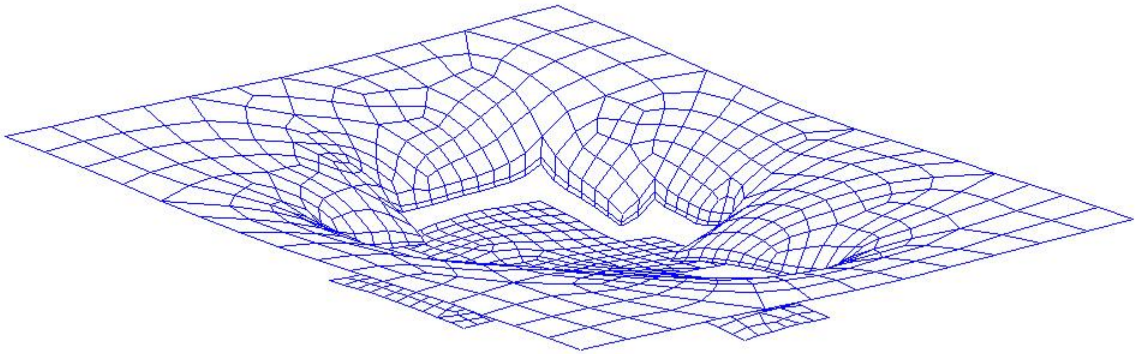
Example of combined slab/pile foundation

- Mesh defines only the surface of the soil and the foundation plate

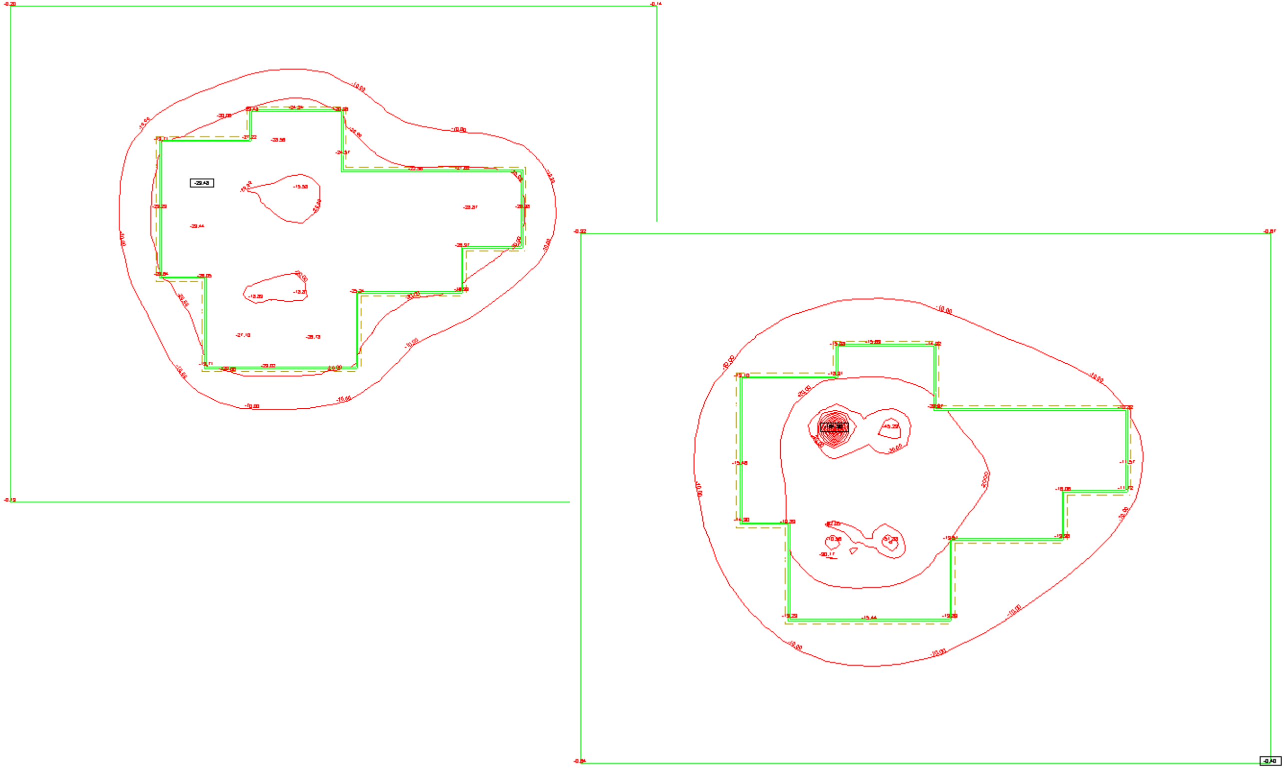


Settlements on Surface

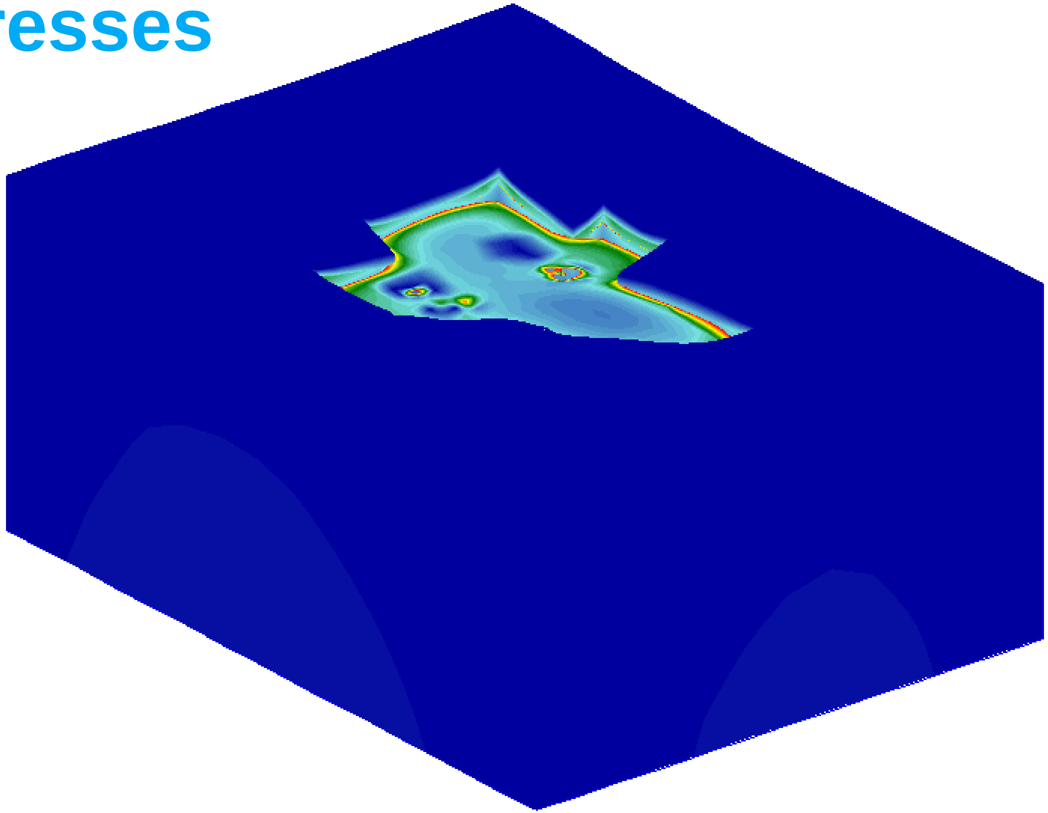
- A small gap between the soil mesh and the slab mesh shows differences in settlements



Stresses in different depths



And a more detailed view on stresses



And a more detailed view on stresses