Industrial Applications of Computational Mechanics Dynamics

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Basic Possibilities

- We may select a solution either
  - in the time domain
  - in the frequency domain
- The transformation between those methods is done with Fourier Analysis
- The Solution in the Time domain is
  - easier to understand,
  - more flexible
  - Needs more computer power
- A complete solution in the frequency domain needs complex functional analysis, but for small damping effects most can be handled in real analysis as well.

Basic Equation in Time

$$m \cdot \ddot{u} + c \cdot \dot{u} + k \cdot u = p(t)$$

- u = Displacement (Verschiebung)
- $\dot{u}$ = Velocity (Geschwindigkeit)
- $\ddot{u}$ = Acceleration (Beschleunigung)
- k = Stiffness => elastic/non linear support force
- m = Mass => Inertia force (Trägheitskraft)
- c = viscous Damping => Damping force
- p(t) = Loading

Discrete Equations in Time

$$m_{ij} \cdot \ddot{u}_i + c_{ij} \cdot \dot{u}_i + k_{ij} \cdot u_i = p_j(t)$$

- Discrete equations are the result of an weighted integration process
- Stiffness matrix k_{ij} is Sparse matrix
- Mass matrix m_{ij} is either
  - Lumped diagonal matrix (ASE)
  - Consistent sparse matrix (DYNA)
- Damping matrix c_{ij} is either
  - A linear combination of m and k-matrix
  - A consistent sparse matrix

Stiffness matrix

- Same as for static analysis
- Important part is the geometric stiffness (initial stress) for many types of structures ! (e.g. string of guitar)
- If not defined / available:
  - Free falling of object
  - Suppressing degrees is therefore not allowed in general
- Fluids with K >0 and G ~ 0
  - only possible for dynamic analysis.
  - Special treatments for „incompressible elements“
  - Stiffness of free surface in fluids !

Mass matrix

- Care for correct units (tons, resp. kNsec2/m)
- Do not forget the permanent part of live loads (ψ_2 = quasi permanent combination)
- Translatoric Masses are in general the same in all directions = uniform tensor.
- Rotational masses have an axis of rotation, thus are tensors with possible off diagonal terms
- Especially if only translatoric masses are used, a sufficient small mesh size is required.
  = subdivide beams / cables
- Degrees of freedom without masses may lead to instabilities in the Eigenvalues.

Damping

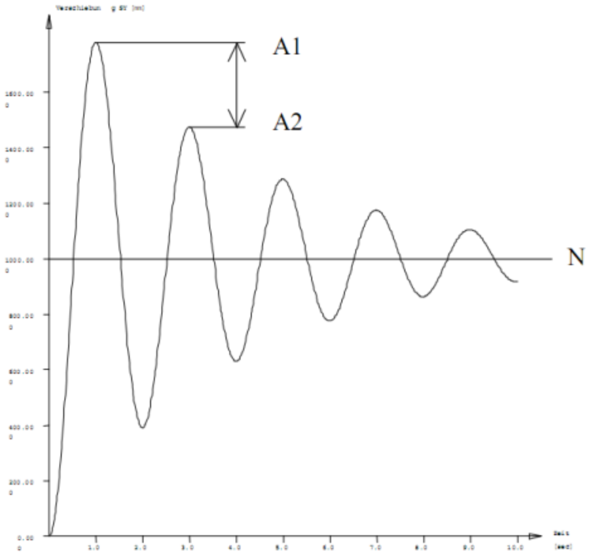
- Damping similar to mass:
  - Moving part in oil / air / water
  - Velocity dependant damping constants: $f = c\ v^{\xi}$
- Damping similar to Stiffness:
  - Material damping, cracking, yielding etc.
- Damping by friction
  - If the displacement is known, an equivalent viscous damper may be used, otherwise a non linear loading has to be analysed
- There is a negative damping possible!
  - Aero-elastic instabilities, Galloping, Flutter

Damping

$$d_i = \frac{\delta}{2\pi} = D + 1/2 \cdot A/\omega_i + 1/2 \cdot B \cdot \omega_i$$

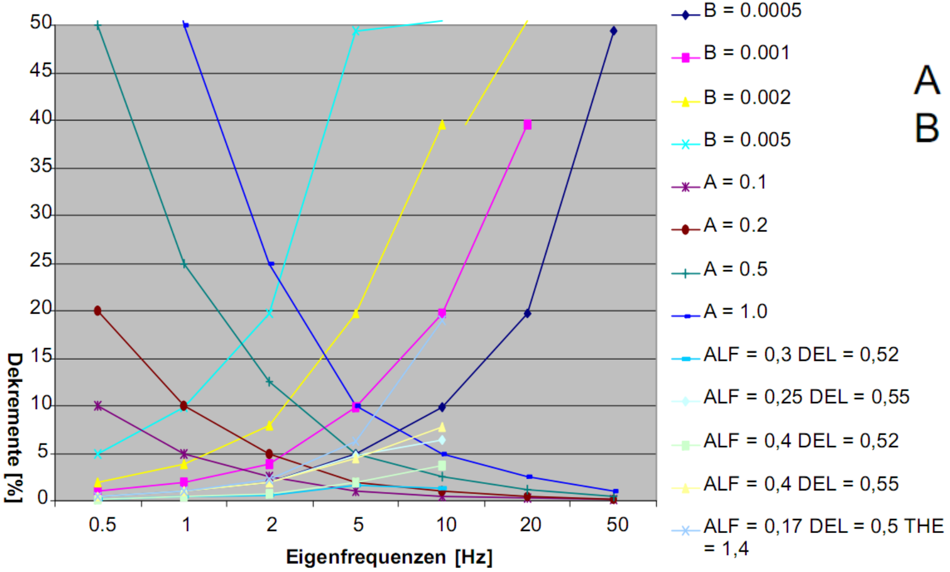
- Modal Damping d or ξ

Estimates	elastic	plastic
• Concrete	1 – 2 %	7 %
• Prestr. Concrete	0.8 %	5 %
• Steel, bolted	1 %	7 %
• Steel, welded	0.4 %	4 %
• Timber	1 – 3 %	
• Brickwork	1 – 2 %	
• Cables	0.1 %	



- logarithmic decrement $\delta = \ln(A1/A2)$
- Rayleigh factors A / B

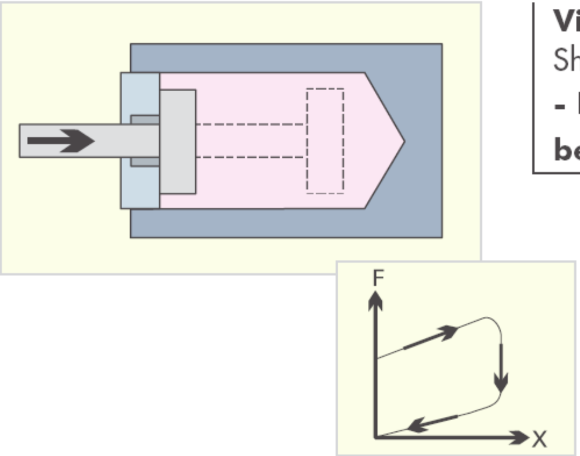
Modal Damping => A,B



$$A = 2*\omega_1*\omega_2(\xi_1\omega_2 - \xi_2\omega_1) / (\omega_2^2 - \omega_1^2)$$
$$B = 2* (\xi_2\omega_2 - \xi_1\omega_1) / (\omega_2^2 - \omega_1^2)$$

Nonlinear Damping

- Hysteresis caused by yielding
- Friction
- Non linear Damping
  (e.g. Jarret Shock Absorber)



Viscosity:
Shock absorber function
- $F = F_0 + KX + CV^{\alpha}$ with α between 0,1 and 0,4

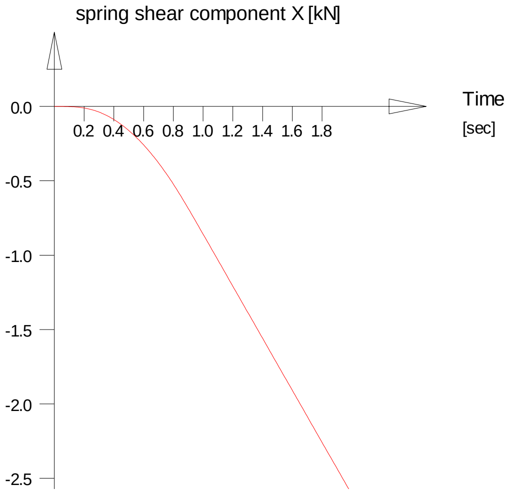
Frictional Damping

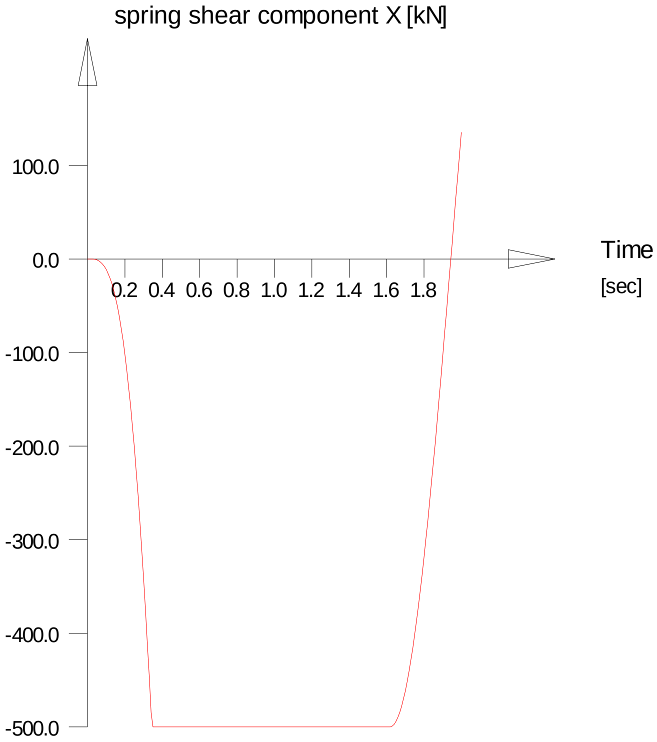
- Frictional force is not a real force but an upper limit
- We need (high) transverse spring constant to get such a force
- Equivalent viscous damping (Flesch equ. 5.24)

$$c_{equ} = \frac{4 \cdot \mu \cdot N}{\pi \cdot \omega \cdot u_{\max}}$$

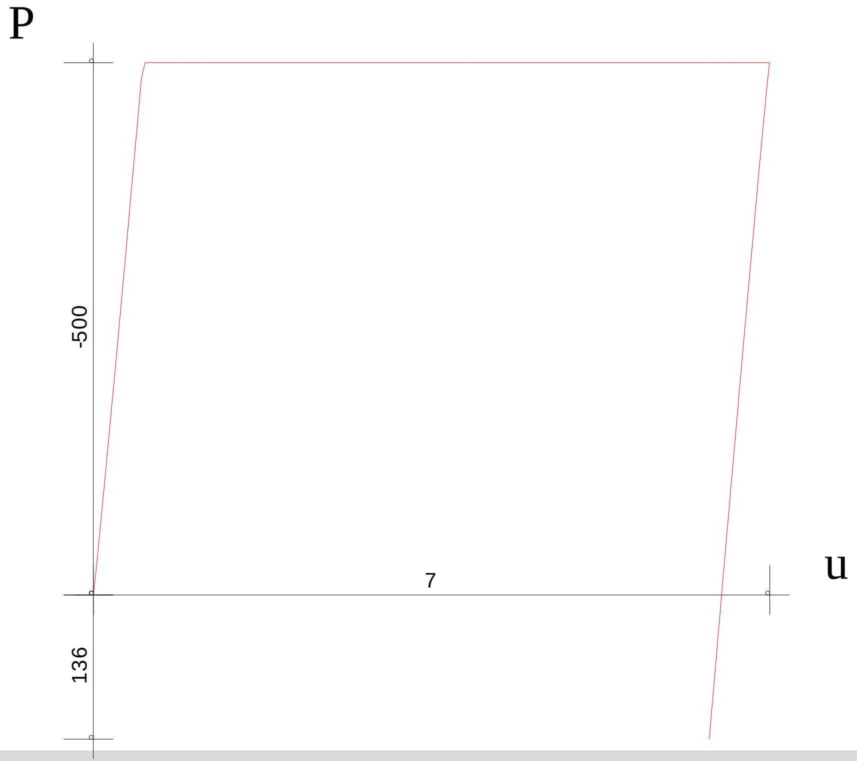
Frictional Damping

```
SPRI 1 PRE -1000. CP 1. CT
   1000000. MUE 0.5
```





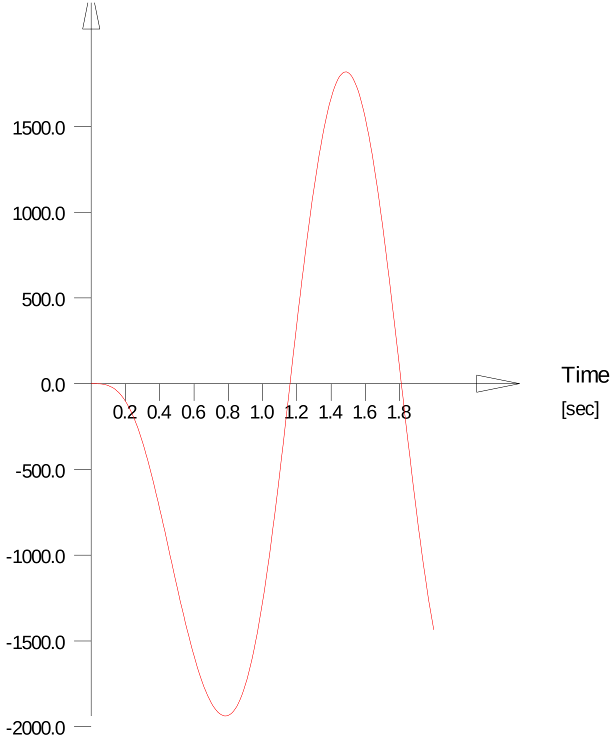
Frictional damping Hysteresis

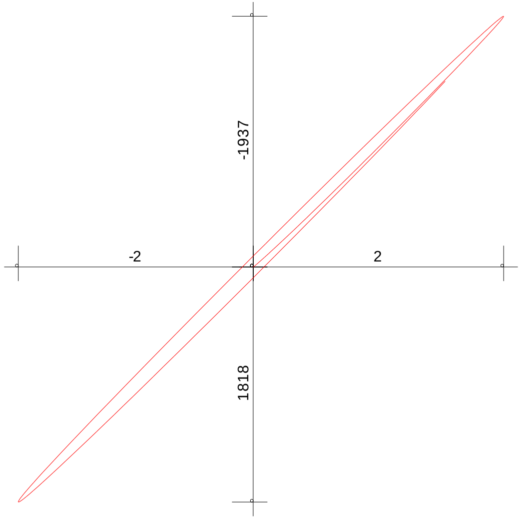


Nonlinear damping

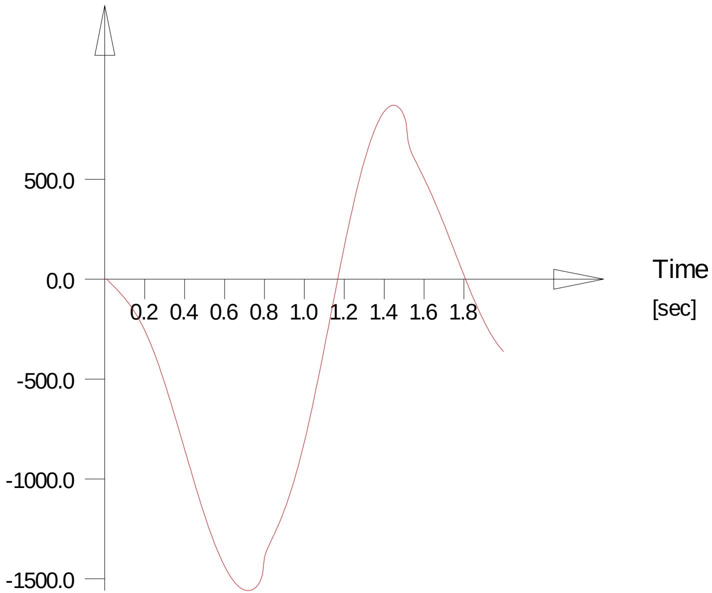
- For all damping effects holds:
  decreasing the time step size does not decrease the damping force !
- Now we have the problem that the given parameters are valid only for some restricted range of data
- V = 0.01 m/sec
  - F = 0.631 * C for α = 0.1
  - F = 0.158 * C for α = 0.4
- V = 2.5 m/sec
  - F = 1.096 * C for α = 0.1
  - F = 1.443 * C for α = 0.4
- Small values of α may become very difficult

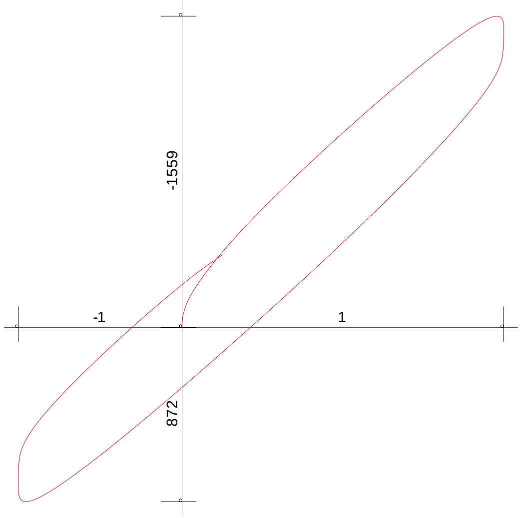
Non linear damping $\alpha = 1.5$



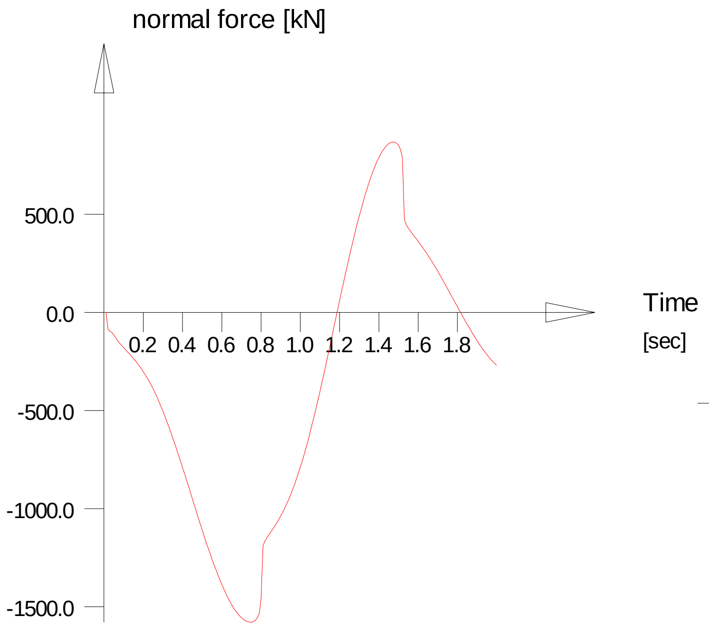


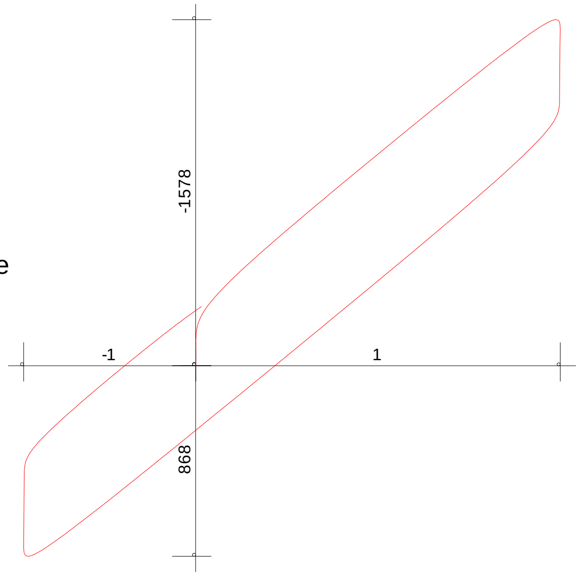
Non linear damping $\alpha = 0.5$



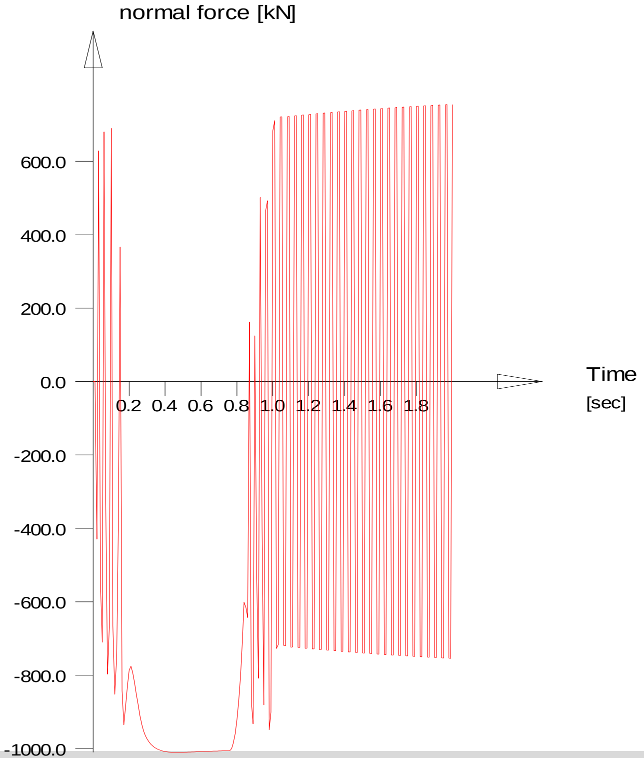


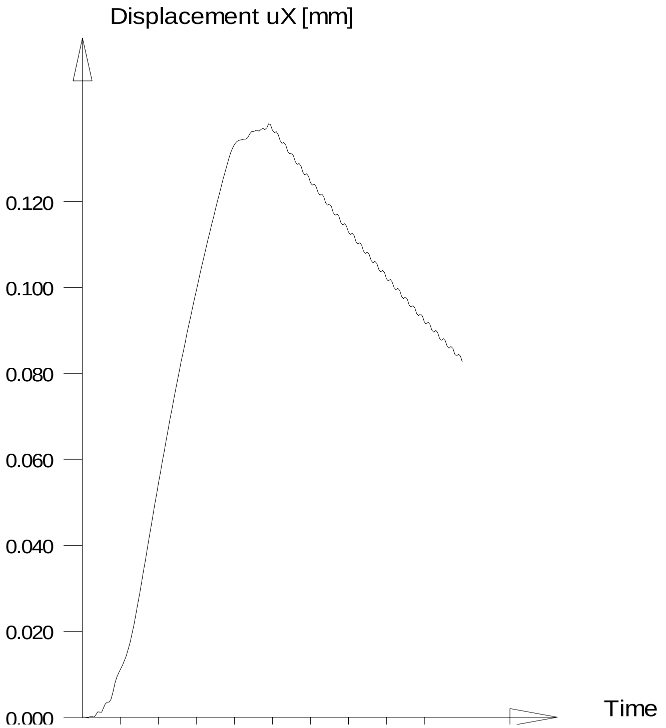
Non linear damping α = 0.2



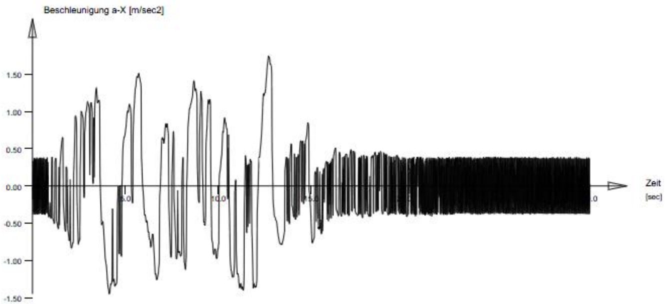


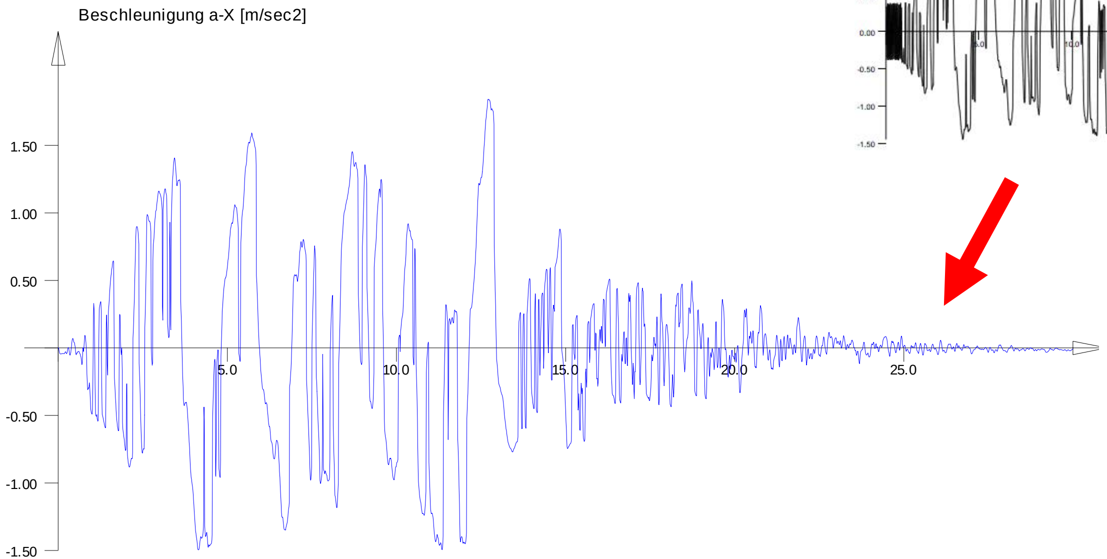
Excessive damping $\alpha = 0.2$





Apply damping only for $v>v_{min}$





Loading

- Constant or variant short time loading - e.g. Impact, discrete earthquakes

  $p(t)$ = discrete short time function

- Harmonic loading - e.g. Machinery

  $$p(t) = P \cdot \sin(\Omega \cdot t - \varphi)$$

- Spectral loading - e.g. Wind, Standard Earthquakes

  $S(\omega)$ = defined Spectra

How to solve dynamic problems

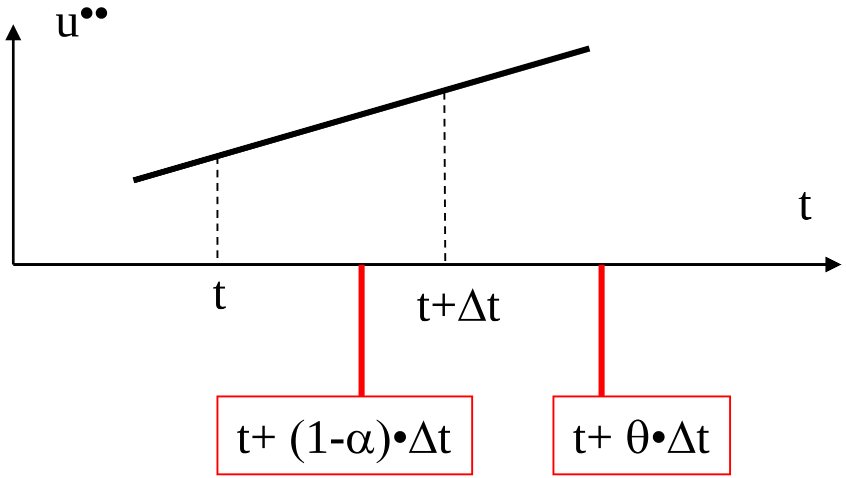
- Direct explicit Integration of the equations
  - Very fast (no matrices are assembled)
  - Allows any type of damping and non linearity
  - Requires very small time step
- Direct implicit Integration of the equations
  - Rather expensive (solving large sparse matrices)
  - Allows any type of damping and non linearity
  - Allows larger time steps
- Modal solution
  - Evaluation of Eigenvalues (costly)
  - Highly efficient and correct solution of equations
  - Harmonic Response very easy (Frequency domain)

Direct Integration

$$\ddot{u} = \ddot{u}(t_o) + \frac{(t - t_o)}{\Delta t} \cdot (\ddot{u}(t_o + \Delta t) - \ddot{u}(t_o))$$

- An assumption about the acceleration within time eg. as above, but other methods possible
- Solve system equations for a given time t
  - Solving for $t = t_o$ = explicit
  - Solving for $t > t_o$ = implicit
- Stability of implicit integration schemes ensures that numerical errors are reduced (damped) within the integration
- Time step should be smaller than 1/4th of highest frequency

Implicit Integration

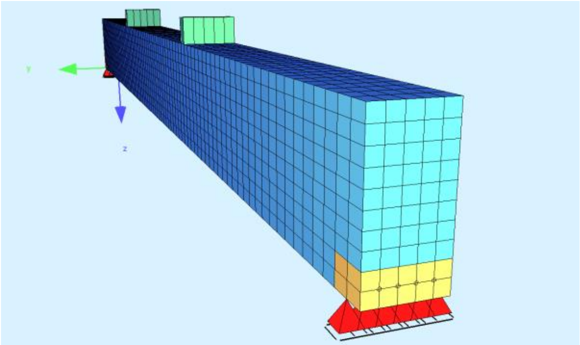


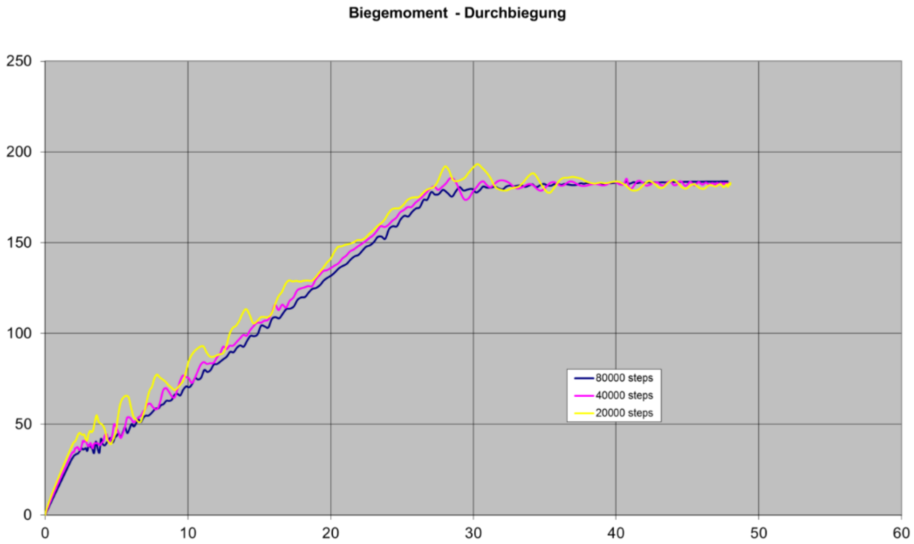
- Solve equations for a time in the future
  - at t + Δt = classical Newmark scheme
  - at t + θ• Δt = Wilson Theta scheme
  - at t + (1-α)• Δt = Hughes alfa scheme

Explicit Integration

- LS-DYNA / PAM-Crash etc.
  Standard method for large short time dynamics
- Using a diagonal mass matrix allows fast solution
- Constraints however
  - Time step has to be smaller than a stability limit
  - Every node needs a mass
  - Many time steps needed, so in general simplified one point integrated elements are used

FEX - Benchmark





Newmark Scheme

A

$$\dot{u}_{t+\Delta t} = \dot{u}_t + \Delta t\left[(1-\gamma)\ddot{u}_t + \gamma\ddot{u}_{t+\Delta t}\right]$$

$$u_{t+\Delta t} = u_t + \Delta t \cdot \dot{u}_t + \frac{\Delta t^2}{2}\left[(1-2\beta)\ddot{u}_t + 2\beta\ddot{u}_{t+\Delta t}\right]$$

U

$$\ddot{u}_{t+\Delta t} = \frac{1}{\beta\Delta t^2}\left[u_{t+\Delta t} - u_t - \Delta t \cdot \dot{u}_t - \frac{\Delta t^2}{2}(1-2\beta)\ddot{u}_t\right]$$

$$\dot{u}_{t+\Delta t} = \dot{u}_t + \Delta t(1-\gamma)\cdot\ddot{u}_t + \frac{\gamma}{\beta\Delta t}\left[u_{t+\Delta t} - u_t - \Delta t \cdot \dot{u}_t - \frac{\Delta t^2}{2}(1-2\beta)\ddot{u}_t\right]$$

Displacement based Integration

$$\left[K + \frac{\gamma}{\beta \Delta t} C + \frac{1}{\beta \Delta t^2} M\right] u_{t+\Delta t} = F_{t+\Delta t} - C\tilde{\dot{u}}_t - M\tilde{\ddot{u}}_t$$

$$\tilde{\dot{u}}_{t+\Delta t} = \frac{\gamma}{\beta \Delta t} \cdot u_t + \left(\frac{\gamma}{\beta} - 1\right) \cdot \dot{u}_t + \Delta t \left(\frac{\gamma}{2\beta} - 1\right) \cdot \ddot{u}_t$$

$$\tilde{\ddot{u}}_{t+\Delta t} = \frac{1}{\beta \Delta t^2} \cdot u_t + \frac{1}{\beta \Delta t} \cdot \dot{u}_t + \left(\frac{1}{2\beta} - 1\right) \cdot \ddot{u}_t$$

- Establish equation for u at t+Δt
- Right-Hand Side with M,C-Matrix only
- Thus numerical effort is favourable

Acceleration based Integration

$$\left[M + \gamma \Delta t C + \beta \Delta t^2 K\right] \ddot{u}_{t+\Delta t} = F_{t+\Delta t} - C \tilde{\dot{u}}_t - K \tilde{u}_t$$

$$\tilde{\dot{u}}_{t+\Delta t} = \dot{u}_t + \Delta t (1-\gamma) \ddot{u}_t$$

$$\tilde{u}_{t+\Delta t} = u_t + \Delta t \cdot \dot{u}_t + \frac{\Delta t^2}{2} (1-2\beta) \ddot{u}_t$$

- Predictor values of velocity and displacements
- Establish equation for acceleration at t+Δt
- Right-Hand Side with C- and K-Matrix
- Thus, numerical effort is higher

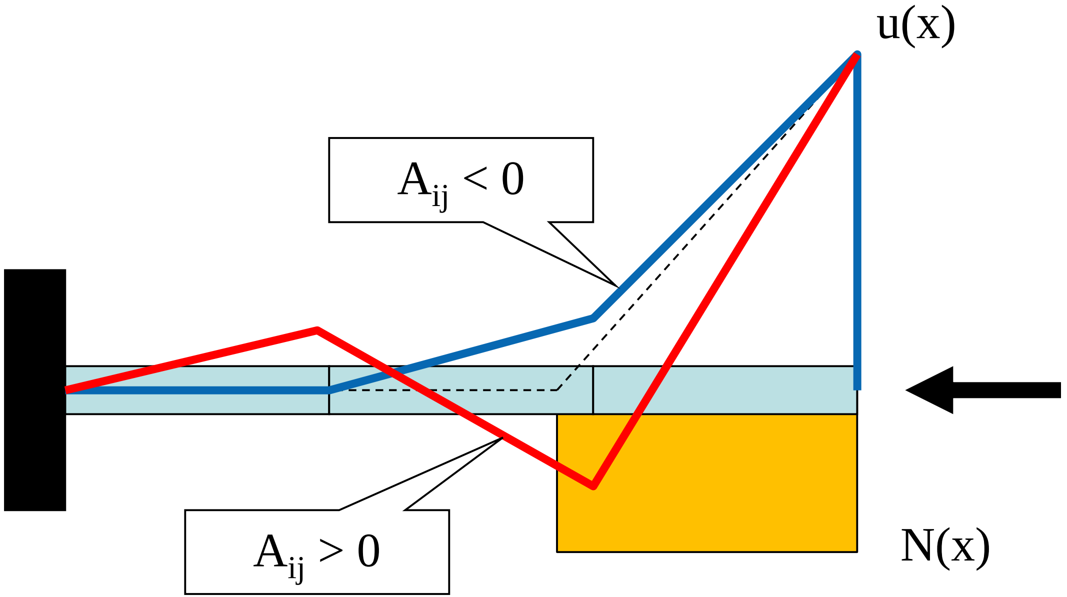
Discrete Maximum Principle

- The off diagonal terms of the stiffness matrix have a negative sign
- The off diagonal terms of the mass matrix have a positive sign
- When combining both matrices to a system of equations:

$$\underline{S} = \left[\underline{K} + \frac{1}{\Delta t^2} \underline{M} \right] = \begin{bmatrix} +\frac{EA}{l} & -\frac{EA}{l} \\ -\frac{EA}{l} & +\frac{EA}{l} \end{bmatrix} + \frac{1}{\Delta t^2} \begin{bmatrix} \frac{A\rho l}{3} & \frac{A\rho l}{6} \\ \frac{A\rho l}{6} & \frac{A\rho l}{3} \end{bmatrix}$$

- For a certain value of Δt the off diagonal terms change sign, giving rise to oscillations in the solution
- Remedy: use lumped matrices

Discrete Maximum Principle

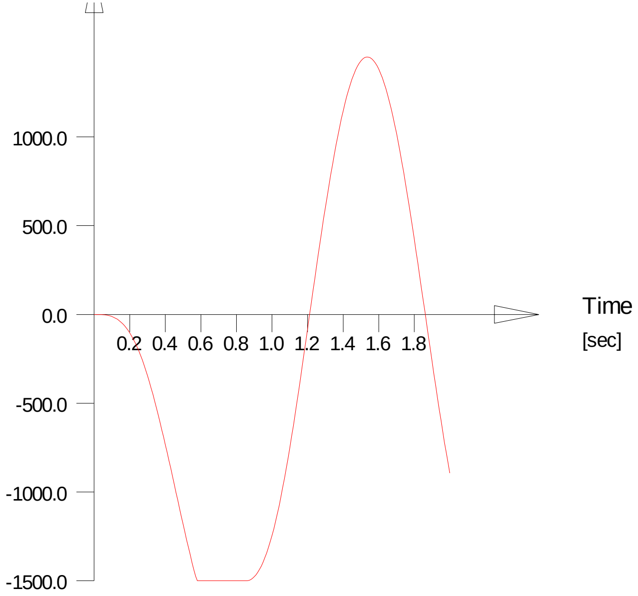


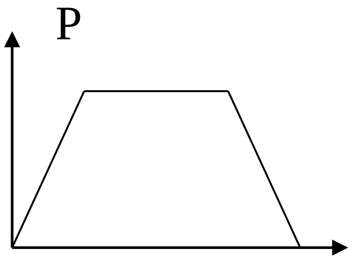
Non linear dynamics

- Most stability criteria are only valid for linear dynamics
- There are examples (pendulum) showing that Newmark schemes may fail for non linear analysis
- Main Problem:
  It is very difficult to establish a nonlinear implicit method
  - Thus use explicit methods or very small time steps
  - Use Iterations during each time step
  - Extrapolate non linear effects for the implicit methods with simple functions.

Example Yielding of a Spring

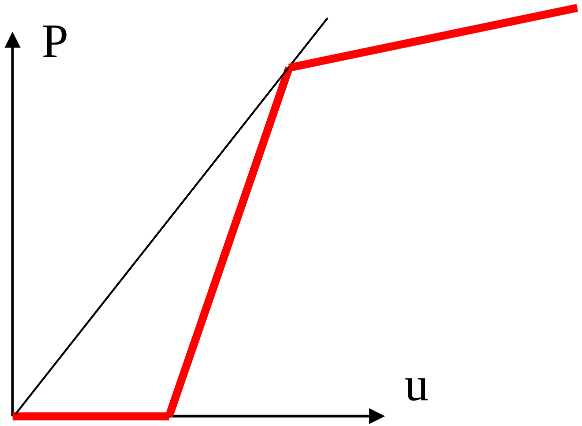
- SPRI CP 1000000. DP 1000. YIEL 1500





Contact of a Spring (GAP)

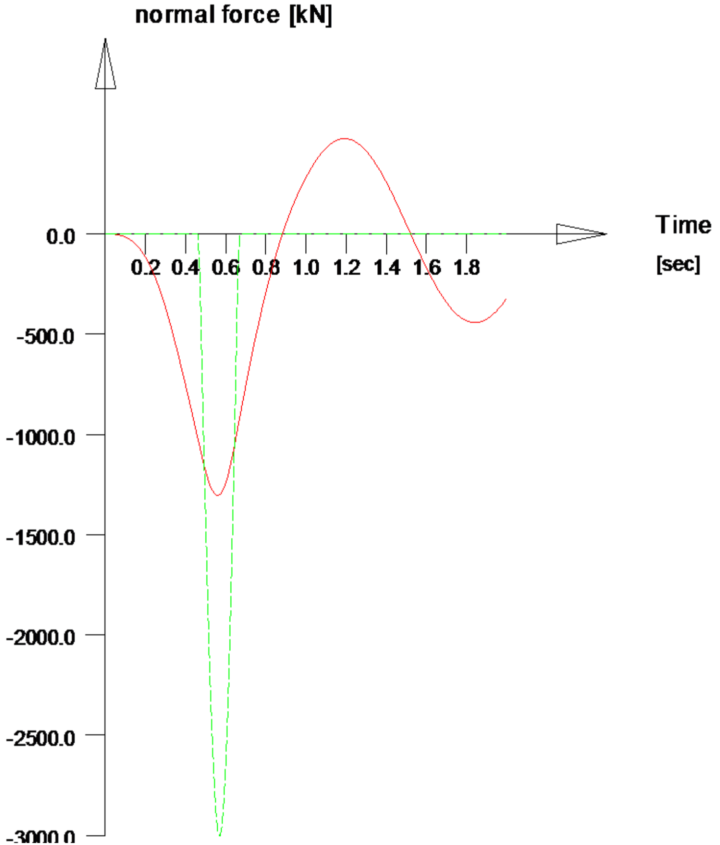
- General Problem: Choose initial stiffness
  - Stiffening is not always stable
  - Softening may need a large number of iterations
  - Good convergence only with Iteration / Line Search



Rather Soft Impact

SPRI 1 CP 1000000.
DP 10000.

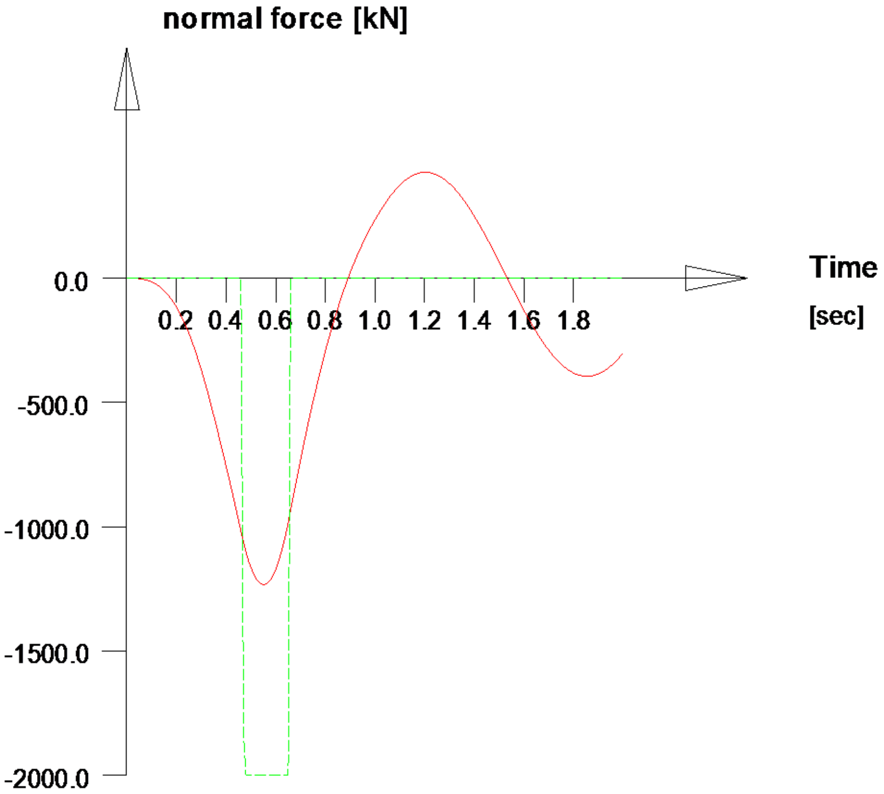
SPRI 2 CP 1E7
GAP 0.001



Soft Impact

SPRI 1 CP 1000000.
DP 10000.

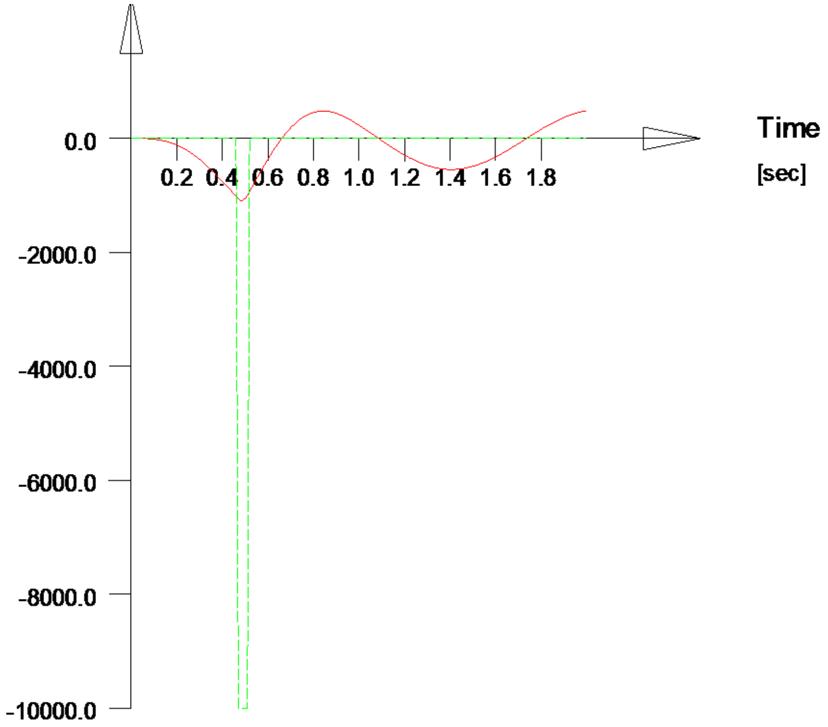
SPRI 2 CP 1E8
GAP 0.001
YIEL 2000



Hard Impact

SPRI 1 CP 1000000.
DP 10000.

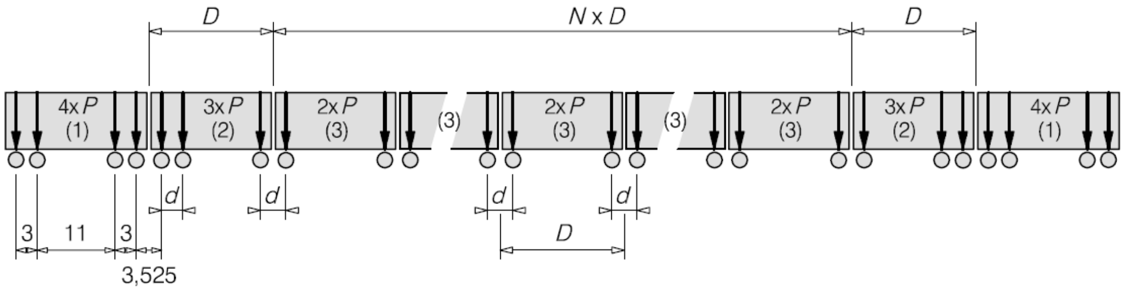
SPRI 2 CP 1E10
GAP 0.001
YIEL 9999



Train Bridge Interaction

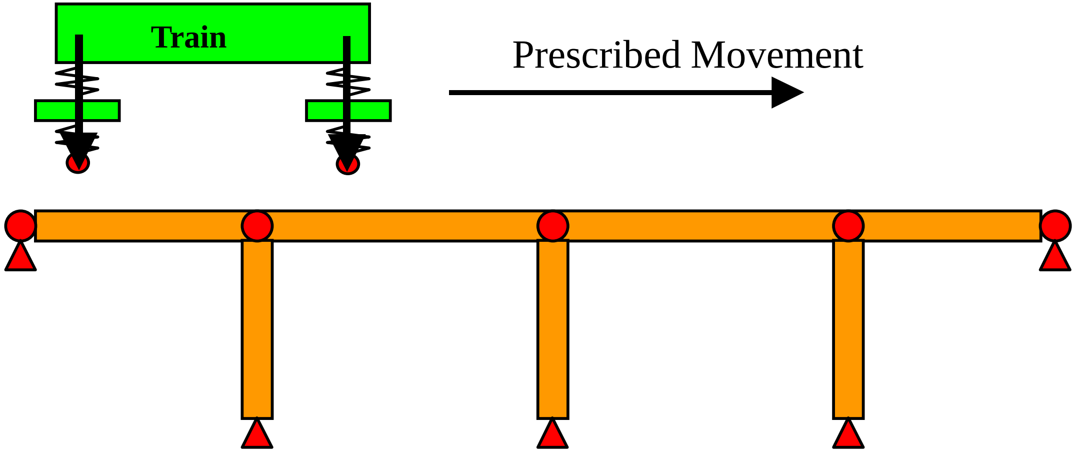
- Dynamic effects caused by a train passing a bridge
  - Resonance of Bridge (short bridges)
  - Fatigue of bridge
  - Comfort of passengers
  - Derailment / Stability of Train
- Somehow a matter of mode
  - Nobody knows the problem in detail
  - So we have to do measurements
  - And analysis ?!

Train Bridge Interaction



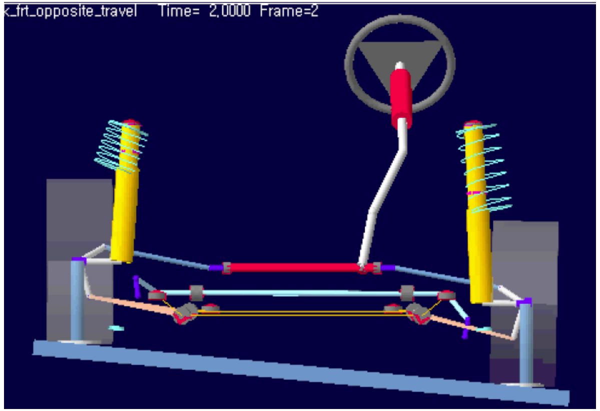
- Which train ?
  - HSLM A1 to A 10
  - UIC
  - ICE2, ICE3, Thalys etc.
  - DIN Fachbericht / Eurocode: Fatigue relevant Trains
- How fast ?
- Bogie, spring and damper properties ?

Task description



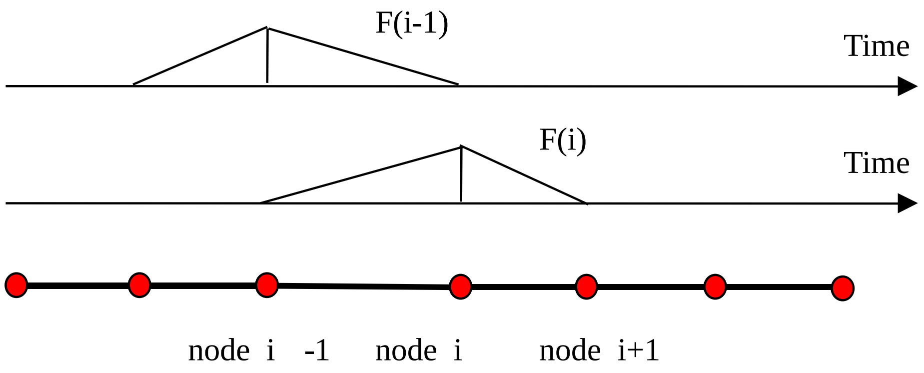
Classification of possible solutions

- Bridge with moving loads
  => classical FE-Dynamics
- Train with prescribed track or pavement
  => classical Multi body dynamics
- Coupling of an elastic FE - Structure with a Multi-Body-Dynamic-Program
- Simulation of two FE-Systems for the train and the bridge with a classical FE-Program



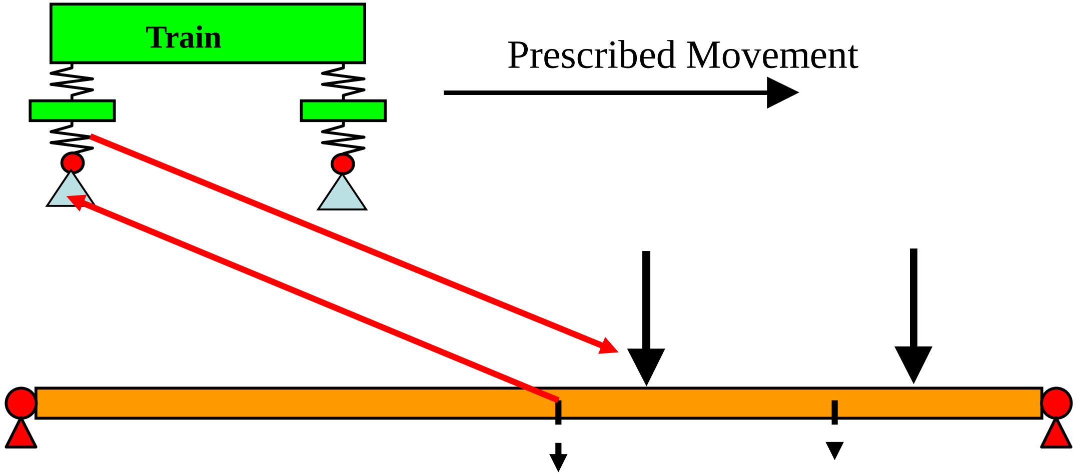
ADAMS
DADS
SIMPACK
RecurDyn
....

The moving load



- For each node and each load there is a function describing which part of the load is acting on which node along the time axis.
- Many nodes times many loads => many time functions
- Automatic realisation possible along a sequence of nodes

Contakt node / Contact point

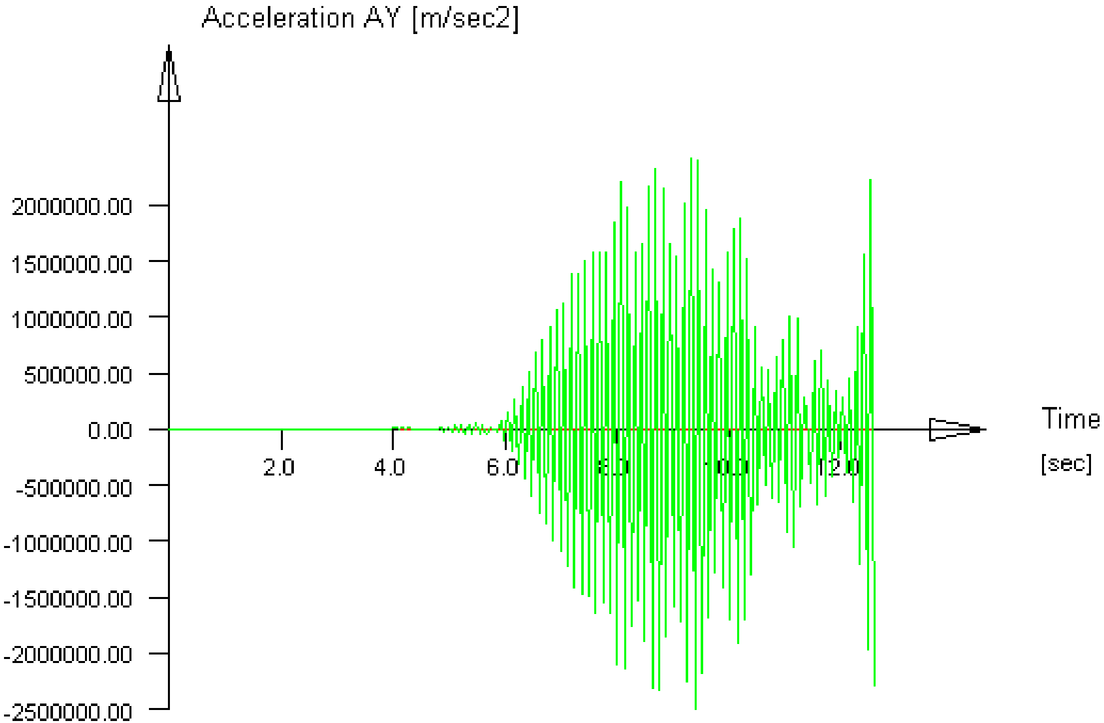


- Deformation in contact point => Prescribed displacement for wheel contact node
- Force in contact spring => Loading for next time step
- Problem: different time values = integration is not consistent
  principal problem also for Multi-Body-Dynamics

Special Problems

- Wheel-Rail-contact
  Hertz Pressure and much more
- Spring and damper properties for primary and secondary spring
- Non linear damper and spring behaviour
- Tilting and Transverse forces
- Bogie as separate body or subdivide it for each axle ?
- Track irregularities / Roughness of pavement easy to include but not available easily.
- Impact if a load enters a predeformed structure
- Centrifugal forces

And what’s that ?



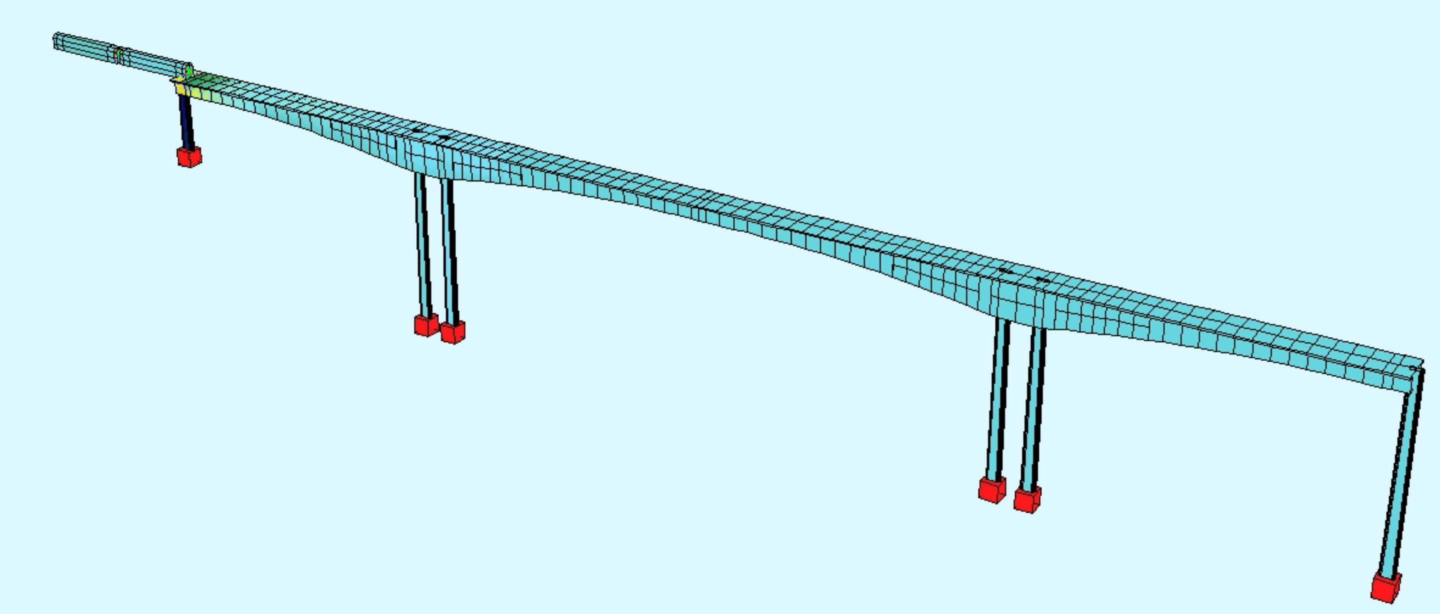
Solutions possible

- Change the integration parameters for the Newmark-Method (delt>0.5)
- Wilson – Theta method
- Hilber-Hughes-Taylor-Alpha-Method
  Should work without numerical damping and is specially suited for non linear dynamics. However there is an higher effort compared with the two methods above

Comparison

Integration scheme	a - wheel	a - coach
Newmark $\delta = 0.5$	4612.47	0.66
Newmark $\delta = 0.6$	27.17	0.54
Newmark $\delta = 0.7$	10.96	0.52
Newmark $\delta = 0.8$	6.41	0.44
Hilber-Hughes Taylor $\alpha = -0.05$	54.31	0.64
Hilber-Hughes Taylor $\alpha = -0.1$	28.07	0.52
Hilber-Hughes Taylor $\alpha = -0.2$	11.78	0.48
Hilber-Hughes Taylor $\alpha = -0.3$	7.96	0.51
Wilson Theta $\Theta = 1.4$	3.59	0.43

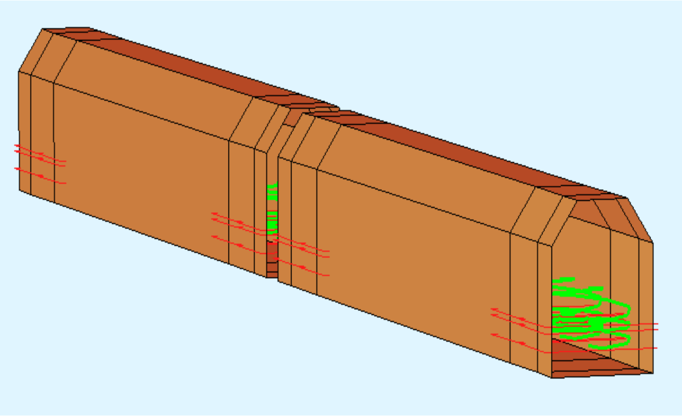
Example



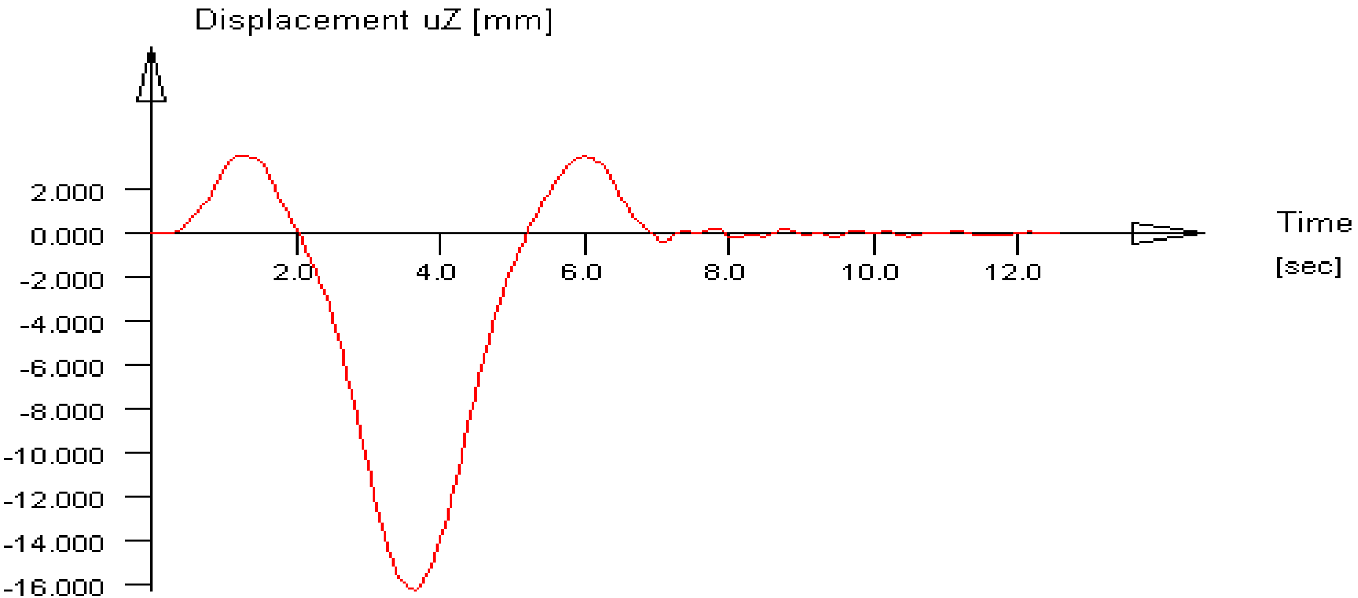
3 spans of 88 / 160 / 88 m Height of pylons 18 / 46 m
Boxed girder sections with heights between 4.5 and 10.5 m
vertical Eigen-frequencies 0.88, 1.67, 2.22 Hertz, transverse from 0.30 Hertz

Train

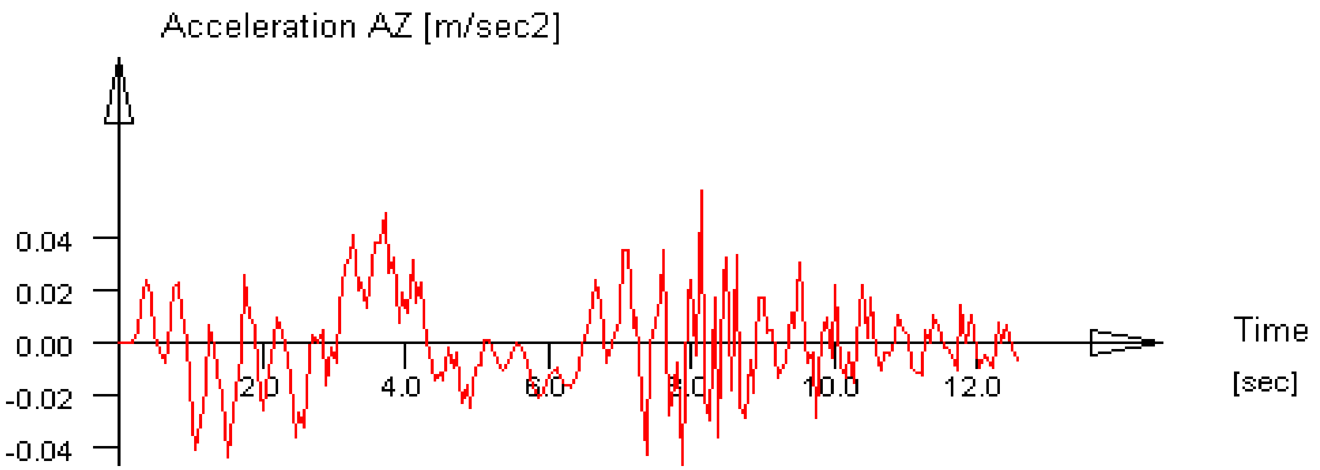
- Two coaches with 25.50 m length with two bogies and four axis.
- Damping properties provided for two velocities => damping with exponential law
- For the safety against derailment one additional torsional spring+damper was sufficient, to get the forces for the individual wheel.
- Speed was 200 km/h.



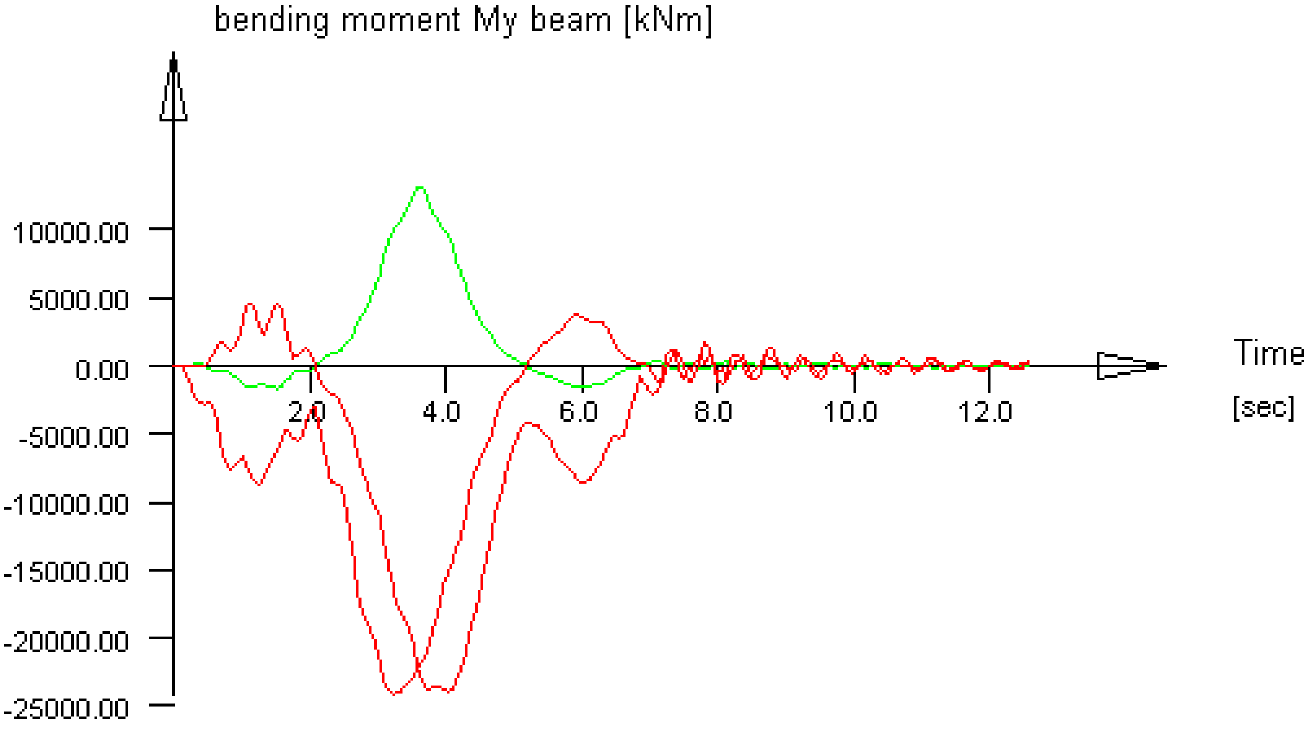
Moving Loads



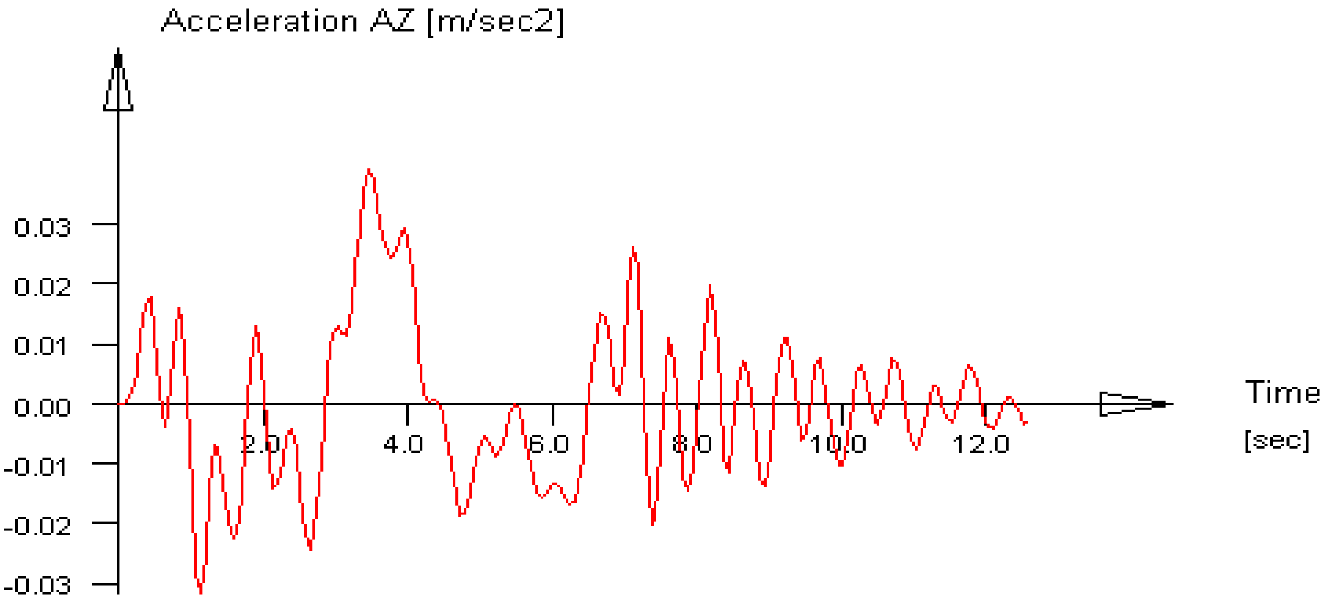
Moving Loads - Acceleration of bridge



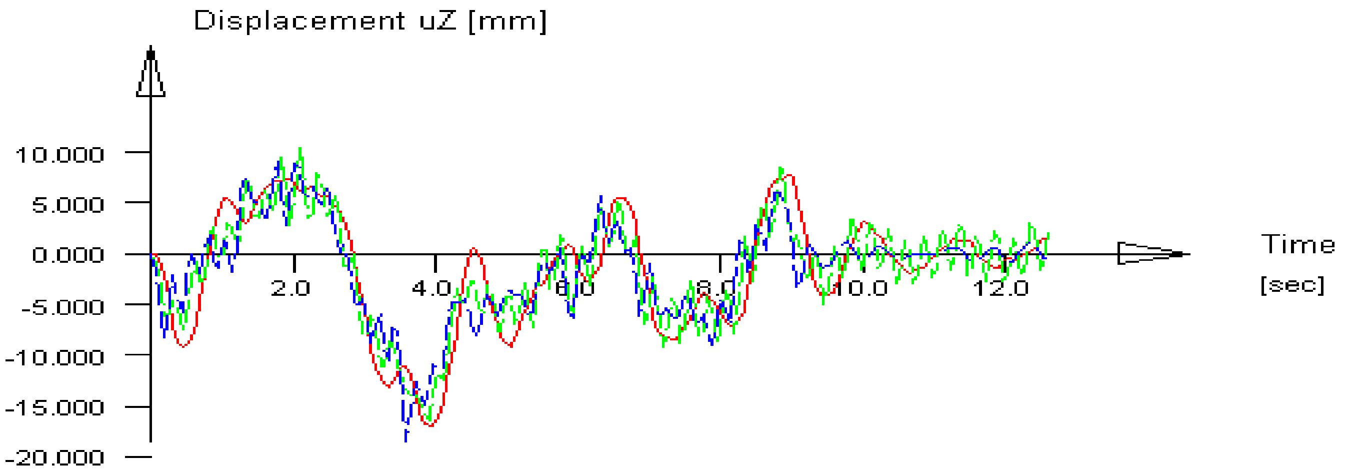
Moving Loads



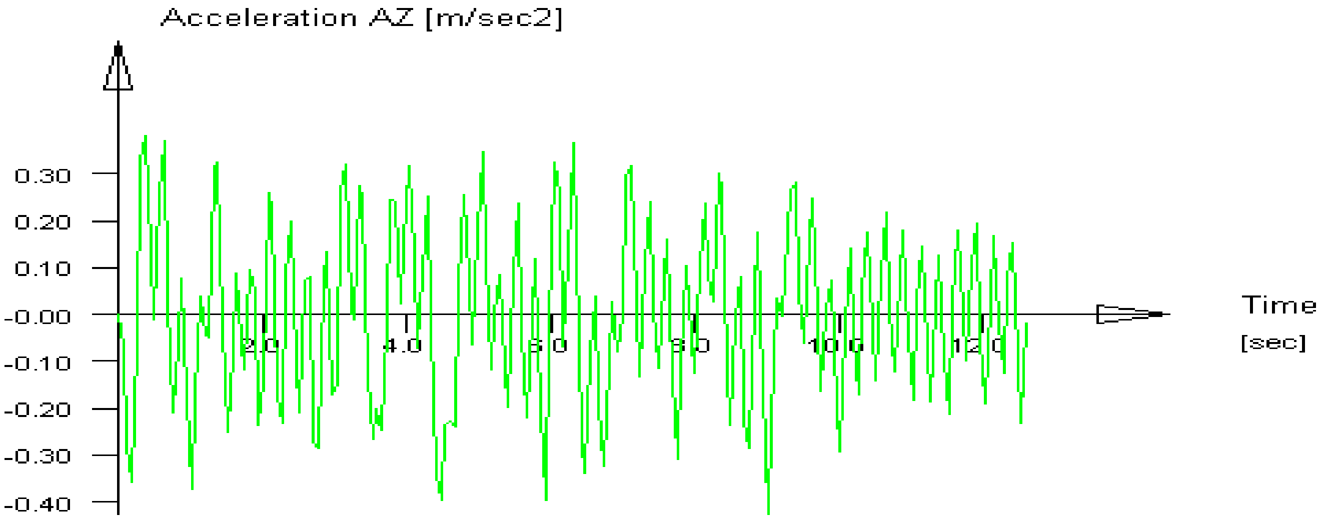
Train + Bridge - Acceleration of Bridge



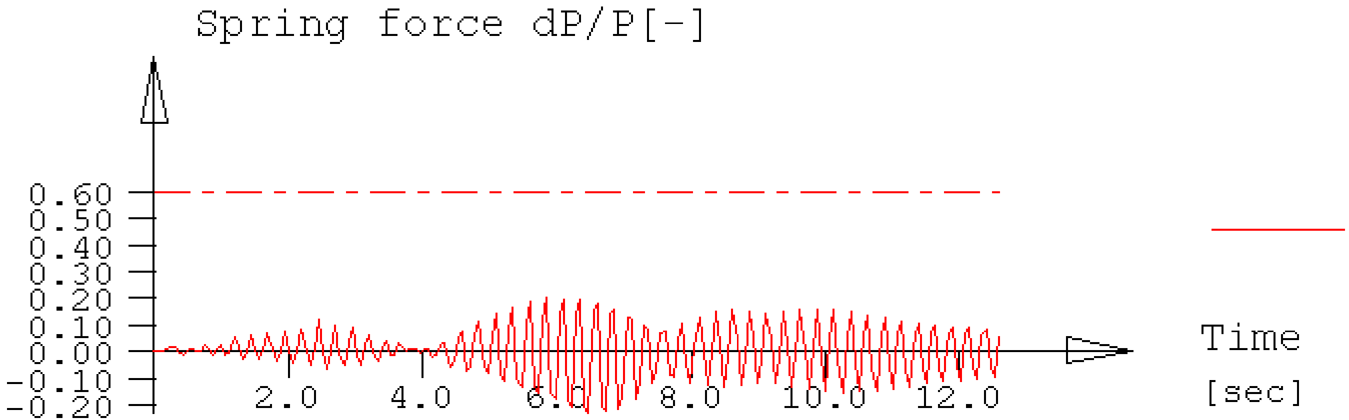
Displacements for the train



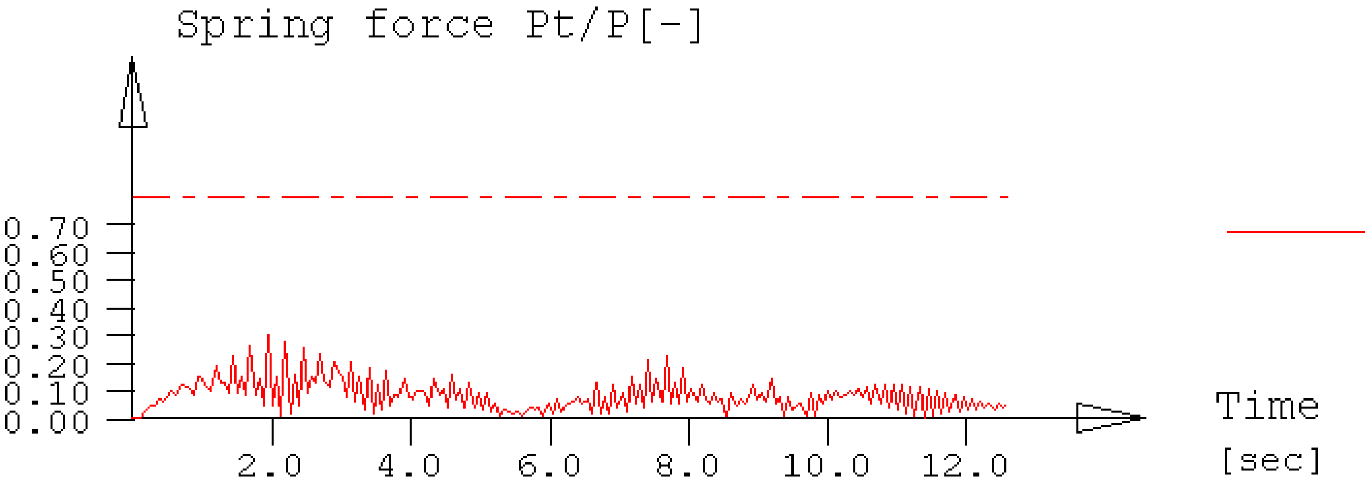
Accelerations for the train



Variation of contact force



Derailment coefficient



Eigenvalues and Modal analysis

- Eigen frequency or frequency of maximum response (resonance) ?
- Real or complex Eigenvalues ?
- What are Eigen forms ?
- What are Buckling Eigenvalues
- General mathematical property of a matrix
- Generalized Eigenvalue-Problem

$$A \cdot X + \lambda \cdot B \cdot X = 0$$

Eigenvalue (→ λ)

Eigenform (→ X)

General Dynamic Eigenvalue problem

$$m \cdot u^{\bullet\bullet} + \lambda_i \cdot c \cdot u^{\bullet} + \lambda_r \cdot k \cdot u = 0$$

- There exist non vanishing solutions for this homogeneous equation leading to a general complex eigenvalue problem.
- Damping c may be neglected in a first step or to be accounted for via a complex eigenvalue solver.
- There exist as many Eigenvalues as the rank of the matrices (but some may be zero)
- Each solution has an Eigenvalue λ_i (but there might be duplicate Eigenvalues)
- Each Eigenvalue has an associated Eigenform Φ_i

Solution strategies

- Solve the Eigenvalue
  - » Solve the characteristic equation for the Eigenvalues
  - » After having obtained λ calculate the Eigenforms (selecting one "arbitrary" entry of X to be 1.0)
- Solve for the Eigenforms and calculate the Eigenvalues from the Eigenforms based on the Energy (Ritz)

$$\lambda = \frac{X^T \cdot A \cdot X}{X^T \cdot B \cdot X}$$

- In all cases the Eigenforms have to be scaled
  - » Either: maximum value = 1.0
  - » Or: Resulting $X^T \cdot B \cdot X$ to 1.0

Eigenvalue Solvers for $Ax = \lambda \cdot Bx$

- General Assumption: A,B positive definite (not always true)
- Classical complete methods (Cyclic Jacobi-Method / Householder)
- Vector-Iteration (gives the lowest eigenvalue): $Ax^{(k)} = \lambda \cdot Bx^{(k-1)}$
- Simultaneous Vector iteration in subspaces
  - Solves for positive and negative eigenvalues
- Lanczos method (a direct solution)
  - Generally the fastest, fails for negative Eigenvalues
- Bisection methods
  - If only a certain range of values is required (not in SOFiSTiK)
- Rayleigh-Quotient-Method
  - Makes best use of iterative Solver, positive eigenvalues only

Transformations

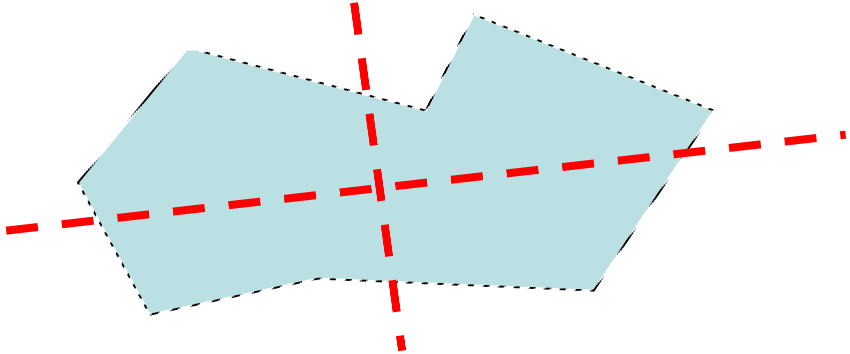
- Any rotated coordinate system may be used to describe a physical or mathematical behaviour:

$$u_{local} = [T] \cdot u_{global}$$

$$u_{global} = [T]^T \cdot u_{local}$$

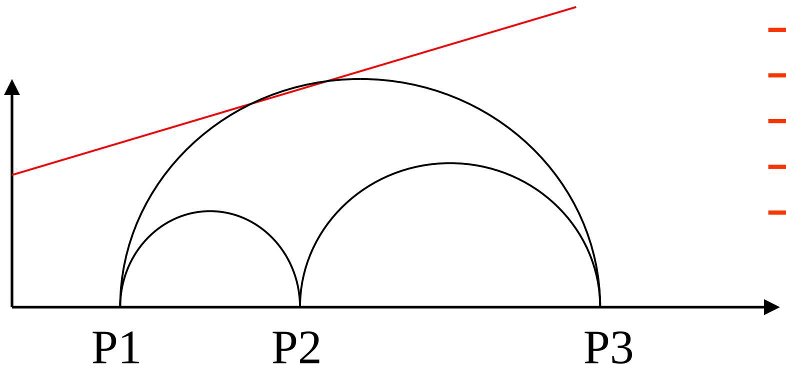
- For a set of n variables the matrix T needs rank n
- Useful properties are:
  - The determinant has the value 1
  - Each row/column has a norm of 1
  - Each row is orthogonal to each other
- Otherwise the transposed is not the inverse!

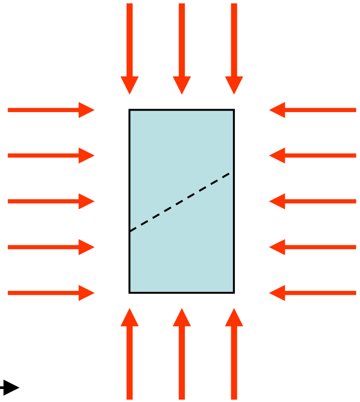
Usefull transformation: Eigenvalues of an Area



- Principal axes of inertia of a cross section
    - The axes are orthogonal
    - The inertias have the lowest and largest value
    - The off diagonal terms are zero
    - The mathematical treatment is simplified by using the bending for the principal axes.

Usefull transformation:
Eigenvalues of a 3D Tensor





- Principal Stresses
- The stresses P1,P2,P3 have the extreme values
- Off diagonal terms (shear) vanishes in principal directions
- The directions of the P-stresses are orthogonal
  but the direction is not of an absolute order.

Transformation to the base of Eigenvectors

- The Eigenforms are a good base of an vector space, as they are orthogonal to each other
- So it is possible to transform the system of equations into that space by two transformations
- As the scaling of the Eigenforms is arbitrary, it is very useful to scale them properly

$$u_{global} = [\Phi] \cdot u_{eigen}$$

$$f_{global} = A \cdot u_{global} = A \cdot [\Phi] \cdot u_{eigen}$$

$$f_{eigen} = [\Phi]^T \cdot f_{global} = [\Phi]^T \cdot A \cdot [\Phi] \cdot u_{eigen}$$

A special kind of transformations

- A transformation may reduce the number of unknowns:
  (e.g. a kind of a 2D shadow instead of a 4D solid)

$$u_{local} = \begin{bmatrix} t_{11} & t_{12} & t_{13} & t_{14} \\ t_{21} & t_{22} & t_{23} & t_{24} \end{bmatrix} \cdot u_{global}$$

$$u_{global} = [T]^T \cdot u_{local}$$

- Some Information will be lost by this transformation
- However it is possible to transform in such a way that the most important information is kept. (e.g. 2D drawings of 3D buildings)

Transformed Equations in the Eigenform Space

$$M \cdot \ddot{u}_{eigen} + C \cdot \dot{u}_{eigen} + K \cdot u_{eigen} = f_{eigen}(t)$$

$$f_{eigen}(t) = [\Phi]^T \cdot p(t)$$

$$M = [\Phi]^T \cdot m \cdot [\Phi] = Diag(1)$$

$$K = [\Phi]^T \cdot k \cdot [\Phi] = Diag(\omega^2)$$

$$C = [\Phi]^T \cdot c \cdot [\Phi] = ?$$

Modal Space or Sub-Space

- Mass and Stiffness matrix are pure diagonal matrices
- It is very often sufficient to use only a few (the lowest eigenvalues) The modal loads may be used for a constant acceleration to show the effectiveness of the subspace
- The C-Matrix is only diagonal for a complex Eigenvalue analysis, or if the damping is a linear combination of the mass and stiffness matrix (Rayleigh-Damping Coefficients)
- The damping is not very well known for most practical problems!

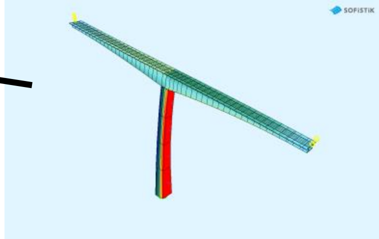
Modal damping example

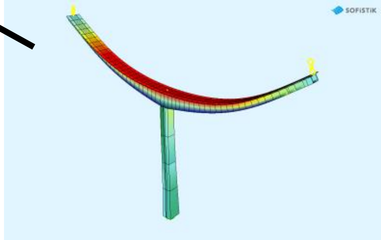
Eigenvalues

No.	LC	λ [rad2/sec2]	error [-]	ω [rad/sec]	f [Hz]	T [sec]	ξ [o/o]	Meff X[o/o]	Y[o/o]	Z[o/o]
1	1	8.6226E-01	0.00E+00	0.929	0.148	6.766	0.774	41.6	0.0	0.0
2	2	1.0219E+00	0.00E+00	1.011	0.161	6.216	0.716	0.0	0.1	0.0
3	3	1.9093E+00	0.00E+00	1.382	0.220	4.547	0.621	0.0	75.1	0.0
4	4	9.0399E+00	0.00E+00	3.007	0.479	2.090	0.540	40.0	0.0	0.4
5	5	1.7350E+01	0.00E+00	4.165	0.663	1.508	0.483	0.4	0.0	28.4
6	6	2.2121E+01	0.00E+00	4.703	0.749	1.336	0.534	0.0	1.1	0.0
7	7	1.0684E+02	0.00E+00	10.336	1.645	0.608	0.980	0.9	0.0	1.7
8	8	1.3609E+02	0.00E+00	11.666	1.857	0.539	1.058	0.0	0.0	0.0
9	9	1.6463E+02	0.00E+00	12.831	2.042	0.490	0.827	0.0	0.0	47.2
10	10	1.9545E+02	0.00E+00	13.980	2.225	0.449	1.484	0.0	14.4	0.0
11	11	4.2102E+02	2.44E-08	20.519	3.266	0.306	1.227	0.5	0.0	19.1
12	12	4.2179E+02	3.89E-07	20.538	3.269	0.306	1.785	0.0	0.0	0.0
13		4.7088E+02	3.23E-08	21.700	3.454	0.290	Σ(Meff)¹	83.4	90.7	96.8
14		8.1033E+02	1.70E-03	28.466	4.531	0.221				

¹ Total effective mass in X-, Y- and Z-direction.

No.	eigenmode number	f	eigenfrequency
LC	load case	T	eigenperiod
λ	eigenvalue	ξ	modal damping ratio
error	relative eigenvalue error	Meff	effective modal mass in X-, Y- and Z-direction
ω	circular eigenfrequency		





Special Remarks

- Every Eigenvalue has its associated Eigenform.
- As the Eigenforms are displacements, any derived information (velocities, accelerations, stresses, sum of reactions may be calculated from those basic values)
- If we have the modal contribution (i.e. the modal response to the modal load) we have a scaling factor for all other results.

Zero Eigenvalues

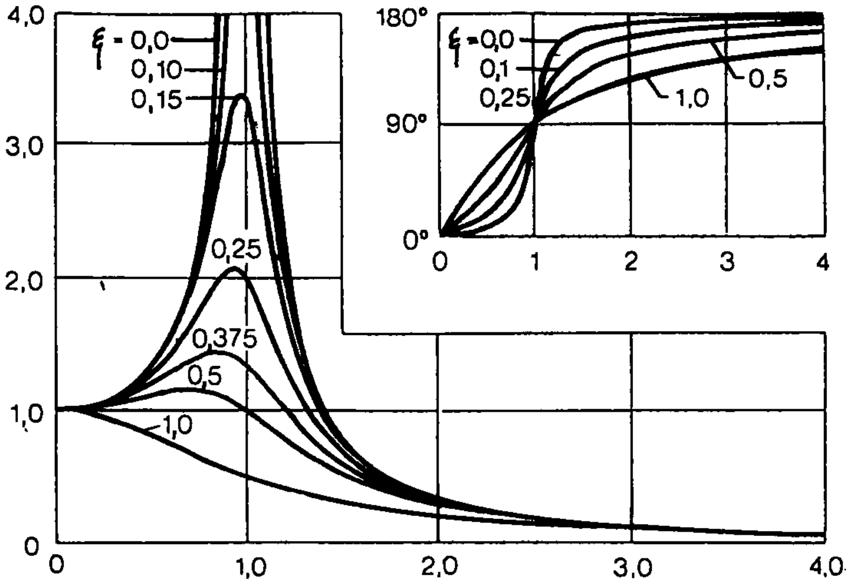
- It happens (input error) or is intended (airplane) to have zero Eigenvalues (= rigid body modes)
- This is a problem for many Eigenvalue solver strategies
- It can be overcome with an Eigenvalue shift

$$A \cdot x = \left(\bar{\lambda} + \lambda_0\right) \cdot B \cdot x$$

$$\left[A - \lambda_0 \cdot B\right] \cdot x = \bar{\lambda} \cdot B \cdot x$$

$$\lambda = \bar{\lambda} + \lambda_0$$

SDOF-Response



- Eigenfrequency ω
- Forced vibration with a sine of frequency Ω (The response has the frequency of the loading Ω !)
- Jump / Transition of phases from high tuning ($\omega > \Omega$, Loading and response have similar directions) to low tuning ($\omega < \Omega$, Loading and response have opposite directions)
- In case of a resonance we have an phase of T/4.

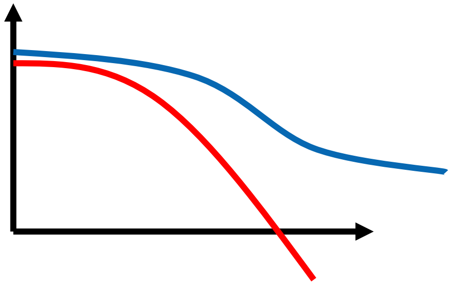
Impedances / Impedance Functions

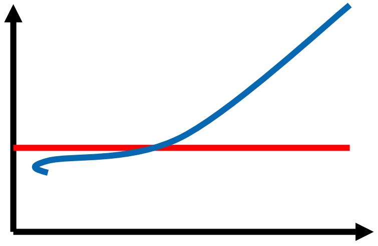
- Dynamic response in complex notation

$$u(t) = A \cdot e^{i(\Omega t - \varphi)} = \mathrm{Re}(u) + i \cdot \mathrm{Im}(u)$$

- Dynamic stiffness with real and imaginary part (

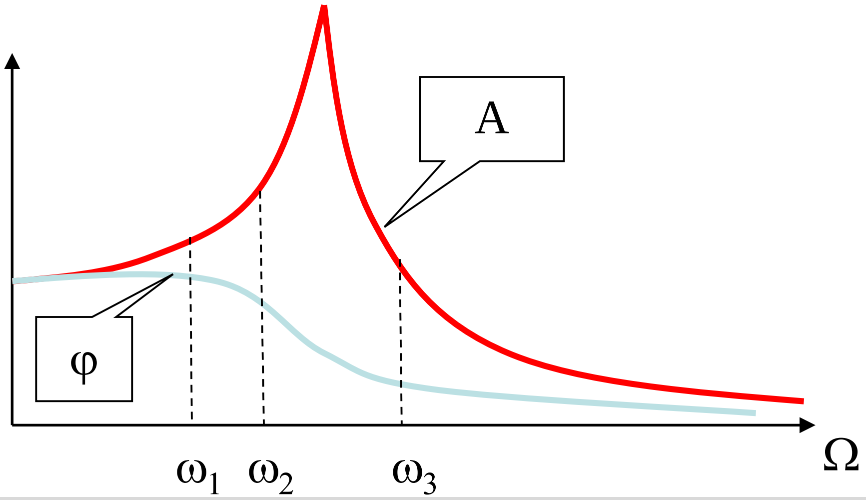
$$K = \frac{P(t)}{u(t)} \quad ; \quad P(t) = K\left[1 + 2\xi i\right] \cdot u(t)$$





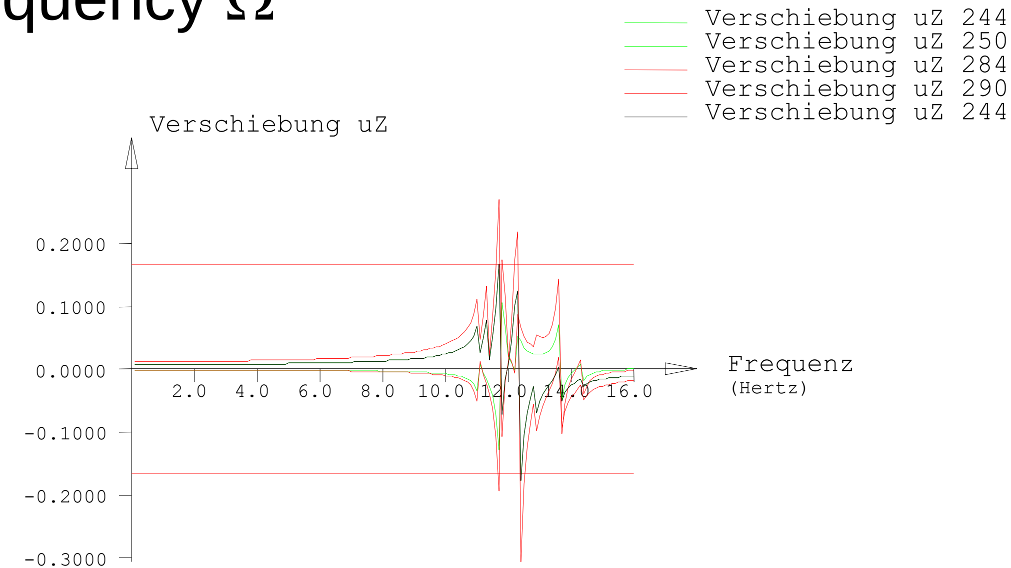
Harmonic response of a MDOF

- Enforced Vibration with a Frequency Ω
- All Eigenforms have an amplitude A, a phase angle φ, but the same frequency Ω
- The total response is the sum of all eigenforms

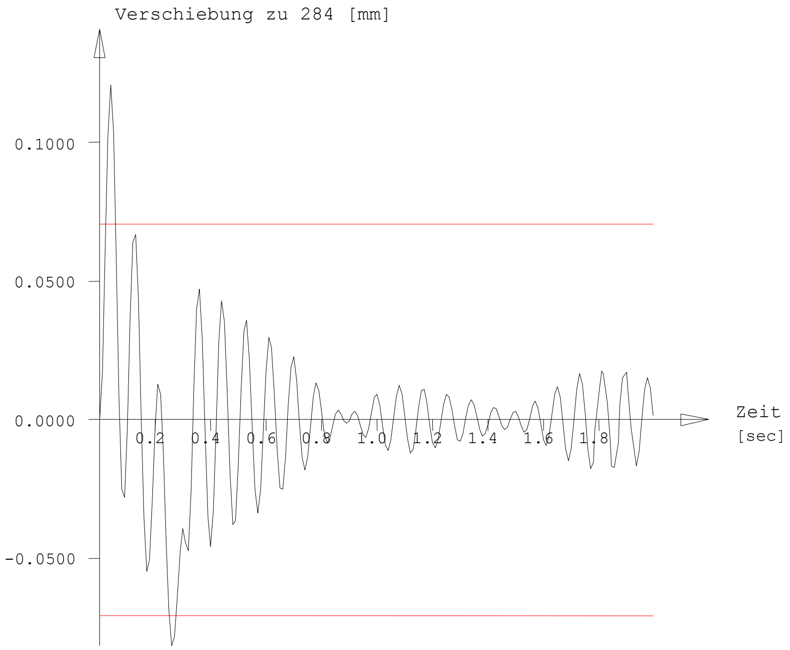


Impedance Functions

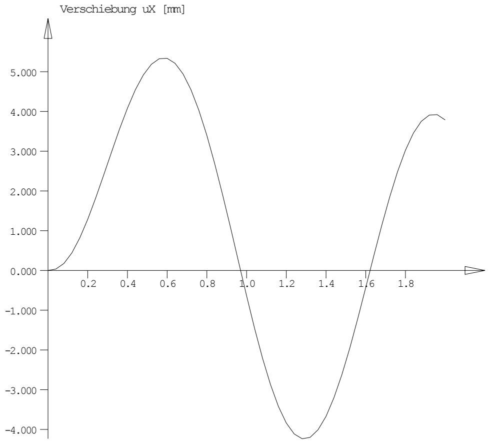
- A function giving the response as a function of the frequency Ω

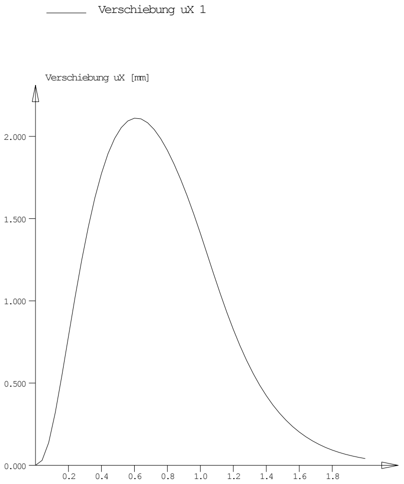


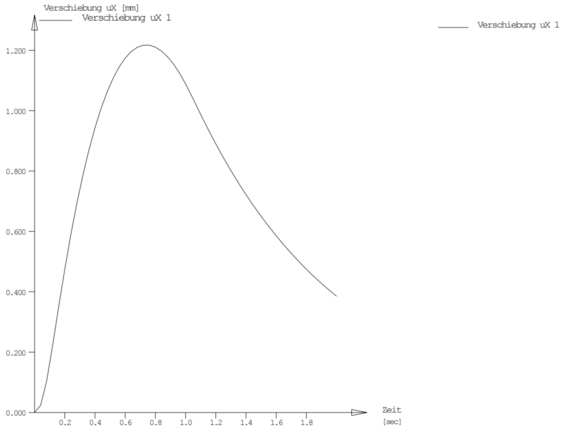
Transient explicit loading



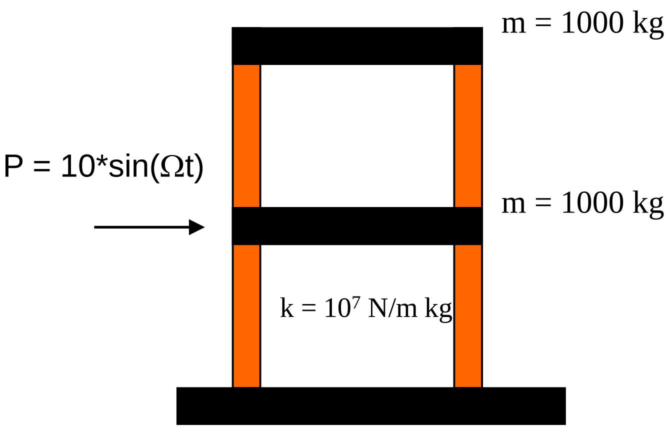
Critical Damping $\zeta = 1.0$







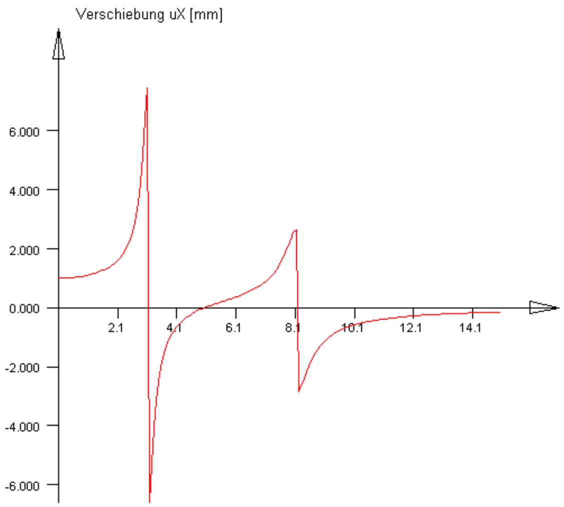
2 DOF System

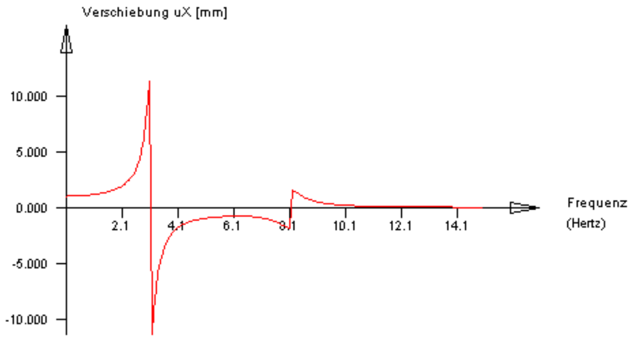


- Eigenforms have the same external Frequency Ω but different modal factors an especially different phases
- Simple phases 0 / 180 degree introduced by sign of modal factors
- Maximum Response may be obtained by a time window of the steady state response
- Maximum Response may be obtained by some mathematics (derivative = zero)

Frequency response for the two storeys

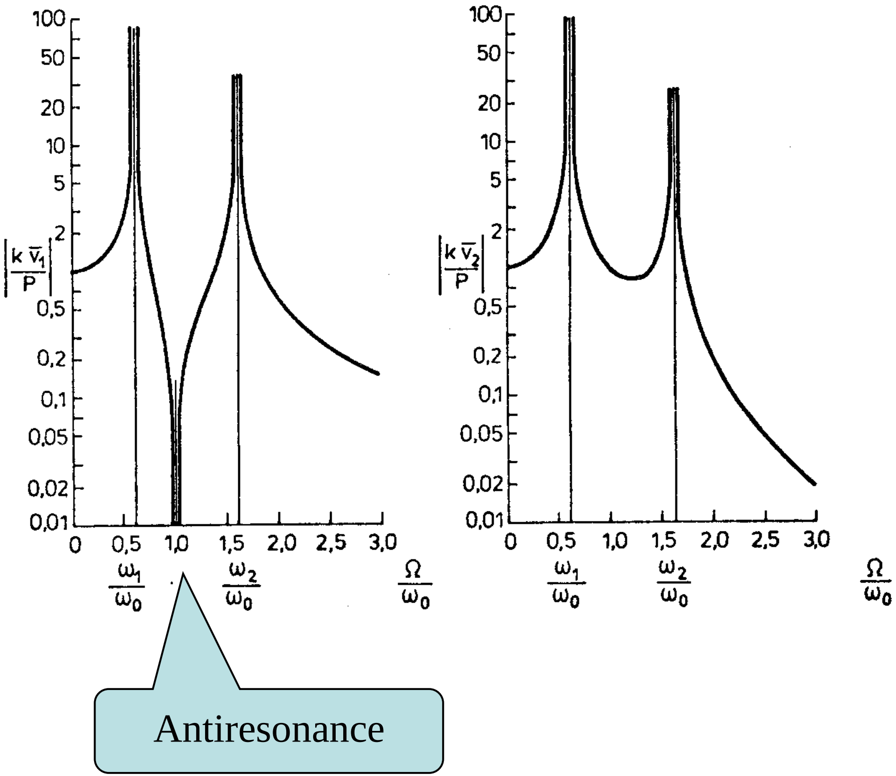
- Response based on simplified phases



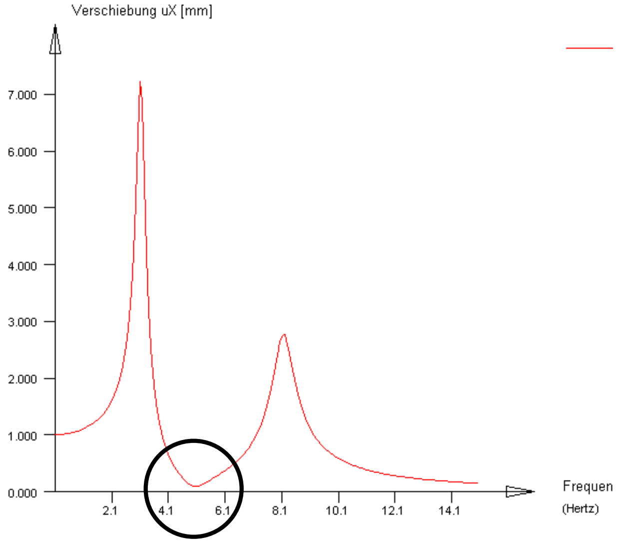


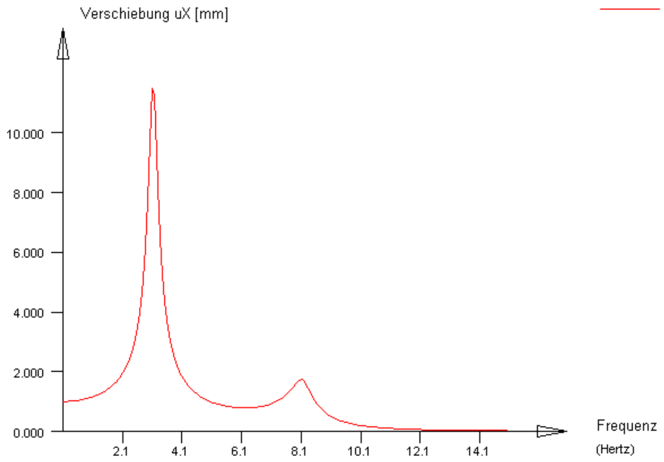
Zero values in Response ?

- It is considerably more effort to account for the correct phases.
- Simulation of Phases with a sign is appropriate for small damping.
- Reference solution available (no Damping):

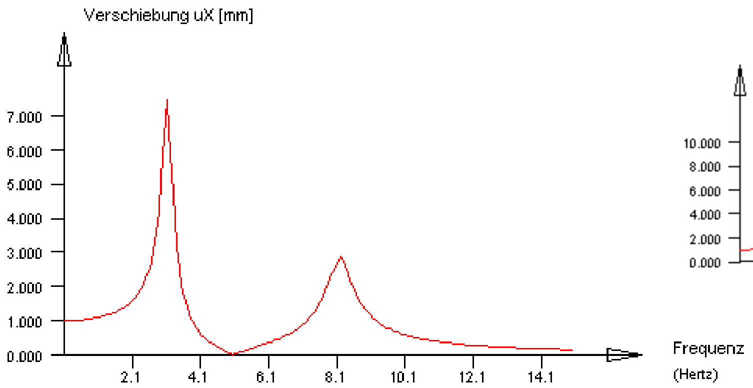


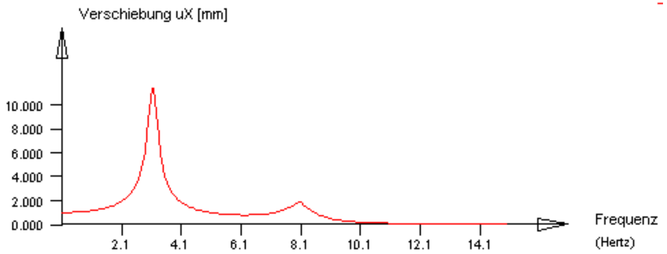
Exact Response





Comparison

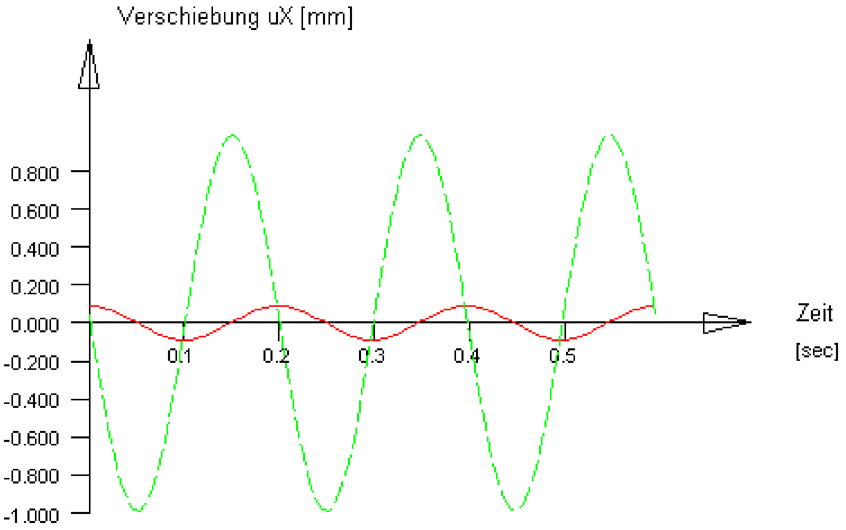




	u-2	u-3
Simplified Phase	7.456 mm	11.344 mm
Exact Phase	7.226 mm	11.491 mm

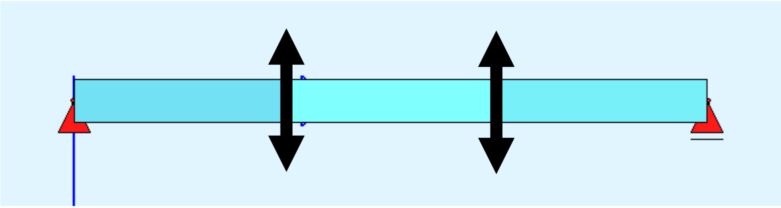
Antiresonance

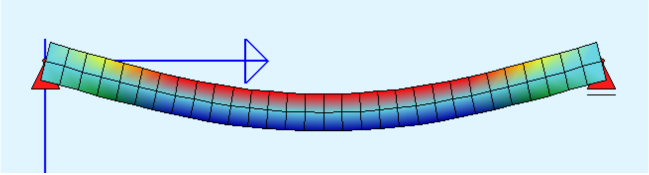
- Minimal movements of the first floor accelerate the second floor in resonance relative to the first floor. Thus we know the frequency of that as the Eigenfrequency of the topmost part of the structure alone.



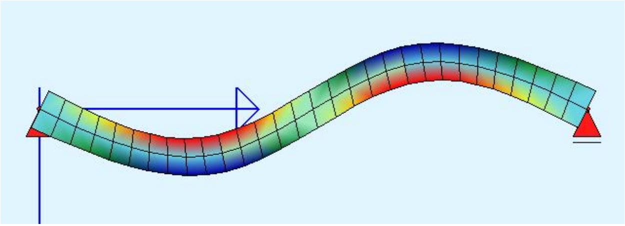
Multiple Periodic Lodings

- Independent “Out of balance” forces





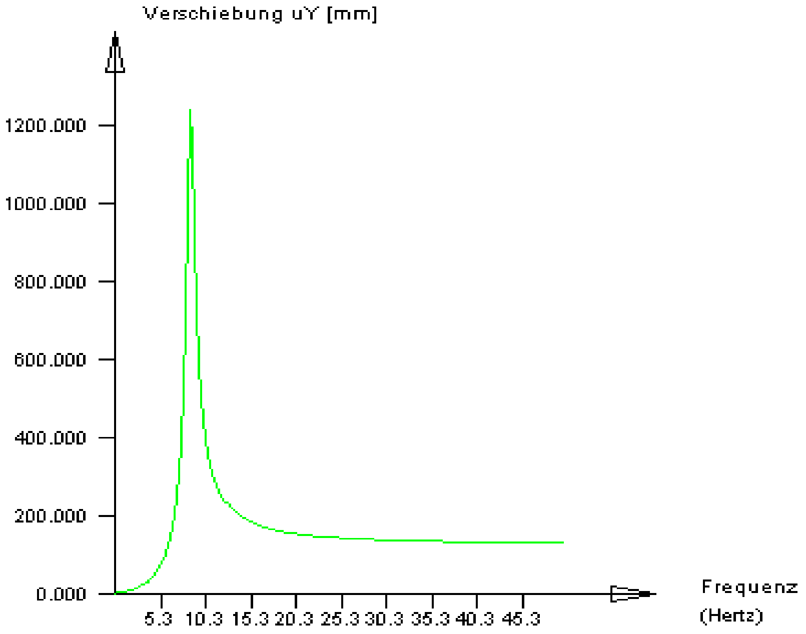
f = 8.55 Hertz



f = 33.11 Hertz

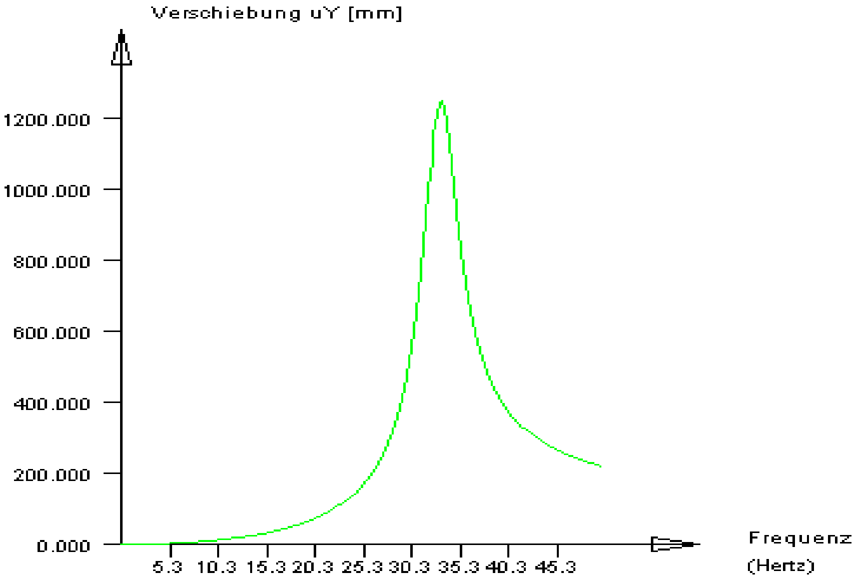
Loading with same Phase

Frequency	u-r = u-l
7 Hz	247
14 Hz	198
21 Hz	148
28 Hz	138
35 Hz	133



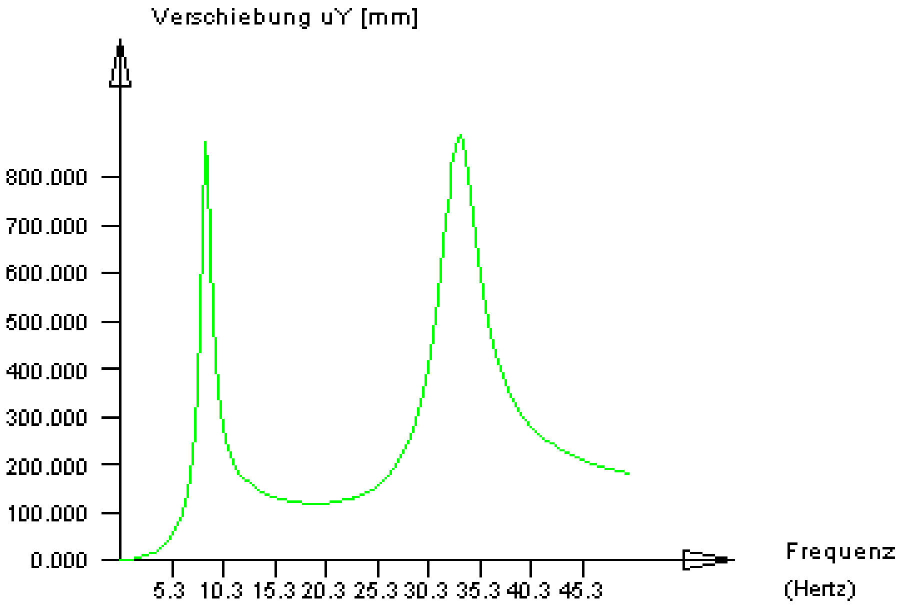
Loading in opposite phase

Frequency	u-r = -u-l
7 Hz	6
14 Hz	27
21 Hz	83
28 Hz	301
35 Hz	883



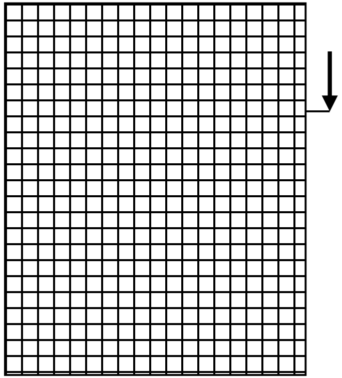
Loading with arbitrary phase (SRSS)

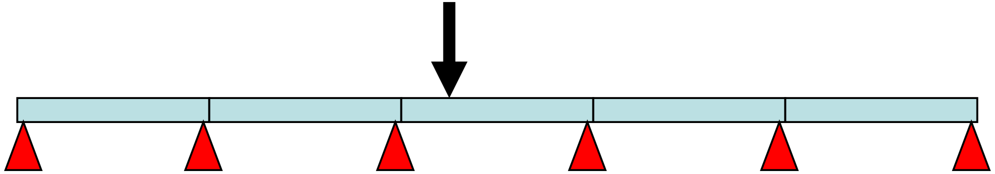
	u-r	u-l
7 Hz	174	176
14 Hz	144	139
21 Hz	128	112
28 Hz	261	204
35 Hz	568	689



Pitfalls for modal analysis

- The representative eigen value has an ordinal number much higher than expected
- A lot of low frequency eigenvalues of cables with low prestress

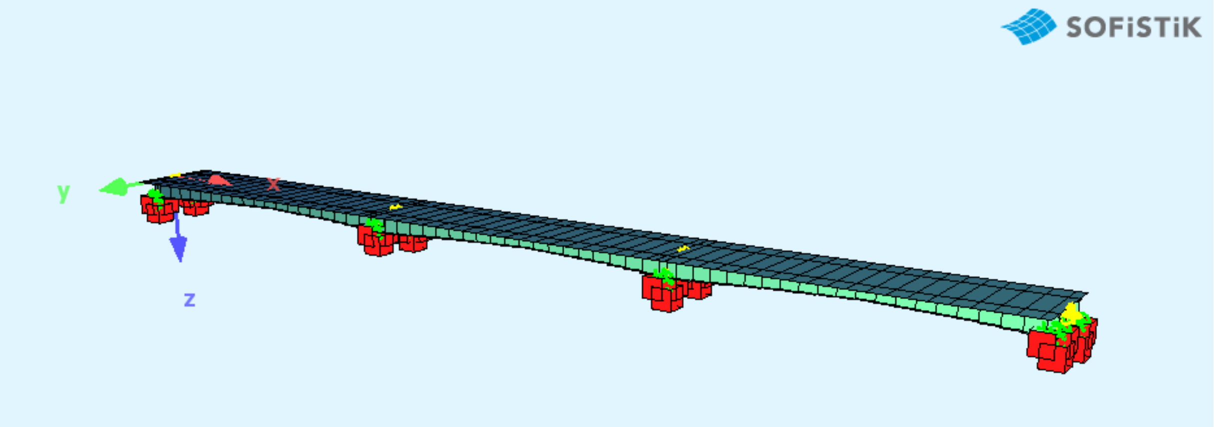




Nonlinear modal Analysis

- Transient Solution in the modal space
- All spring elements may act in a nonlinear way and create residual forces
- Very old, very fast method, introduced by Wilson
- Drawback:
  - Does not work if the used Eigenforms have no contribution for the nonlinear effects.
  - This occurs if the linear stiffness is significantly higher than the nonlinear behavior.

Example Friction Pendulum



- Analysis time for 10000 time steps
  - Non linear direct 21 sec 88 mm
  - Non linear modal 2 sec 66 mm (30 EV)
    72 mm (50 EV)
    85 mm (100 EV)

Man induced vibrations

- Exciting forces of 40 to 80 kg Mass with frequencies between 1.6 and 2.4 Hertz
- Rule of thumb to avoid frequencies below 5 Hertz is not sufficient
- Rule of thumb to avoid frequencies below 8 Hertz is to expensive
- Criteria for acceptance of accelerations 0.4 m/sec^2
- Criteria for velocities
- KSB-values according to many national and international design codes
- Literature: Prof. Hugo Bachmann

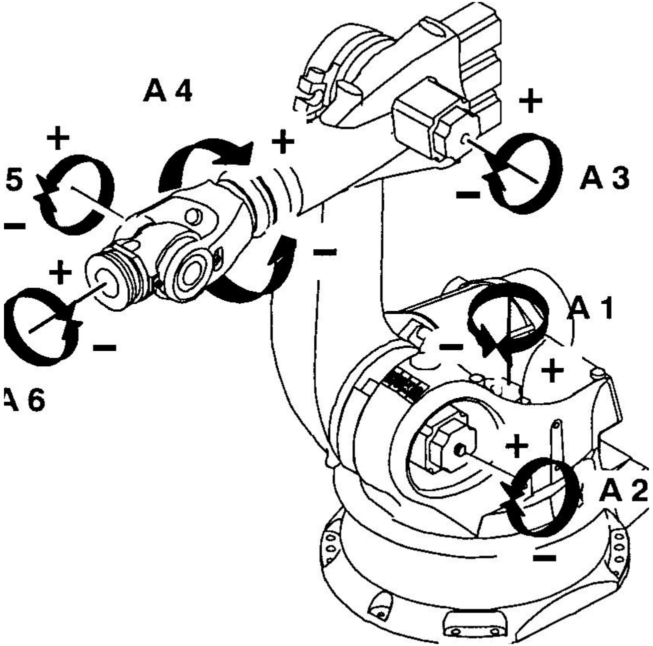
Traffic densities

	Weak traffic Here: event traffic Number of pedestrians: 60 Group size: 2-4 P Density: 0,2 P/m^2
	Exceptionally dense traffic Here: opening ceremony traffic Density: > 1,5 P/m^2

Traffic densities (HIVOSS)

Traffic Class	Density d (P = pedestrian)	Description	Characteristics
TC 1*)	group of 15 P; d=15 P / (B L)	Very weak traffic	(B=width of deck; L=length of deck)
TC 2	d = 0,2 P/m²	Weak traffic 	Comfortable and free walking Overtaking is possible Single pedestrians can freely choose pace
TC 3	d = 0,5 P/m²	Dense traffic 	Still unrestricted walking Overtaking can intermittently be inhibited

Problem: Acting Robots



Problem: Acting Robot

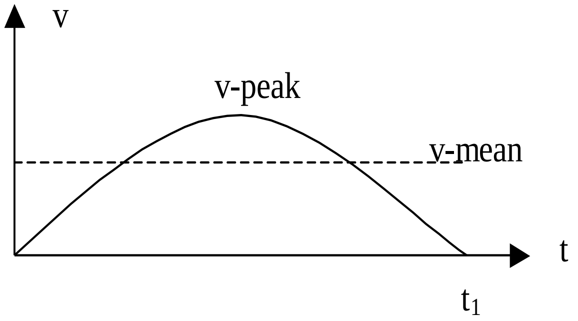
- Large accelerations allow faster production cycles
- Fast movements will create acceleration forces
- There is very sensitive equipment in the same room
- If the fixing of a robot is moving itself the movements of the tip are larger by an amplification function
- Special Circumstances
  - Placement in the 2nd flour !
  - A composite slab is very light

Problem: Acting Robot

- Forces are unknown
  But values in manufactures specifications were three times higher than last model
- Movements and Synchronisation of robots is unknown
- Seeing a robot moving reveals considerably amplitudes
- Pushing at a robots arm reveals a rather soft construction of the robot itself
- Technical descriptions are not available in detail

Estimate the accelerations

- We have 84° /sec as speed with a load for the axis A2 and A3
- Accelerations for these axis may then reach 90 to 270 ° /sec²



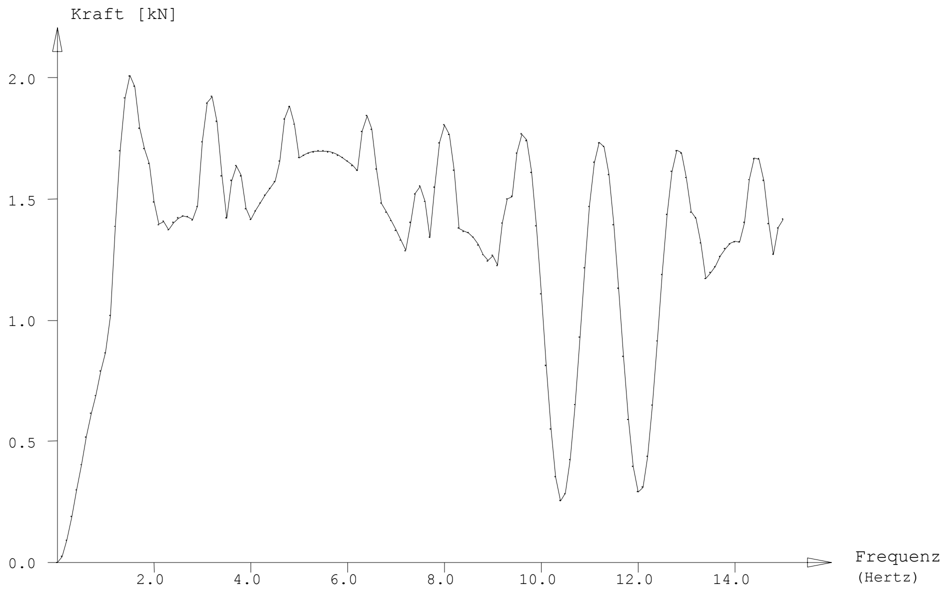
$$v = \xi(\xi - 1)$$
$$a = (2\xi - 1)/t_1$$
$$s = v_{mean} \cdot t_1$$

Frequencies

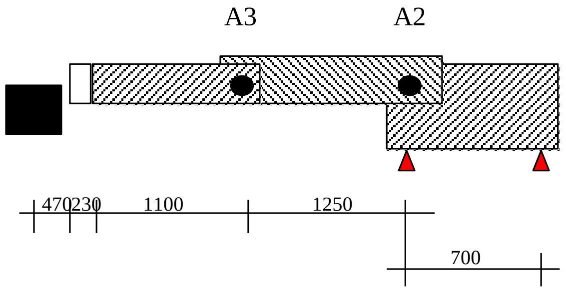
$a = 90\,°/sec^2$	$t_1 = 0.933$ sec	$s_1 = 78.4\,°$
$a = 180\,°/sec^2$	$t_1 = 0.622$ sec	$s_1 = 52.2\,°$
$a = 270\,°/sec^2$	$t_1 = 0.311$ sec	$s_1 = 26.1\,°$

- The higher the accelerations, the shorter the period
- Highest frequency at 1/0.311 = 3.21 Hertz
- Expected Eigenfrequency of slab 6 - 10 Hertz
- So we have a high tuning

Impact function
0.622 sec 1.0 P



Estimate the real forces



	Acceleration referenced on Axis A2				Force	Moment
	Angle	Radius	vertical	Mass	vertical	Axis A2
	[°/sec²]	[m]	[m/sec²]	[kg]	[kN]	[kNm]
Carousell	0	0	0	511	0	0,00
Lever	180	0,625	1,96	252	0,49	0,31
Arm	360	1,800	7,38	157	1,16	2,09
Hand	360	2,465	11,56	109	1,26	3,10
Pay load	0	3,050	15,23	210	3,20	9,75
			Sum	**1239**	**6,11**	**15,25**

Lever	a1=0.625*3.14=1.96 m/sec²
Arm	a2=1.25*3.14+0.55*3.14*2=7.38 m/sec²
Hand	a3=1.25*3.14+(1.1+0.23/2)*3.14*2=11.56 m/sec²

Estimate the real forces

- Dynamic Loading varies between 2.8 and 6.1 kN
- The static loading would be a vertical sum of 12.4 kN and a Moment of 12.76 kNm.
  Do we have to expect 50 % dynamic loading ?
- For the design of the fixing the manufacturer has specified vertical forces of 24 kN and a tilting moment of 49 kNm. These loads include however extreme events like an emergency stop.

Uncertainties

- How often will we have extreme forces
  - Empirical hope: only rare ?
- Repetition frequency of movements
  - Empirical: slower movements on a more regular base
  - Intermissions of movement
- Synchronisation
  - Energetic Criteria: Square root of N
  - Pedestrian Bridges: up to 25 % of people may act in phase (lock in effect should not occur for robots)

Assumptions

- One single robot acting unfavourably with a maximum impact force and an empirical Synchronisation factor of 2.0
- Periodic movements with a mean acceleration of a single robot with a factor of 10 (based on 100 robots) and an efficiency factor of 50 % to account for the distributed loading and a factor of 0.25 for true synchronisation over a longer period
  => yielding a factor of 1.25

Measurements

- The robots have provisions to avoid overloading of the gears, reducing accelerations for large loadings or levers
- Although the existence of such a software was not denied, any values about the quantity effect were not available.
- Measurements have been made.
- With some tricks it was possible to measure the effects.
- Measured forces emergency stop: 23.8 kN
  Values from manufacturer: 24.0 kN

Observations

- The arm of the robot is not very stiff. If one pushes with the hand or simulates an emergency stop, large amplitudes may be observed.
- For the standard case the electronic tries to avoid any sudden acceleration. There are only rather smooth movements.
- The estimated forces are larger than the real forces. Further it was observed, that the Axis A2 and A3 never worked in the same sense. Accelerations / movements were always in opposite rotational directions.

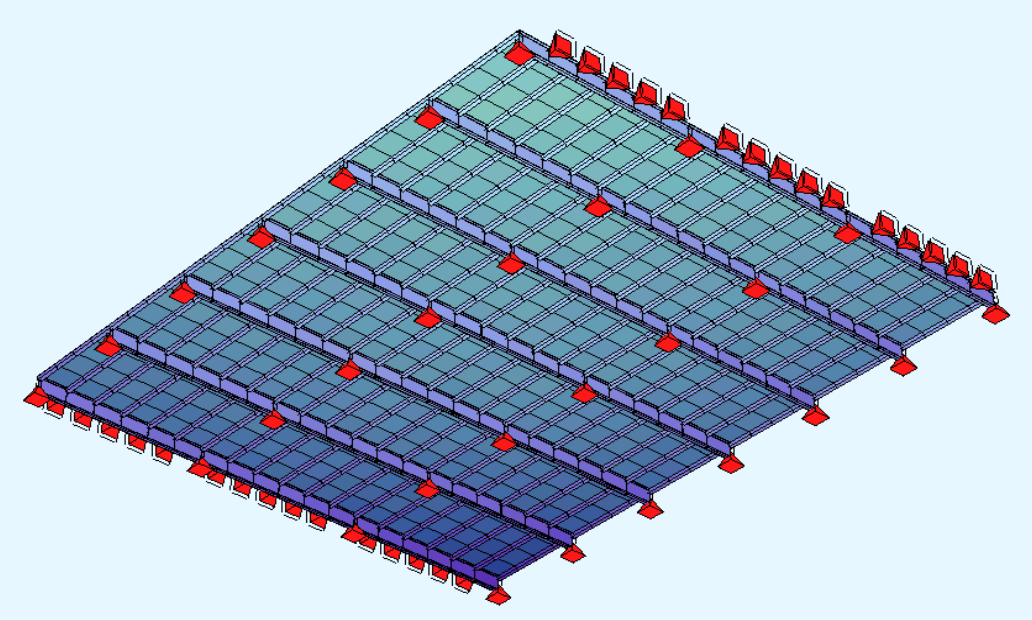
Scaled Measurements

- **Maximum Acceleration at arm 12 m/sec²**
- **Emergency stop with 35 m/sec²**

	Acceleration referenced on Axis A2				Force	Moment
	Angle	Radius	vertical	Mass	vertical	Axis A2
	[°/sec²]	[m]	[m/sec²]	[kg]	[kN]	[kNm]
Carousell	0	0	0	511	0	0,00
Lever	158	0,625	1,73	252	0,43	0,27
Arm	316	1,800	6,49	157	1,02	1,83
Hand	316	2,465	10,16	109	1,11	2,73
payload	0	3,050	13,39	100	1,34	4,08
			Sum	**1129**	**3,90**	**8,92**

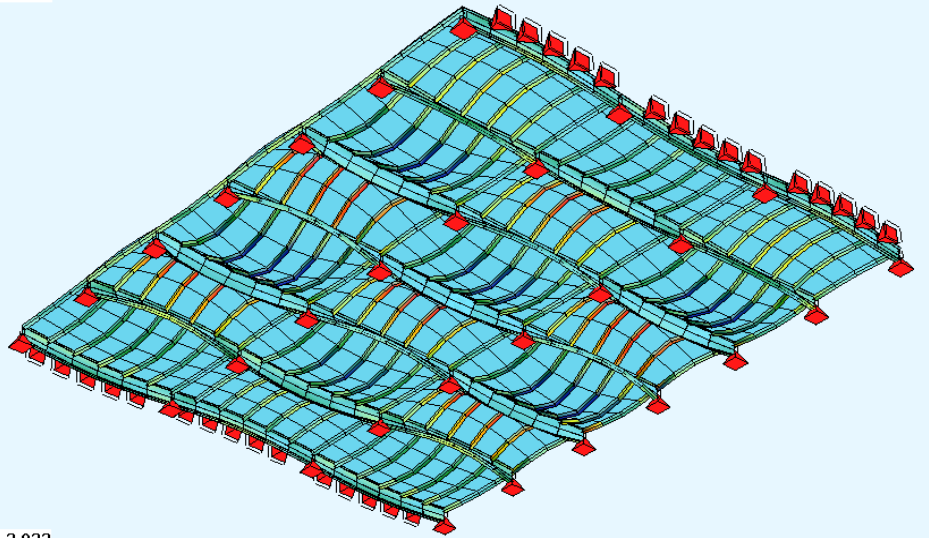
Lever	a1=0.625*2,76=1.73 m/sec²
Arm	a2=1.25*2,76+0.55*2,76*2=6.49 m/sec²
Hand	a3=1.25*2,76+(1.1+0.23/2)*2,76*2=10.16 m/sec²

Analysis

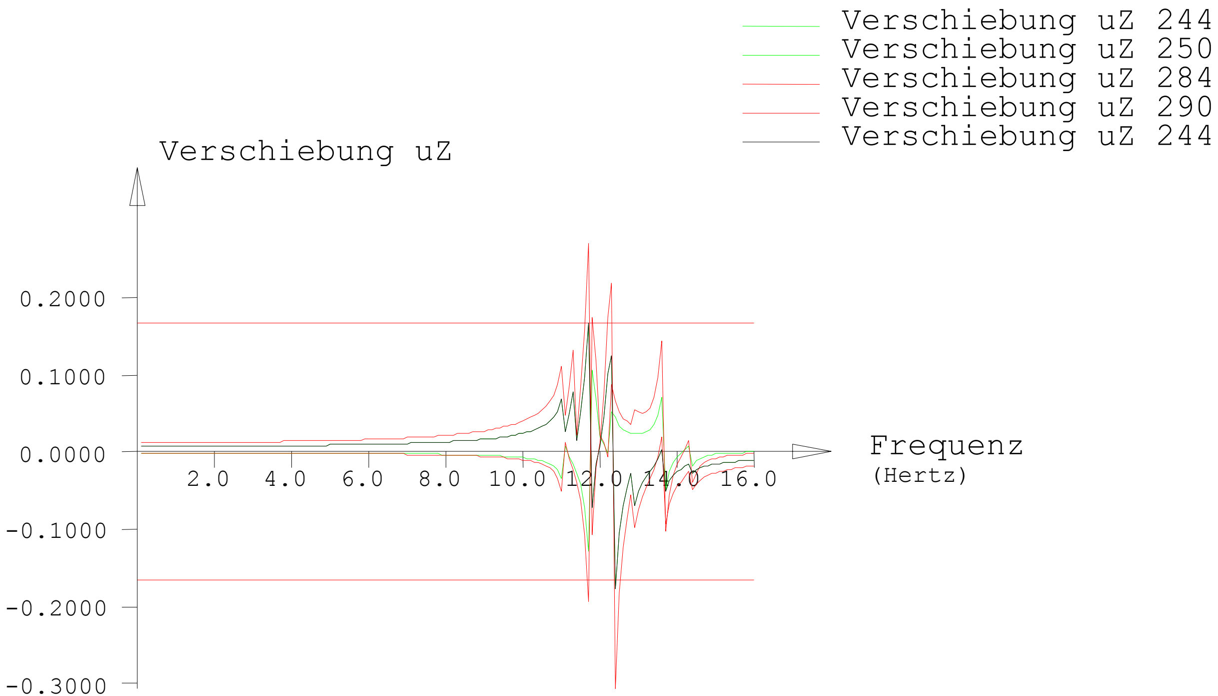


Eigen-Frequencies

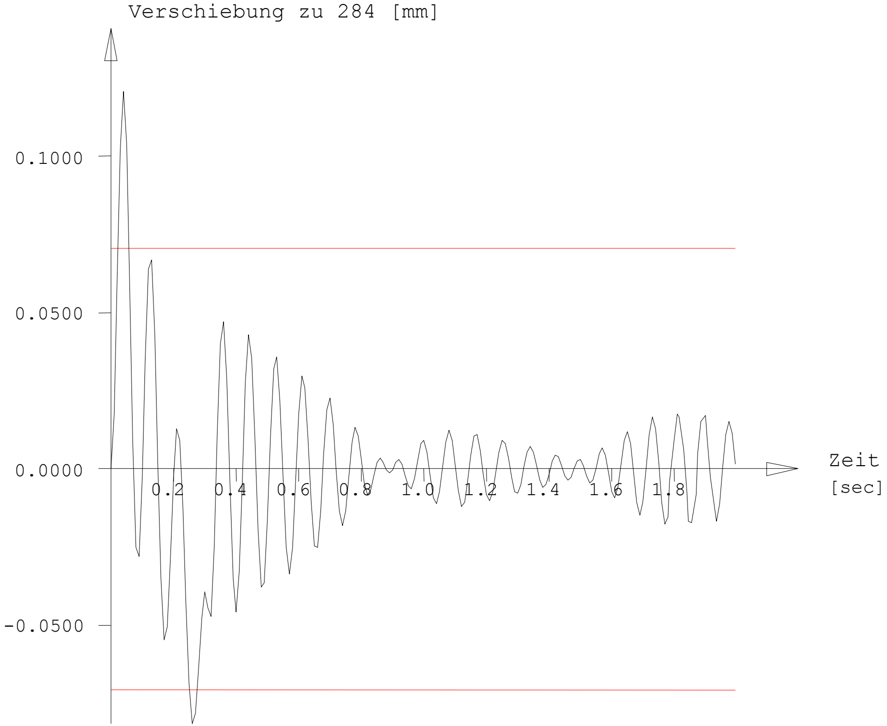
```
EIGENFREQUENCIES [Hertz]
 1   10.819
 2   10.920
 3   11.018
 4   11.186
 5   11.294
```



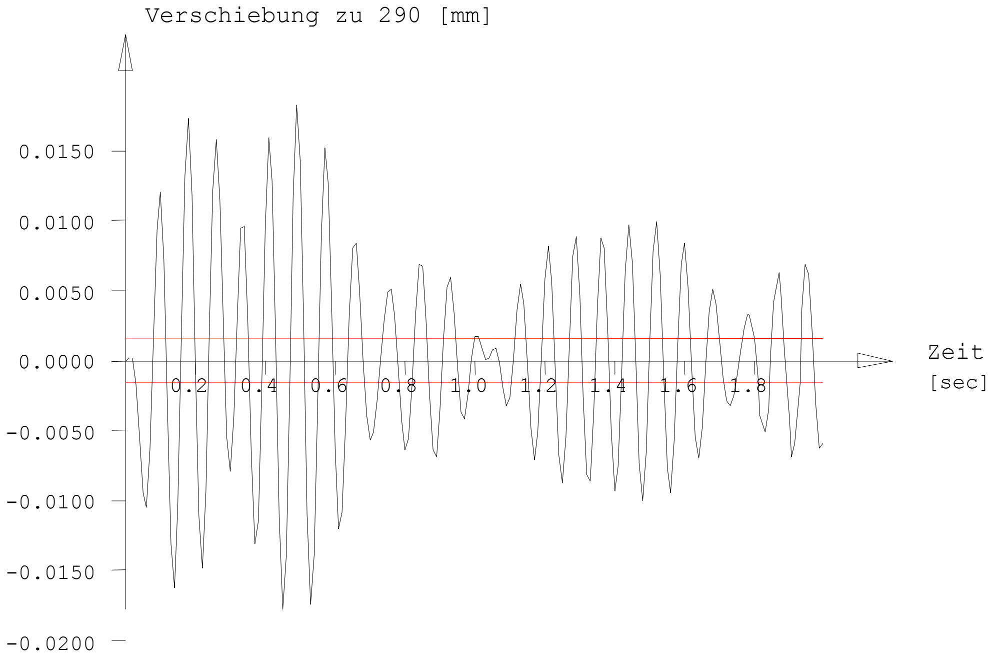
Frequency dependant steady state loading



Impact loading at centre



Impact loading in neighbouring span



Assessment

- For a one time loading we have an impact factor of 1.8 in the same span and values by a factor of 7 smaller from neighboured spans.
- As may be seen from the response, the slabs will move within their own eigen frequencies. So we have to select the impact factors for the robots for these frequencies between 10 and 12 Hertz. Those values are between 2 and 5, for some rare cases up to 9.

Assessment

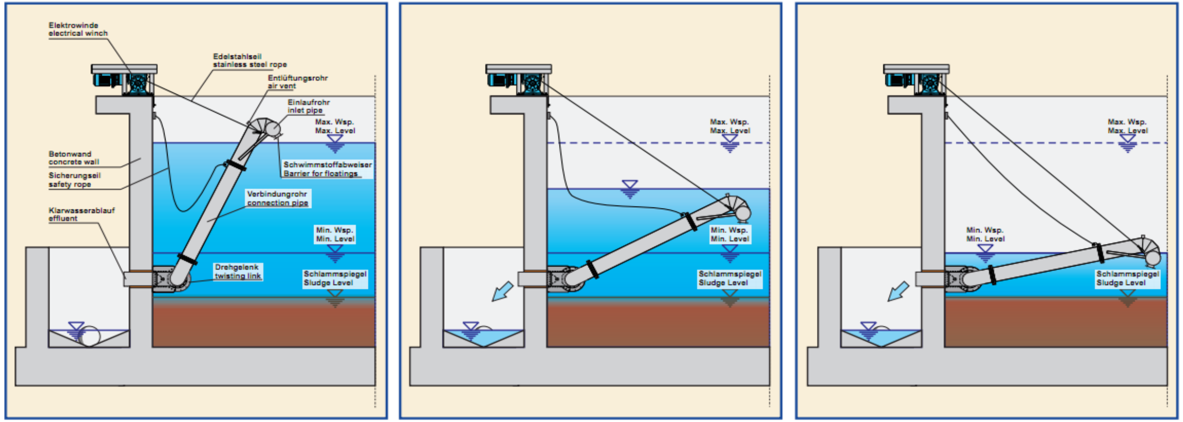
- Loading by a single robot enlarged with a resonance factor of 2.0
  Moving at robot base 0.12 mm

- Loading from a steady state robot movement with a factor of 4 from the frequency response
  Moving at robot base 1.00 mm

- Assessed displacement from robot movements
  at any point in the slab 0.24 mm

- Expected Movement of the robots hand with an amplification factor of 6
  1.44 mm

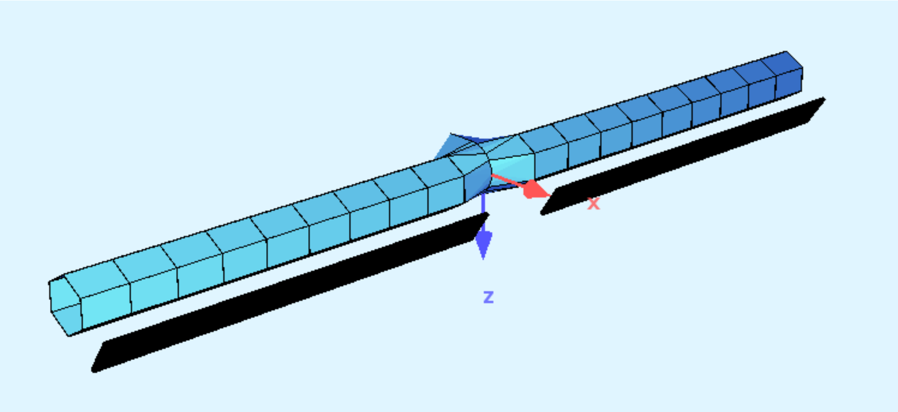
Damage within a sewage decanter



- 550 kg mass
- Speed is only 2m/min, but stop is always sudden
- => Impulse 18.3 [kgm/sec] = 0.183[kNsec]

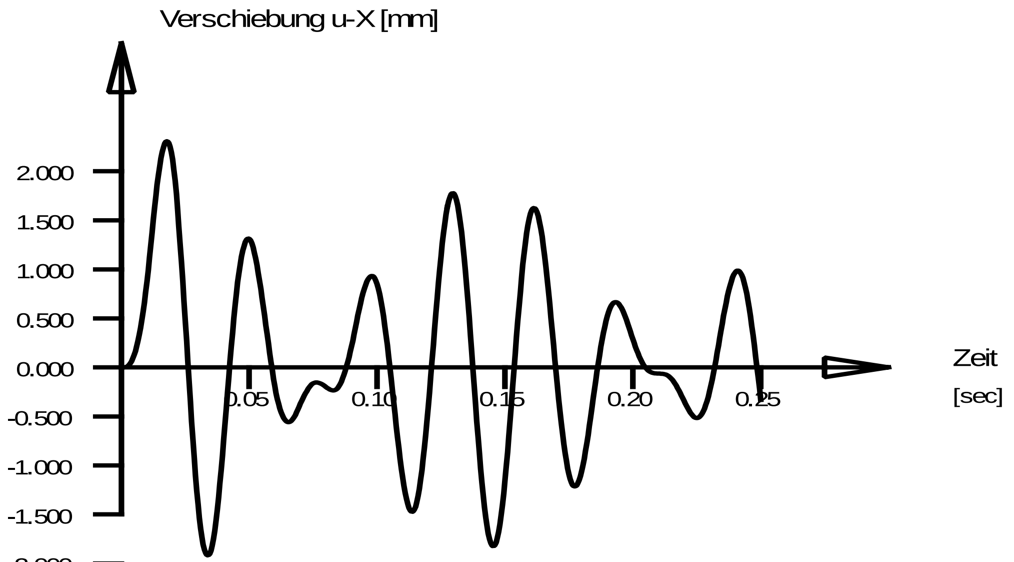
The damaged part

Beam Analysis

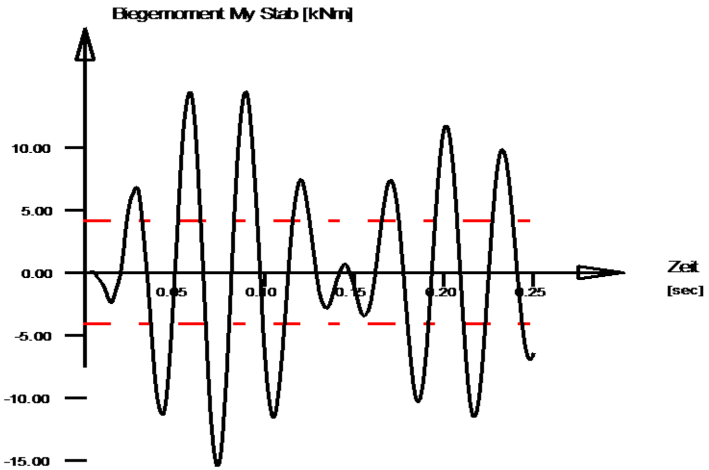


• Eigenfrequencies	27.6 / 35.1 Hertz
• Static moment self weight	3.10 kNm
• Max. static moment	4.12 kNm
• Dynamic Displacements estimate	1.9 mm

Calculated Displacements

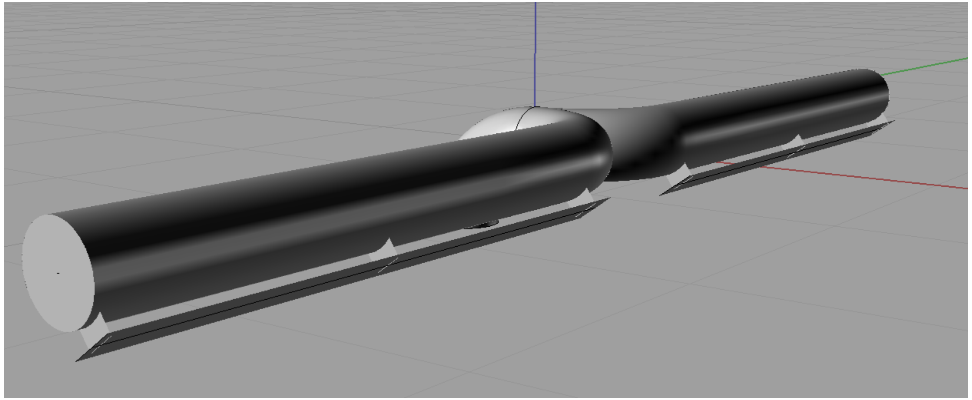


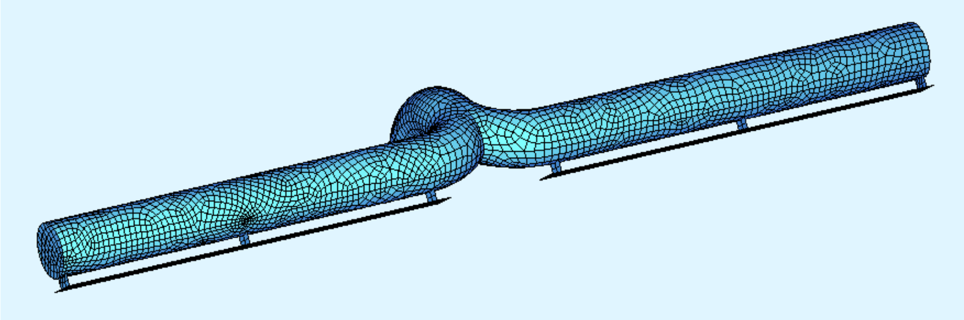
Moments and Stresses



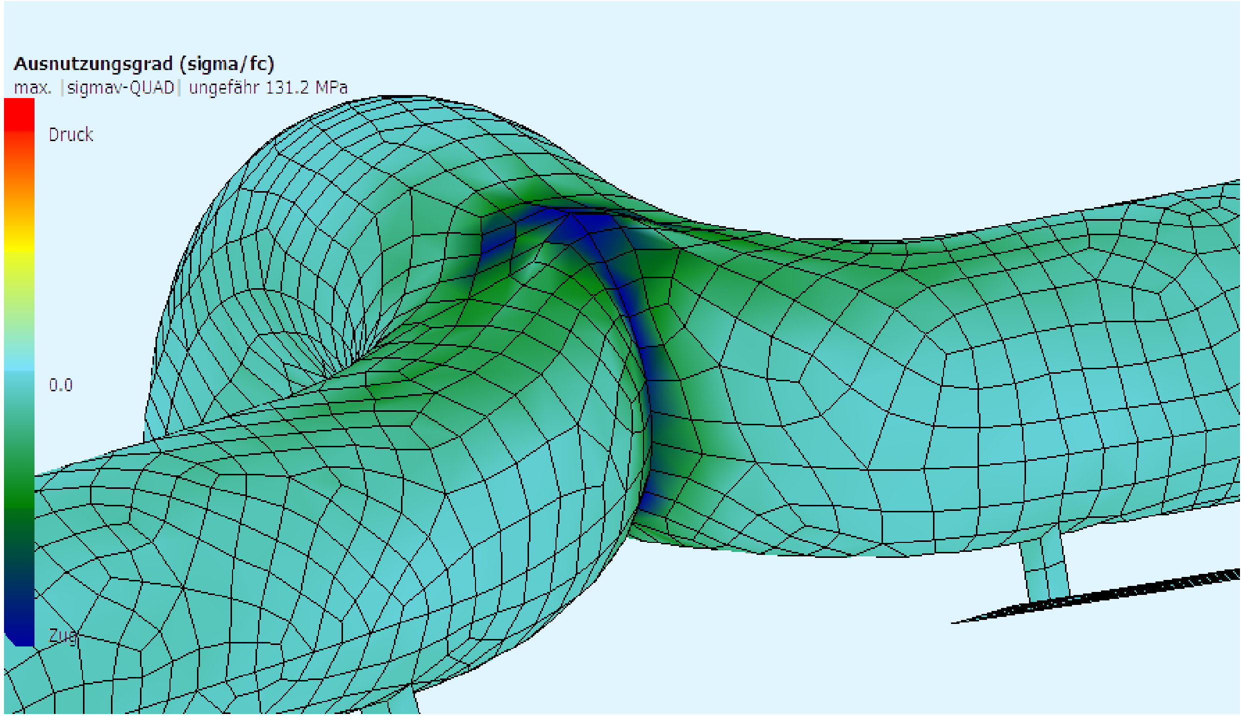
- Maximum in horizontal pipe 45 Mpa
- Maximum in junction 103 MPa

FEM Analysis

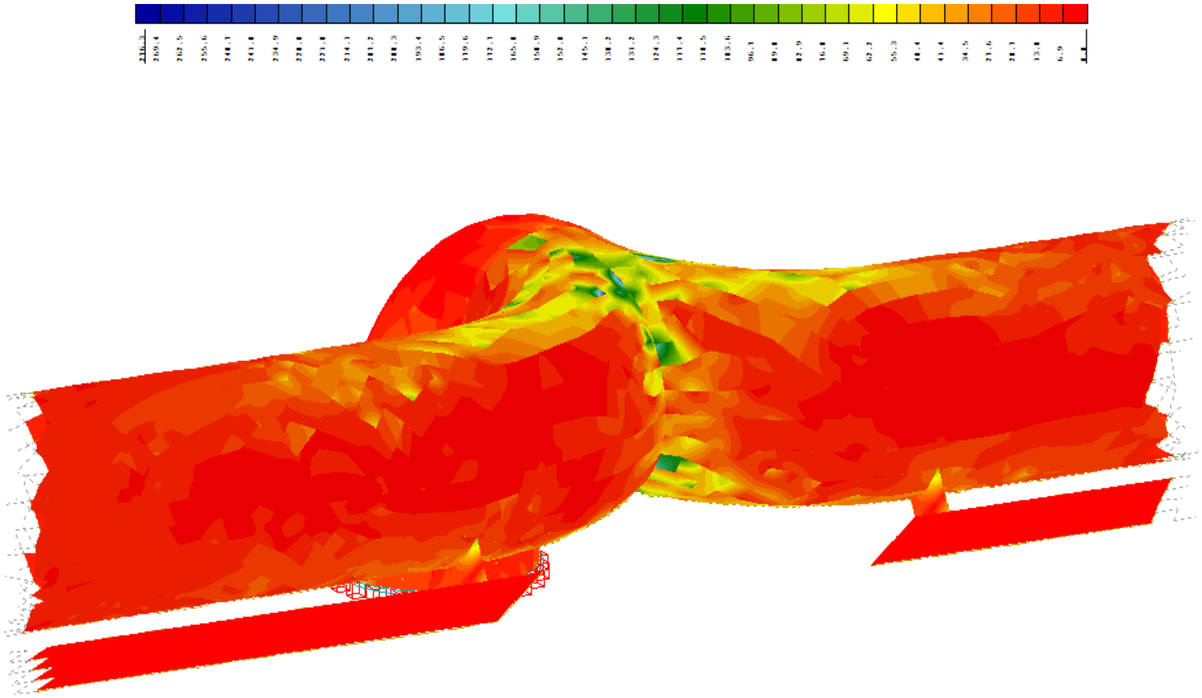




Static Analysis



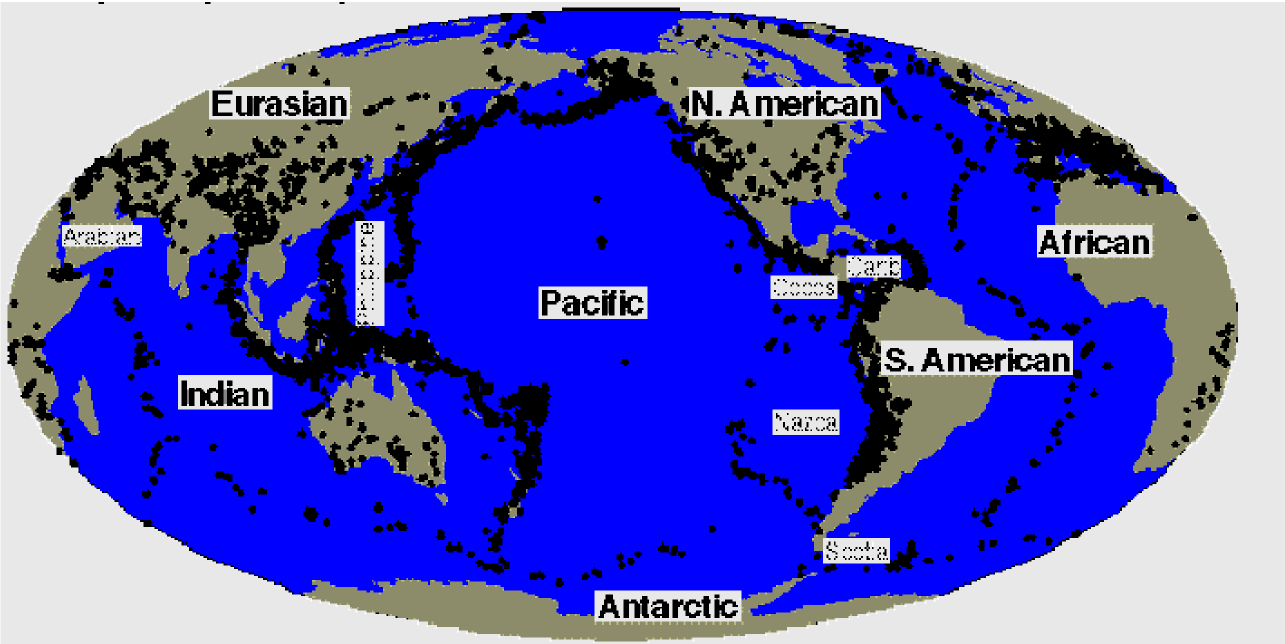
Dynamic Analysis (σ = 230 MPa)



Nonlinear modal Analysis

- Transient Solution in the modal space
- All spring elements may act in a nonlinear way and create residual forces
- Very old, very fast method, introduced by Wilson
- Drawback:
  - Does not work if the used Eigenforms have no contribution for the nonlinear effects.
  - This occurs if the linear stiffness is significantly higher than the nonlinear behavior.

Earthquakes caused by plate tectonics



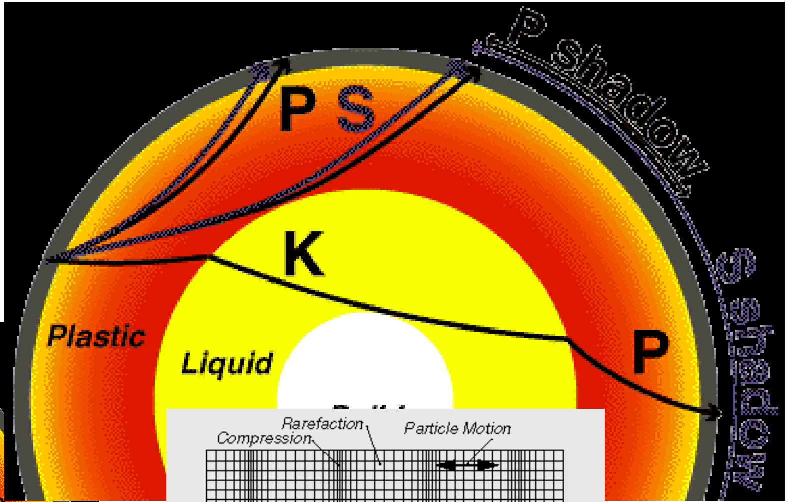
(from the USGS)

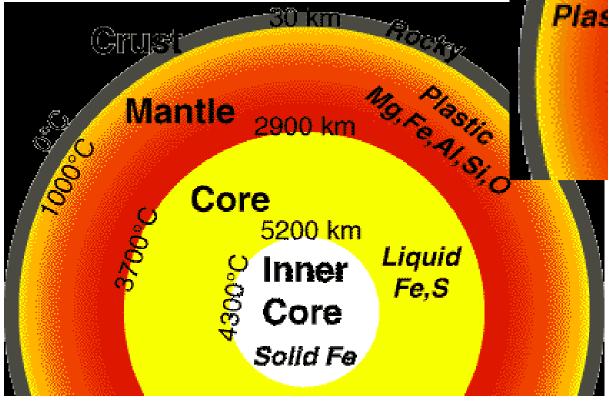
Magnitudes

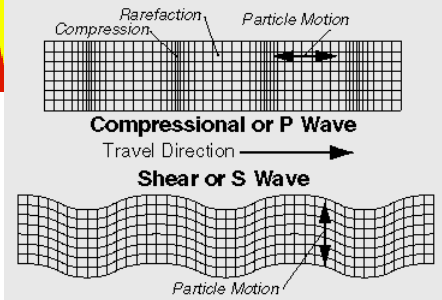
Richter Magnitude	TNT Energy	Example
-1.5	6 ounces	Breaking a rock on a lab table
1	30 pounds	Large Blast at a Construction Site
2	1 t	Large Quarry or Mine Blast
4	1000 t	Small Nuclear Weapon
5	80 000 t	Little Skull Mtn., NV Quake, 1992
6	1 million t	Double Spring Flat, NV Quake, 1994
7	32 million t	Hyogo-Ken Nanbu, Japan Quake, 1995; Largest Thermonuclear Weapon
8	1 billion t	San Francisco, CA Quake, 1906
9	32 billion t	Chilean Quake, 1960
10	1 trillion t	San-Andreas type fault circling Earth
12	160 trillion t	Fault Earth in half through center

Seismic waves

- Compressive P
- Shear S
- Rayleigh R
- Love L

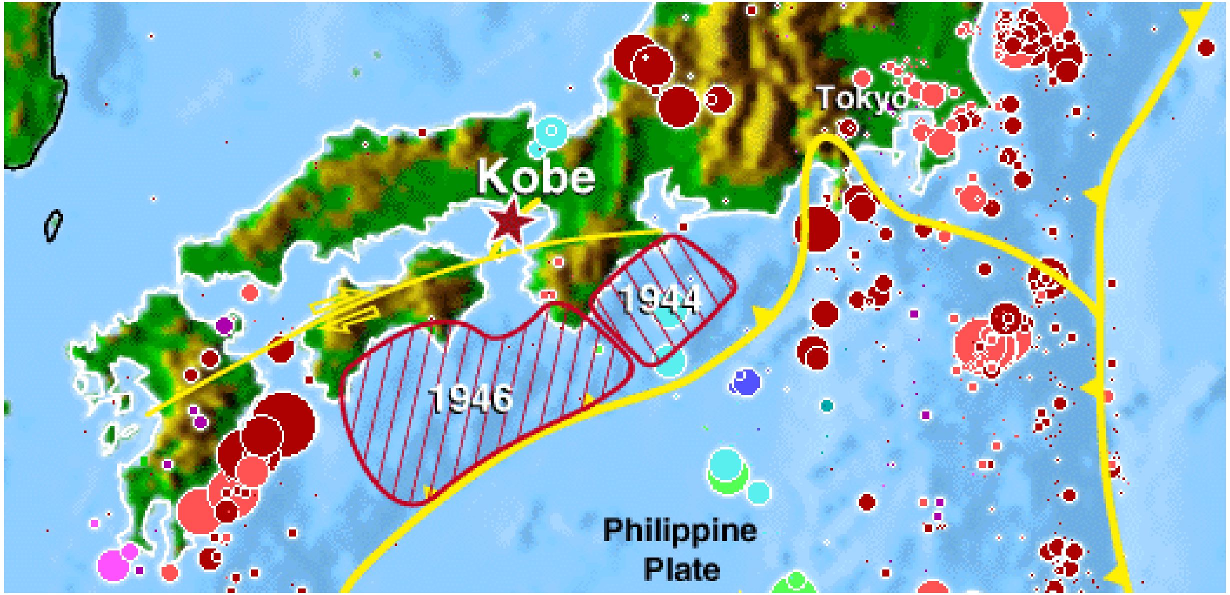






Something happens between

- The waves are refracted and changed by all soils between the epicentre and the site of our building

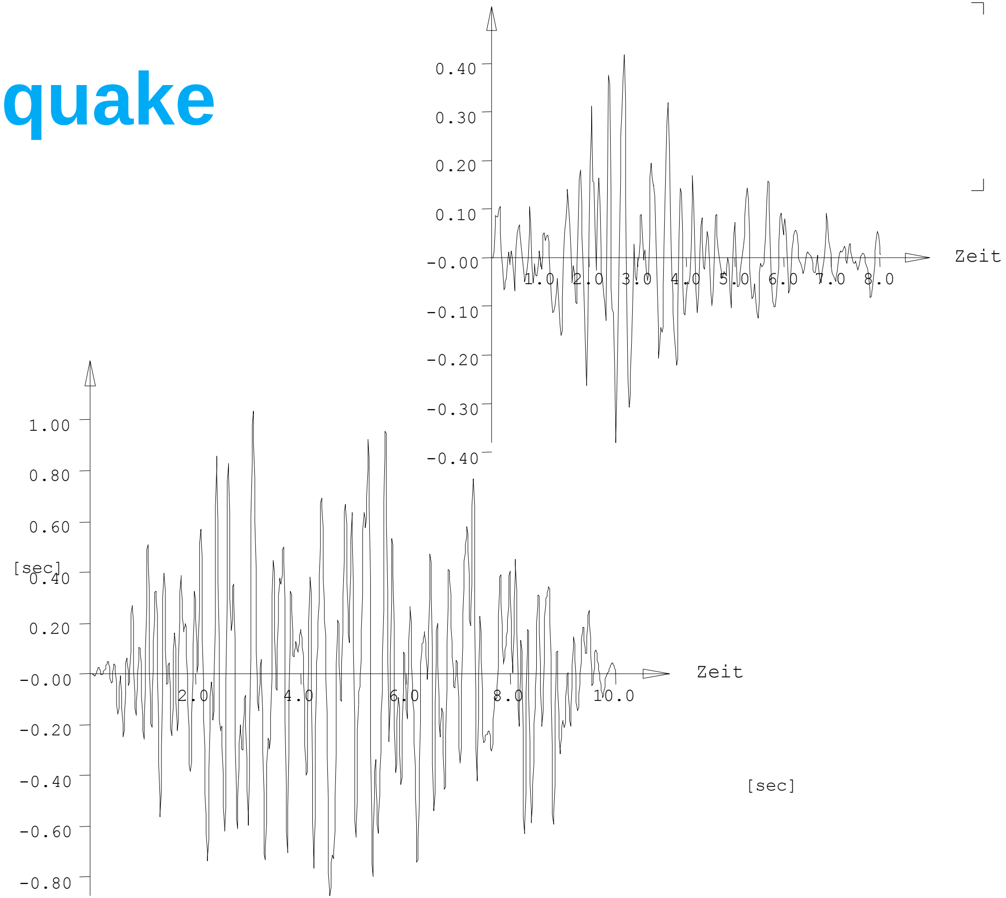


Primary effects

Secondary effects

Loading by an earthquake

- Acceleration as a function of time
  - Measured
  - Artificially generated



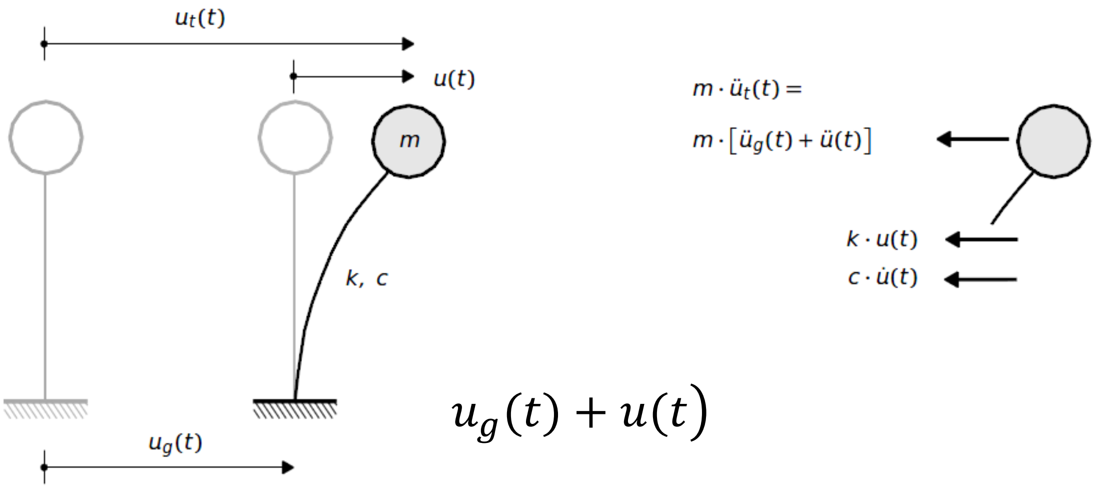
Earthquake Analysis

- There are a lot of measurements of earthquake accelerations
- You may generate artificial accelerations from a given spectra
- Problem is the check/conversion to a displacement history
- The standard analysis method is to switch to a moving coordinate system and apply the loads as mass times acceleration
- This gives sometimes raise to understanding the meaning of the resulting acceleration
- Problem of coherence for different base accelerations
  - To be solved with different displacement histories
  - To be solved with different influence functions for the accelerations
- Problem of eccentricity (given in design codes)

Earthquake Analysis

- Methods to do the analysis
  - Lateral Force Method
    - Very crude, only for preliminary analysis
  - Multimodal Spectral Analysis
    - Most widely used
    - Application to nonlinear systems limited
  - Non-linear Static Method (PUSHOVER)
    - Takes into consideration performance of the structure
    - Transition between Linear Static and Direct Dynamic
  - Direct Dynamic Analysis (linear and nonlinear)
    - Correct, computationally extensive, only for structures of high importance

The moving coordinate system



$$u_g(t) + u(t)$$

$$m \cdot \ddot{u}_t(t) + c \cdot \dot{u}_t(t) + k \cdot u_t(t) =$$

$$m \cdot \left[\ddot{u}_g(t) + \ddot{u}(t)\right] + c \cdot \dot{u}_t(t) + k \cdot u_t(t) = 0$$

Dynamic Equation of SDOF

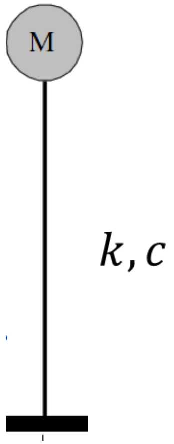
$$m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g(t)$$

$$\ddot{u} + 2\xi\omega\dot{u} + \omega^2 u = -\ddot{u}_g(t)$$

Analytical solution – only for a certain type of loading.

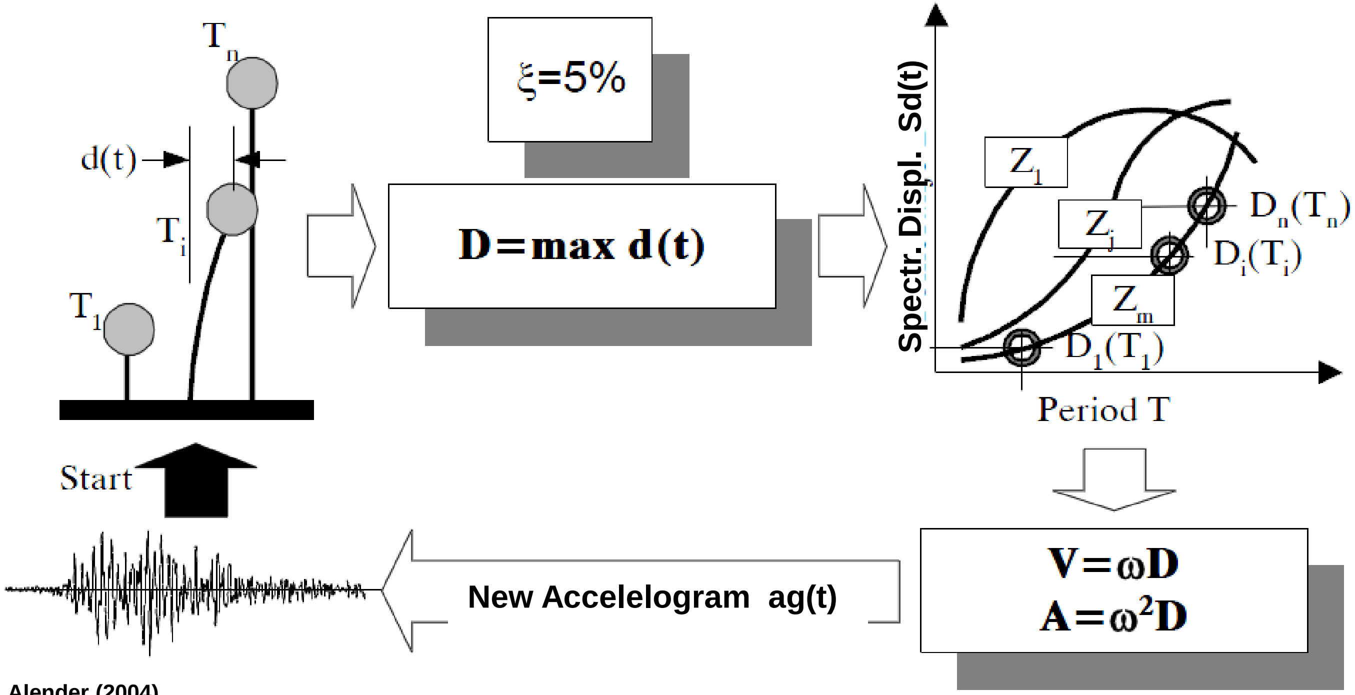
For a general loading – integration of Duhamel‘s integral:

$$u(t) = -\frac{1}{\omega_d}\int_0^t \ddot{u}_g(t)e^{-\xi\omega(t-\tau)} sin\omega_d(t-\tau)d\tau$$



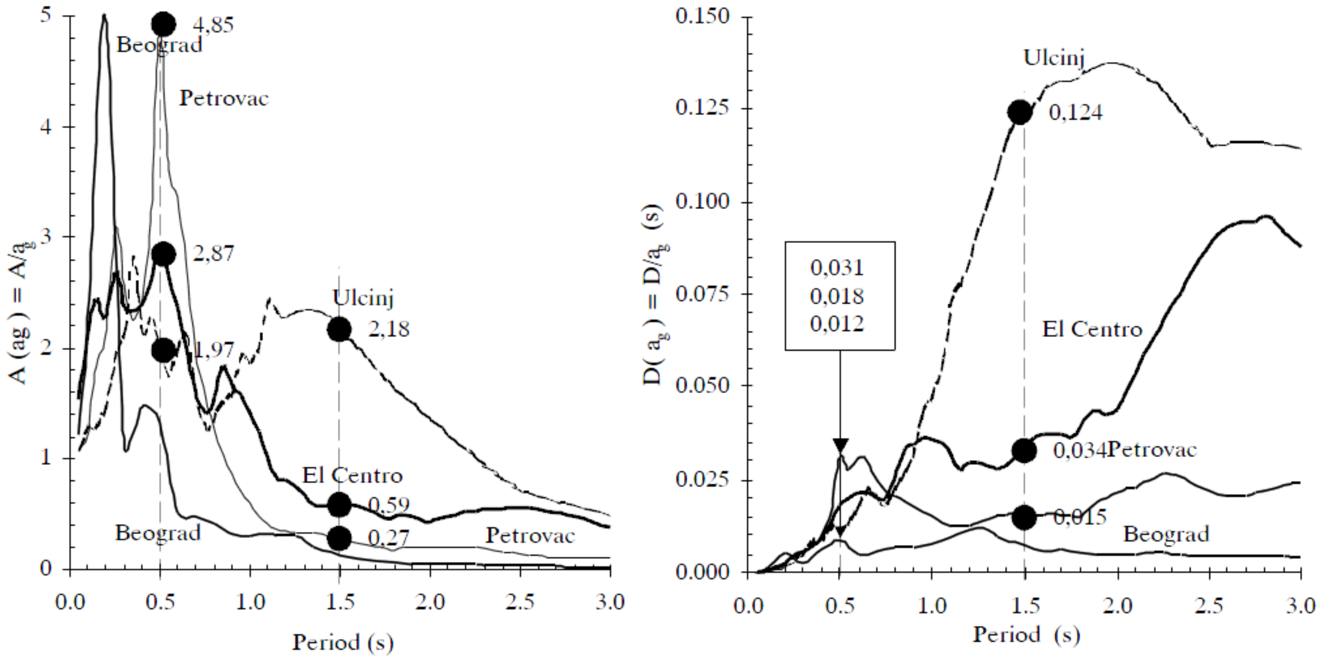
We are not interested in entire time-history, mostly in maximum values (displacements, forces) => SPECTRUM.

How to obtain Response Spectrum?



V. Alender (2004)

Elastic Response Spectra (a) Pseudo acceleration (b) Displacement



V. Alender (2004)

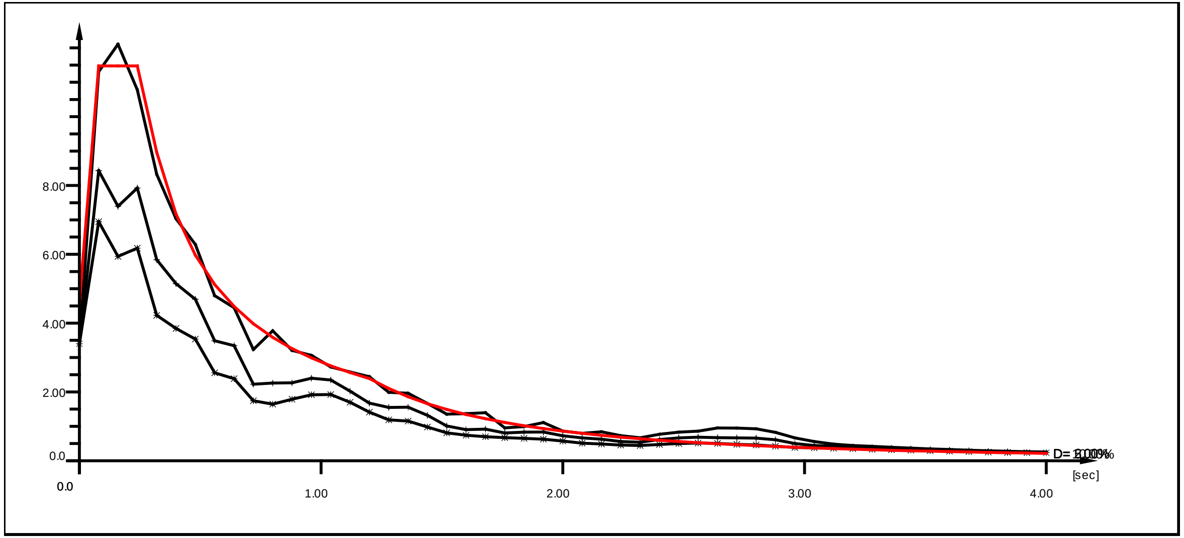
SOFiSTiK

Response Spectra

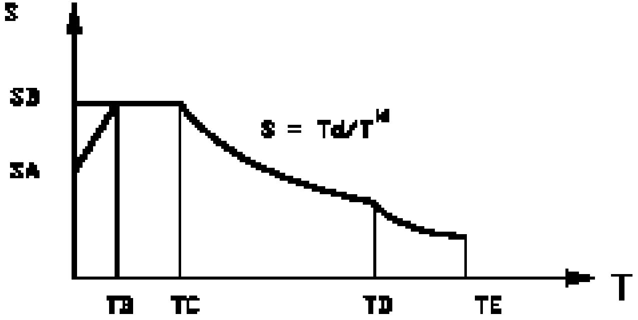
- A response spectra gives the maximum answer of a SDOF (single degree of Freedom) system to a given loading like an earthquake acceleration.
- The answer may be any value, but in general it is the acceleration
- However two things got lost:
  - The phase respective the time of the maximum
  - The sign of the maximum value

Response Spectra

- The design codes provide response spectra which are an envelope to many observed or generated events



General Response Spectra (EC)

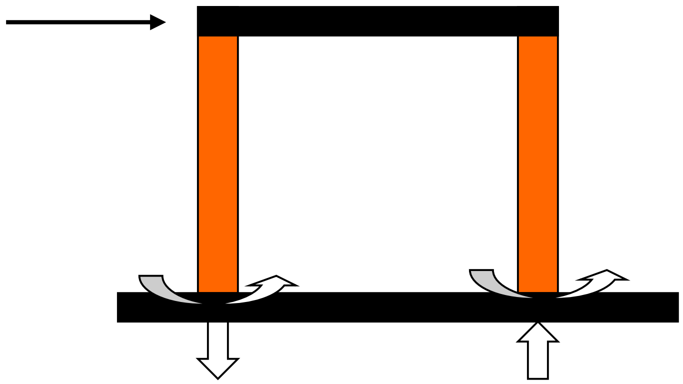


DYNA61.

$0 < T < TB$	$S = \left[S_A + \frac{T}{T_B}\cdot(S_B - S_A)\right]$
$TB < T < TC$	$S = S_B$
$TC < T < TD$	$S = S_B\cdot\left[\frac{T_C}{T}\right]^{k1} \geq S_{min}$
$TD < T < TE$	$S = S_B\cdot\left[\frac{T_C}{T_D}\right]^{k1}\cdot\left[\frac{T_D}{T}\right]^{k2} \geq S_{min}$
$TE < T$	$S = 0$

Energetic Superposition

- We have the individual response of n Eigenforms but we do not know how to combine them
- SRSS – (Square Root of Sum of Squares) Method
  - The target result is always positive (alternating load cases!)
  - But what about the corresponding forces ?



Sign of Results

- Value of target for the case of absolute addition:

$$sum_j = \sum_i |s_{ij}|$$

- Alternate Solution:

$$SUM = \sum_i f_i \cdot S_i \quad ; \quad f_i = \begin{cases} +1 & \textit{für} \quad s_{ij} \geq 0 \\ -1 & \textit{für} \quad s_{ij} < 0 \end{cases}$$

Sign of Results

- Value of target for the case of SRSS:

$$sum_j = \sqrt{\sum_i s_{ij}^2}$$

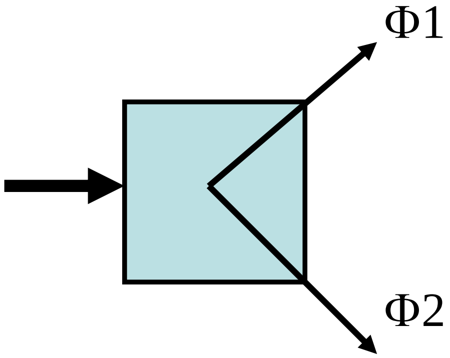
- Alternate Solution:

$$SUM = \sum_i f_i \cdot S_i \;\; ; \;\; f_i = \frac{s_j}{\sqrt{\sum_i s_{ij}^2}}$$

CQC - Method

$$q = \sqrt{\sum \sum q_i \cdot \rho_{ij} \cdot q_j}$$

$$\rho_{ij} = \frac{8\sqrt{d_i d_j}\,(d_i + r \cdot d_j) \cdot r^{\frac{3}{2}}}{(1 - r^2)^2 + 4 d_i d_j r \cdot (1 + r^2) + 4(d_i^2\, d_j^2) r^2}$$



Method	Loaddirection	Transverse
History	100.2	4.9
SRSS	78.8	71.4
SUM ABS	127	116
SUM	127	6.1
CQC	100.8	6.0

Directional superposition

- EN 1998-1 4.3.3.5.1

  (1) In general the horizontal components of the seismic action shall be taken as acting simultaneously.

  (7) When using non-linear time-history analysis and employing a spatial model of the structure, simultaneously acting accelerograms shall be taken as acting in both horizontal directions.

- An error in assumption ?
  - Adding a second (even reduced) acceleration in a transverse direction is equivalent to a one directional earthquake of larger magnitude in that other direction !

Directional superposition (old style)

- Classical:

```
PROG MAXIMA
KOPF EXTREMA FOR EARTHQUAKE USING RESPONSE SPECTRA
KOMB 3 STAN
301 A4  1.3 $ earthquake X
301 A4 -1.3
304 A4  1.3 $ earthquake Y
304 A4 -1.3
307 A4  1.3 $ earthquake Z
307 A4 -1.3
301 A4  1.3*0.7 ; 304 F  1.3*0.7
301 A4 -1.3*0.7 ; 304 F  1.3*0.7
301 A4 -1.3*0.7 ; 304 F -1.3*0.7
301 A4  1.3*0.7 ; 304 F -1.3*0.7
     etc...
```

Directional with SRSS (EN 1998-1 4.3.3.5)

a. The structural response to each component shall be evaluated separately, using the combination rules for modal responses given in 4.3.3.3.2.

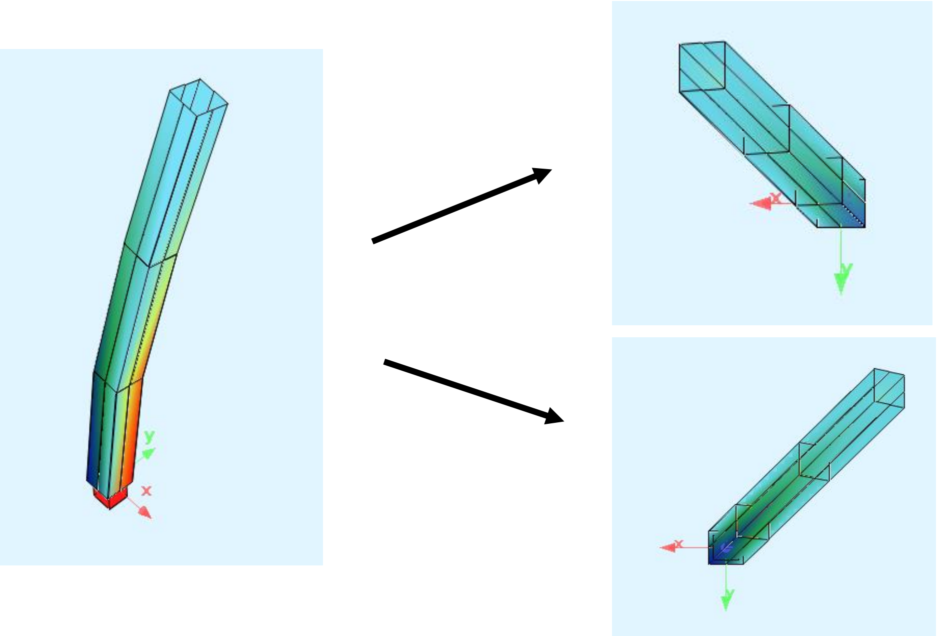
b. The maximum value of each action effect on the structure due to the two horizontal components of the seismic action may then be estimated by the square root of the sum of the squared values of the action effect due to each horizontal component. More accurate models may be used for the estimation of the probable simultaneous values of more than one action effect due to the two horizontal components of the seismic action.

c. As an simplified alternative with combination of effects:

$$E_{Edx} \text{ “+” } 0.30\, E_{Edy} \quad \text{and} \quad E_{Edy} \text{ “+” } 0.30\, E_{Edx}$$

Example for a better SRSS solution

- Column with diagonal (multiple) Eigenvalues



Eigenform 1
MY = -1633
MZ = +2412

Eigenform 2
MY = -2412
MZ = -1633

Acceleration in Diagonal Directions of Eigenvalues

- Acceleration

  in two load cases for +45° and -45°

- Result are maximum moments:

Load case 11:	My	33.32	MZ	- 33.32
Load case 12:	My	33.32	MZ	33.32

- Perfect Symmetry has been achieved only by CQC-Method !
- Combination of both directions with signed SRSS:

Load case 21:	My	47.12	MZ	0.00
Load case 22:	My	0.00	MZ	47.12

Intermission

- We did not request for the maximum corner stress but for the maximum moment My.
- This is obtained by an acceleration in the local z-axis. But then the acceleration and the forces in the transverse direction are zero!
  Thus directional load cases identical to 21 and 22
- So if we add the two diagonal directions by the SRSS-Method we have to provide for the correct sign of the superposition according to the superposition of the Eigenvalues!

Other results possible?

- Combining the maximum absolute values (?):

load case 33	My	47.12	Mz	47.12
load case 34	My	47.12	Mz	47.12

- Combining the $E_x \oplus 0.3 \cdot E_y$ on LC 11 and 12:

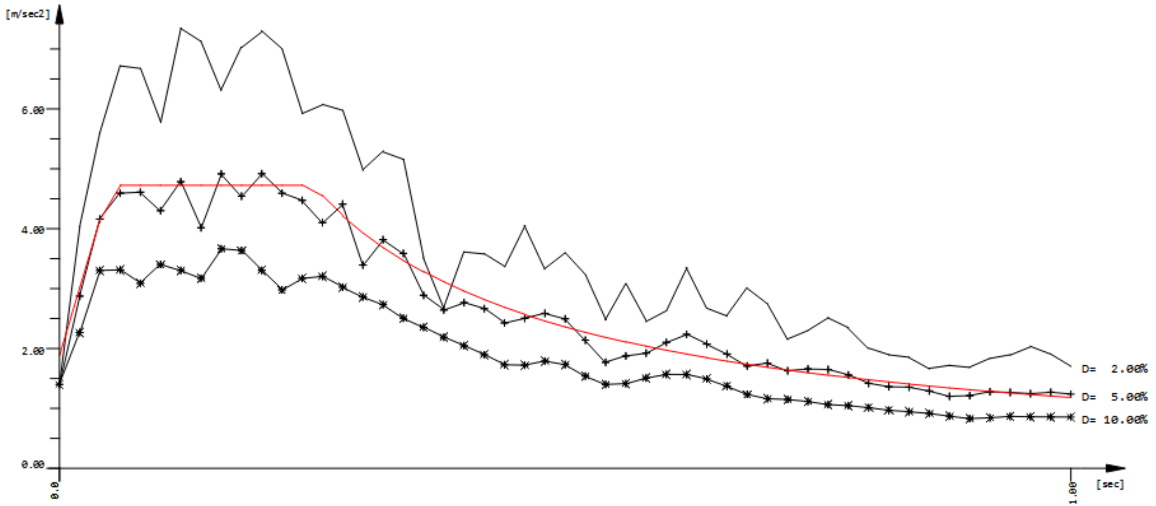
load case 33	My	43.32	Mz	23.32
load case 34	Mz	43.32	My	23.32

- Combining the $E_x \oplus 0.3 \cdot E_y$ on LC 21 and 22:

load case 33	My	47.12	My	10.00
load case 34	Mz	47.12	My	10.00

Direct integration with Time Histories

- Generate artificial earthquakes from design code response spectra



How many Histories?

- design code EN 1998: at least 3, recommended 5
- Newer recommendations of the ASCE require 11

Number of Collapses	Likelihood for Various P[C|MCE$_R$] Values				
	0.05	0.10	0.15	0.20	0.30
0 of 11	93%	74%	51%	30%	7%
1 of 11	7%	23%	36%	38%	21%
2 of 11	0%	3%	11%	22%	29%
3 of 11	0%	0%	2%	8%	24%
4 of 11	0%	0%	0%	2%	13%
5 of 11	0%	0%	0%	0%	5%

Comparison for given Example

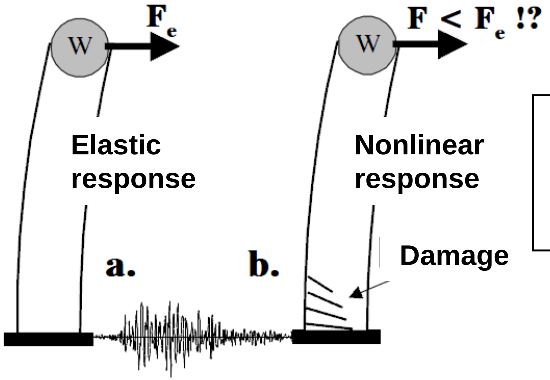
	max M_y	zug. M_z	min M_y	zug. M_z	max M_z	zug. M_y	min M_z	zug. M_y
Transient 1	58,53	15,30	-66,77	-18,65	57,42	-19,29	-53,82	-17,57
Transient 2	62,36	-21,17	-49,95	22,20	59,06	6,66	-53,26	-6,11
Transient 3	46,06	7,07	-42,61	-3,50	41,78	8,29	-44,60	-11,81
Mean(1-11)	55,14	-0,16	-57,03	3,47	55,75	4,66	-56,56	-10,81
SRSS	47,52	0,00	-47,52	0,00	47,52	0,00	-47,52	0,00

Alternate approaches

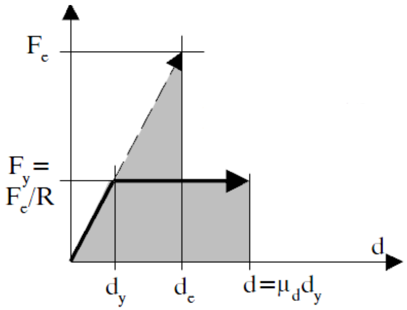
- Drawbacks
  - Statistical Results are quite better now but still not perfect
  - No non linear effects so far
  - Phase differences may become important for larger structures
- Possible Extensions
  - Direct Time Integration with many artificially generated earthquakes
  - Push-Over-Analysis

Why go NONLINEAR?

- Earthquake is a displacement, not a force!
- Earthquake is a probabilistic event – Should we invest into something that might not even happen (*Tp=475 years*)?
- Earthquake is a powerful force of nature (*ag=1.0g*)
- Price to be paid for nonlinearity – Ductility!

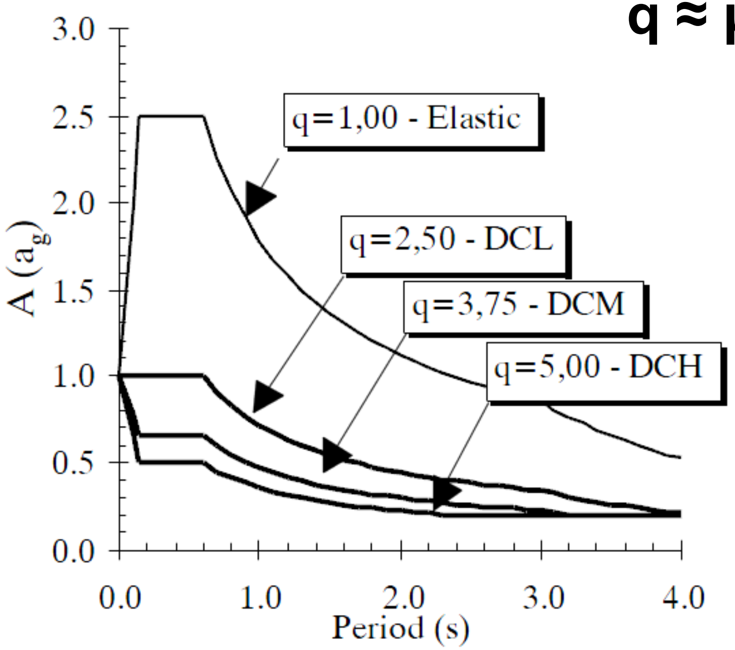


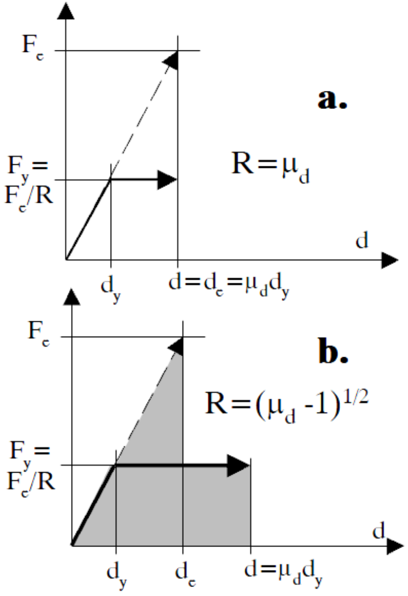
PARADOX?
Smaller force –
Higher
damage



Nonlinear (Design) Response Spectrum

q ≈ μd





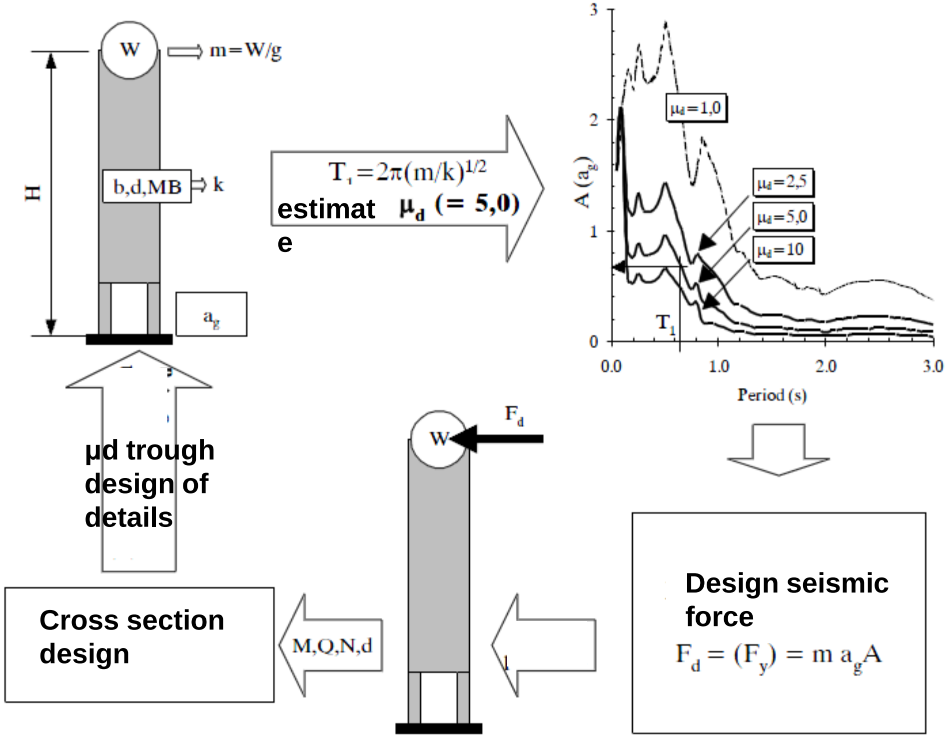
V. Alender (2004)

Behaviour factor q

Table 6.2: Reference values of behaviour factors for systems regular in elevation

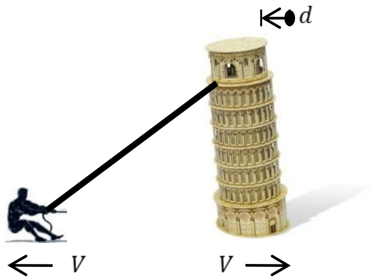
STRUCTURAL TYPE	Ductility Class	
	DCM	DCH
a) Moment resisting frames	4	$5\alpha_u/\alpha_1$
b) Frame with concentric brancings		
Diagonal bracings	4	4
V-bracings	2	2,5
c) Frame with eccentric bracings	4	$5\alpha_u/\alpha_1$
d) Inverted pendulum	2	$2\alpha_u/\alpha_1$
e) Structures with concrete cores or concrete walls	See section 5	
f) Moment resisting frame with concentric bracing	4	$4\alpha_u/\alpha_1$
g) Moment resisting frames with infills		
Unconnected concrete or masonry infills, in contact with the frame	2	2
Connected reinforced concrete infills	See section 7	
Infills isolated from moment frame (see moment frames)	4	$5\alpha_u/\alpha_1$

Concept of calculation of the structures using Nonlinear Response Spectrum



V. Alender (2004)

PUSHOVER Analysis



- **Static nonlinear procedure**

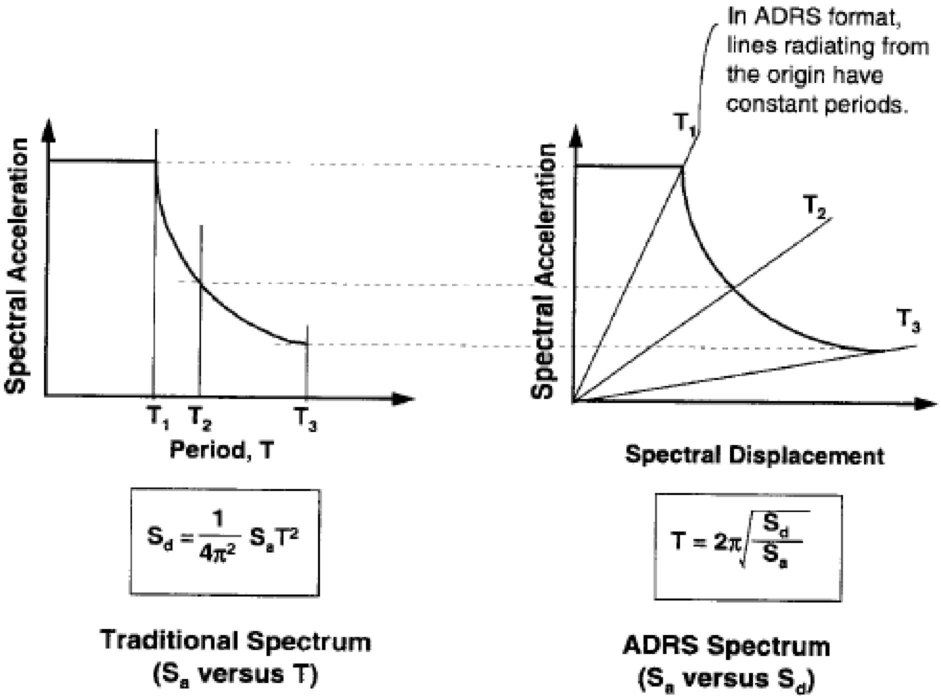
- **Comprises of two steps**
  1) Determination of the capacity of the structure under a specified loading patern
  2) Determination of the performance of the structure under a chosen demand (Earthquake) and a given capacity

- **In earthquake (and dynamic in general) analysis demand depends also on the capacity of the structure (stiffnes, ductility, etc.).**

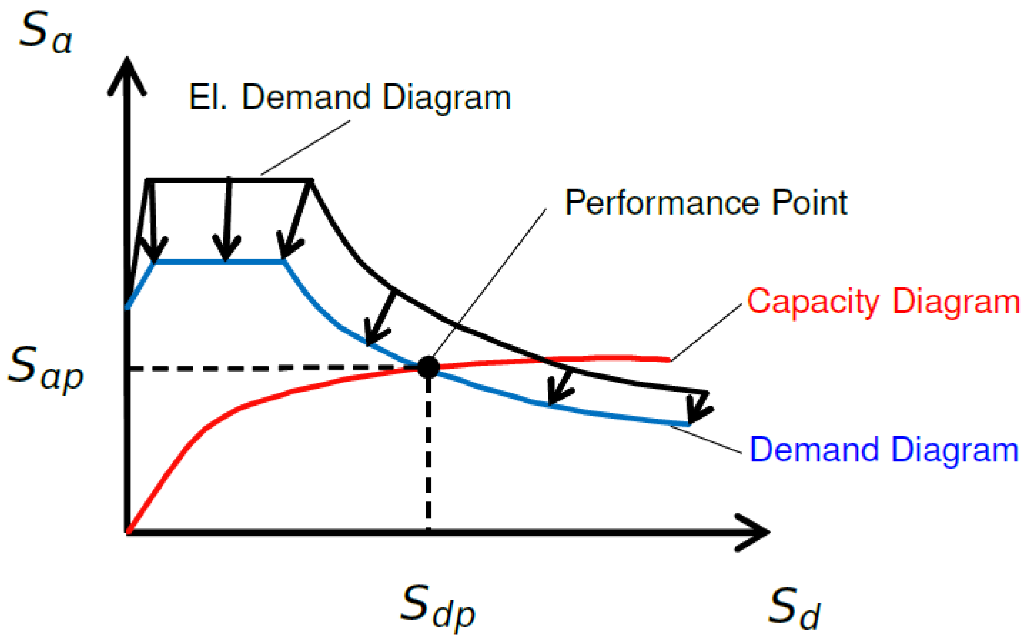
ADRS - Spectra

- Converting the Time-Scale into a spectral displacement (based on the harmonic relation)

$$S_d = \frac{T^2}{4\pi^2} \cdot S_a \cdot a$$

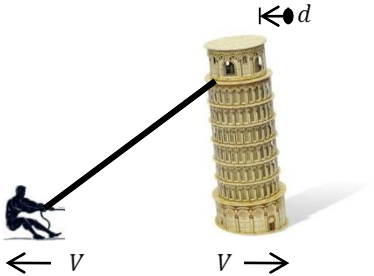


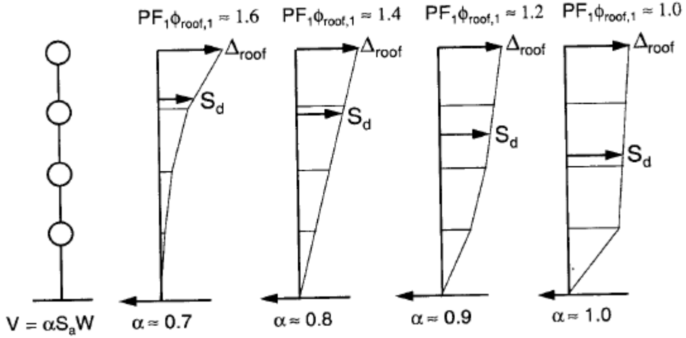
Performance Analysis



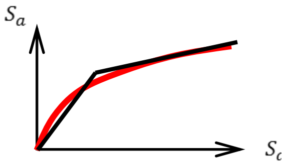
Push-Over-Loading

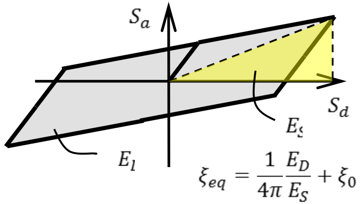
- Apply a representative horizontal acceleration





From Static to „Dynamic“





Push-Over-Curve

- Base Shear Force as a function of a characteristic roof displacement
- Convert to the ADRS-Format

$$S_a = \frac{V}{\alpha \cdot W} = \frac{\sum m_i \cdot \phi_i \cdot a_g}{\alpha \cdot \sum m_i \cdot g}$$

$$S_d = \frac{\Delta_{roof}}{PF \cdot \phi_{roof}}$$

Conversion

- Simplification

$m_i\ \phi_i^2 = 1.0$

$$PF = \frac{\sum m_i \cdot \phi_i}{\sum m_i \cdot \phi_i^2}$$

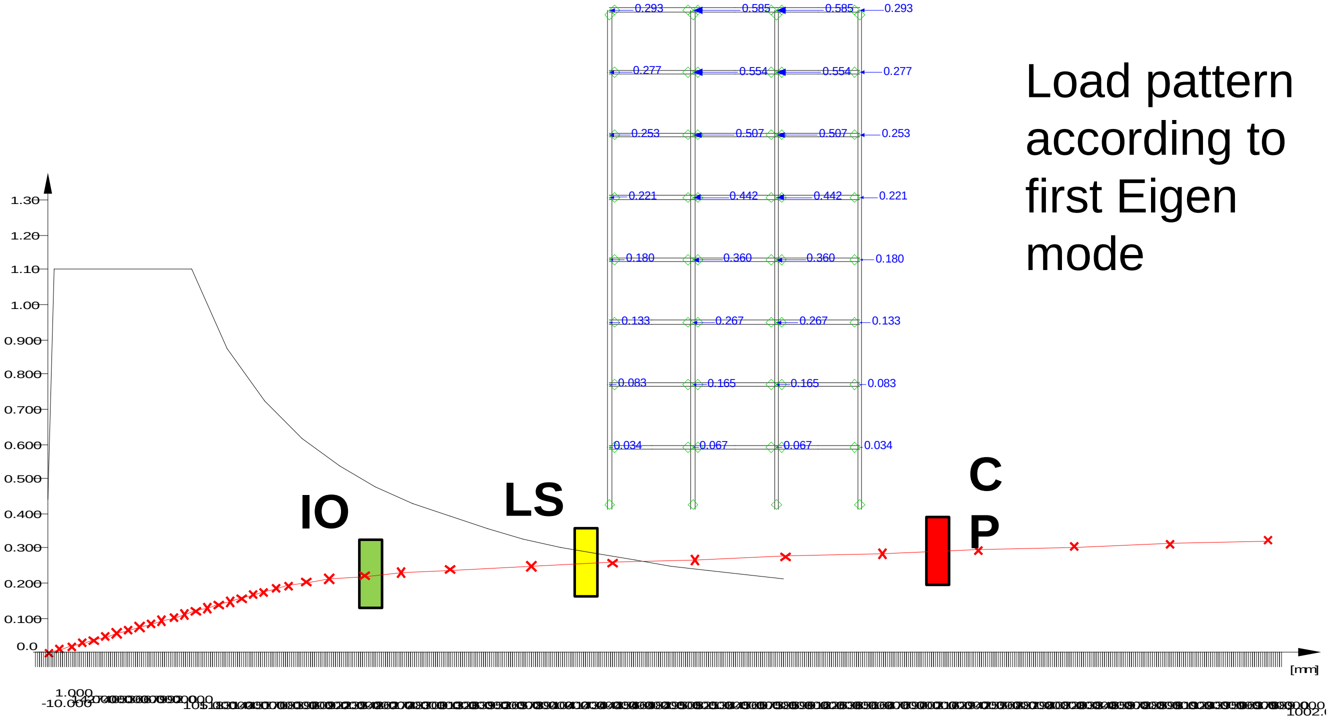
Total

$$\alpha = \frac{\left[\sum m_i \cdot \phi_i\right]^2}{\left[\sum m_i\right] \cdot \left[\sum m_i \cdot \phi_i^2\right]}$$

$S_a\ g$

$$S_a \cdot g = \frac{a_g}{\sum m_i \cdot \phi_i}$$

Capacity curve (Spectrum)



Load pattern according to first Eigen mode

SOFiSTiK

Structural Performance Levels and Damage

Table C1-3 Structural Performance Levels and Damage[1, 2, 3]—Vertical Elements

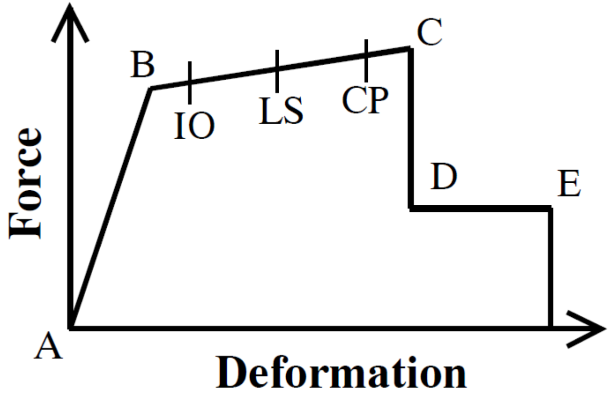
Elements	Type	Structural Performance Levels: Collapse Prevention S-5	Life Safety S-3	Immediate Occupancy S-1
Concrete Frames	Primary	Extensive cracking and hinge formation in ductile elements. Limited cracking and/or splice failure in some nonductile columns. Severe damage in short columns.	Extensive damage to beams. Spalling of cover and shear cracking (<1/8" width) for ductile columns. Minor spalling in nonductile columns. Joint cracks <1/8" wide.	Minor hairline cracking. Limited yielding possible at a few locations. No crushing (strains below 0.003).
	Secondary	Extensive spalling in columns (limited shortening) and beams. Severe joint damage. Some reinforcing buckled.	Extensive cracking and hinge formation in ductile elements. Limited cracking and/or splice failure in some nonductile columns. Severe damage in short columns.	Minor spalling in a few places in ductile columns and beams. Flexural cracking in beams and columns. Shear cracking in joints <1/16" width.
	Drift	4% transient or permanent	2% transient; 1% permanent	1% transient; negligible permanent
Steel Moment Frames	Primary	Extensive distortion of beams and column panels. Many fractures at moment connections, but shear connections remain intact.	Hinges form. Local buckling of some beam elements. Severe joint distortion; isolated moment connection fractures, but shear connections remain intact. A few elements may experience partial fracture.	Minor local yielding at a few places. No fractures. Minor buckling or observable permanent distortion of members.
	Secondary	Same as primary.	Extensive distortion of beams and column panels. Many fractures at moment connections, but shear connections remain intact.	Same as primary.
	Drift	5% transient or permanent	2.5% transient; 1% permanent	0.7% transient; negligible permanent

FEMA

Investor (owner) choses a performance level for a given Earthquake (50-50,20-50,5-50)

SOFiSTiK

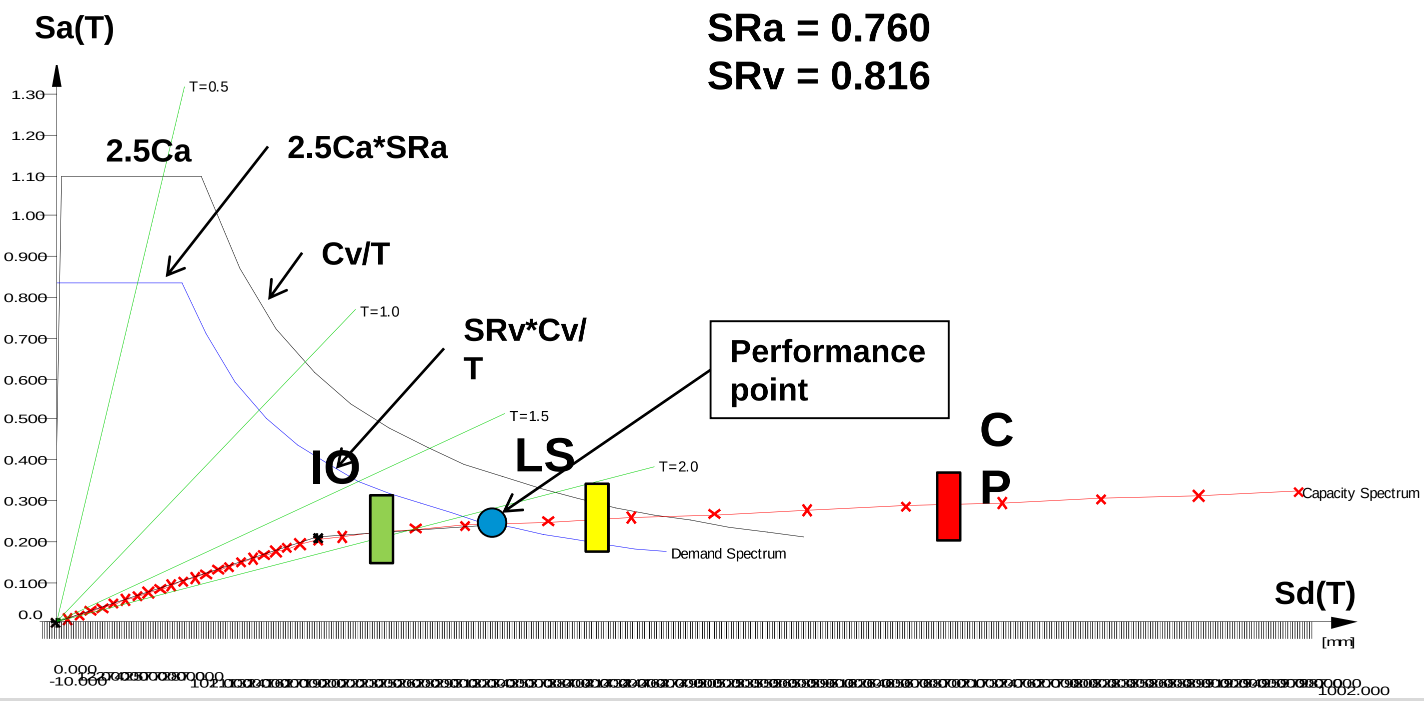
Element Perfomance Levels



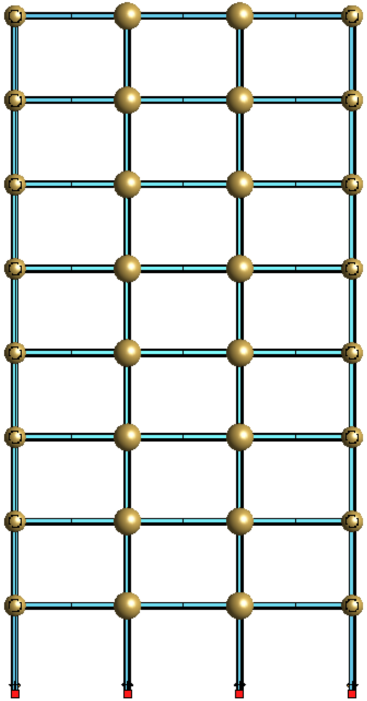
Generalized force-deformation relation for steel elements or components

SOFiSTiK

Capacity and Demand Spectra (ADRS)

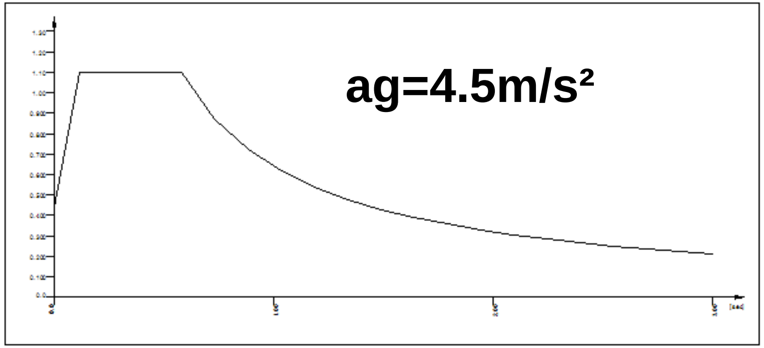


Two-Dimensional 8-Storey Frame



Response spectra   Uniform Building Code: Z 4 SD

D[-]	SA[-]	SB[-]	MIN[-]	TB[sec]	TC[sec]	TD[sec]	TE[sec]	K1[-]	K2[-]	A[m/sec2]
0.0500	0.440	1.100	0.000	0.116	0.582	3.000	0.000	1.000	1.000	10.00
	Zone = 4			ah =*	1.000	av =*	1.000			



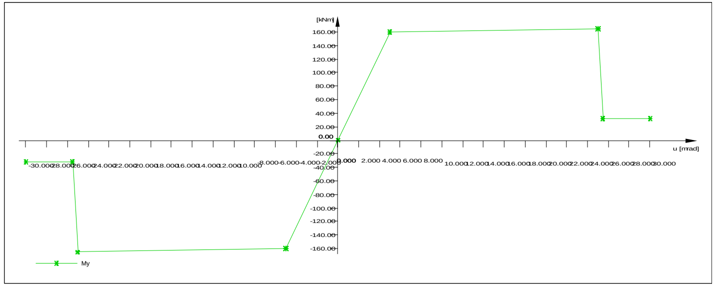
Loads acting on Nodes

Node	A-X	A-Y	A-Z	A-RX	A-RY	A-RZ
	[m/sec2]	[m/sec2]	[m/sec2]	[1/sec2]	[1/sec2]	[1/sec2]
0	12.27					

H=8x3m   S 235   Column HEA 240
L=3x4m   Beam   IPE 240

Plastic Hinges

- **Properties according to FEMA (Table 5-6)**



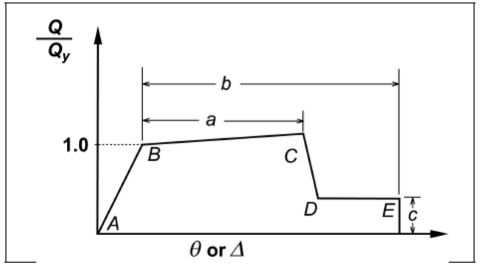
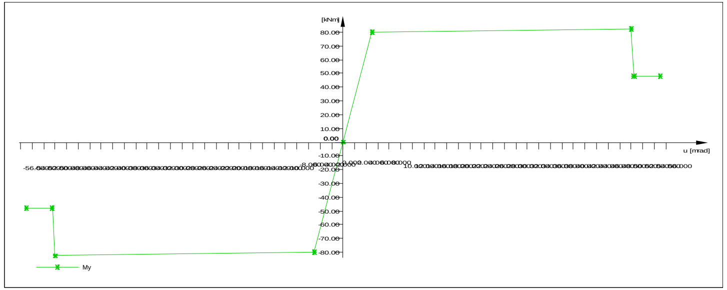


Figure 5-1 *Generalized Force-Deformation Relation for Steel Elements or Components*

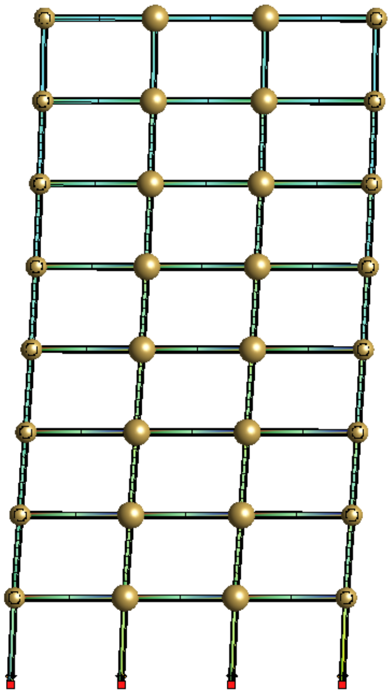


Beams: $\theta_y = \frac{ZF_{ye}l_b}{6EI_b}$ $\quad Q_{CE} = M_{CE} = ZF_{ye}$

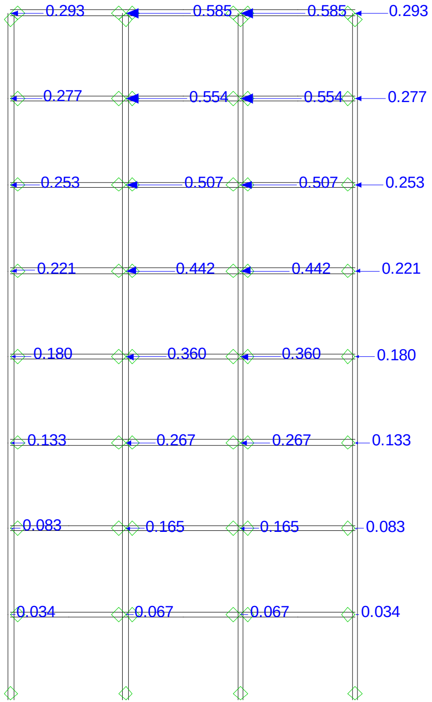
Columns: $\theta_y = \frac{ZF_{ye}l_c}{6EI_c}\left(1 - \frac{P}{P_{ye}}\right)$

$$Q_{CE} = M_{CE} = 1.18ZF_{ye}\left(1 - \frac{P}{P_{ye}}\right) \le ZF_{ye}$$

Force-Controlled Pushover



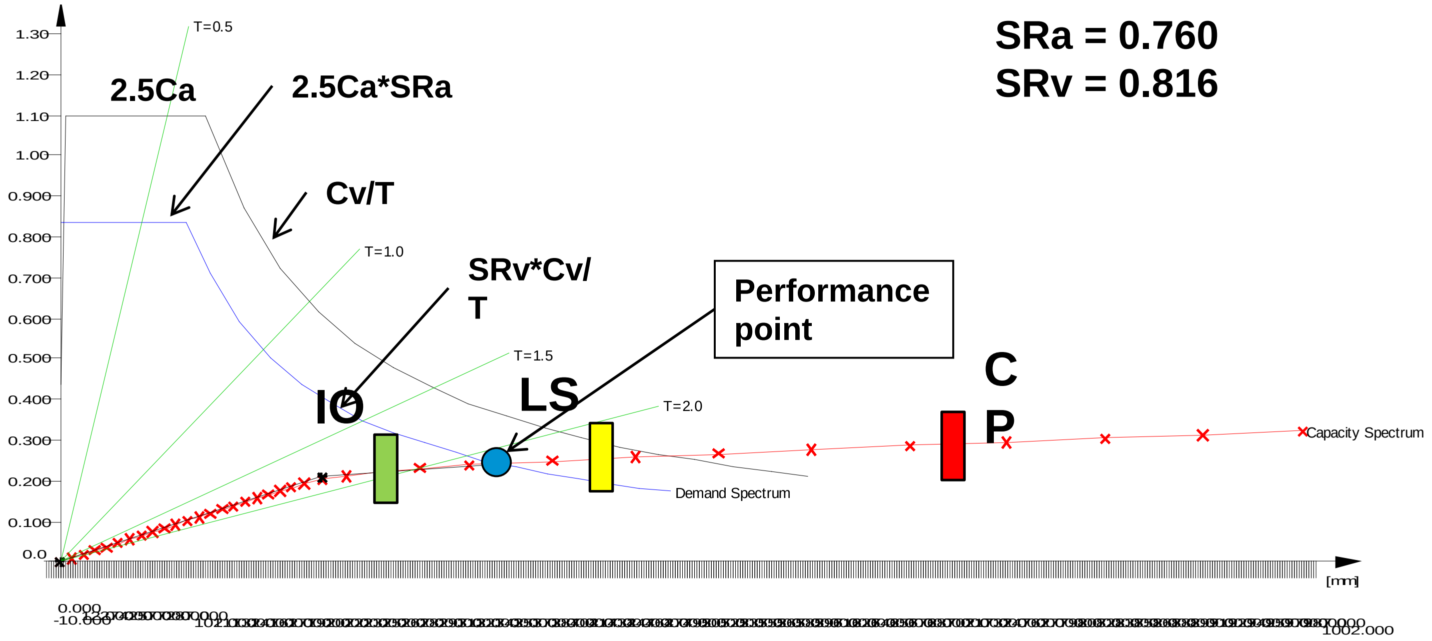
T1=1.8 Hz, modal mass=81%



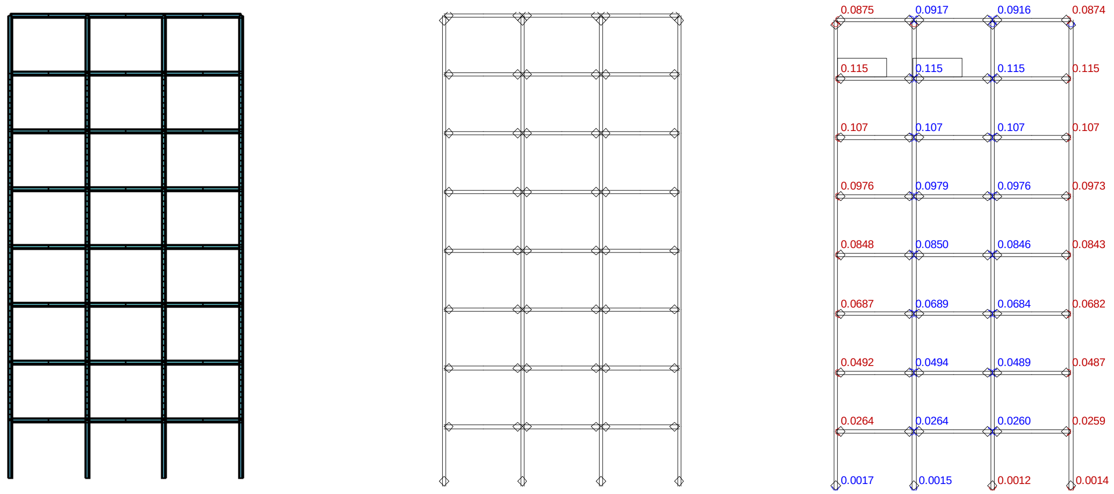
Load patern according to the first natural mode

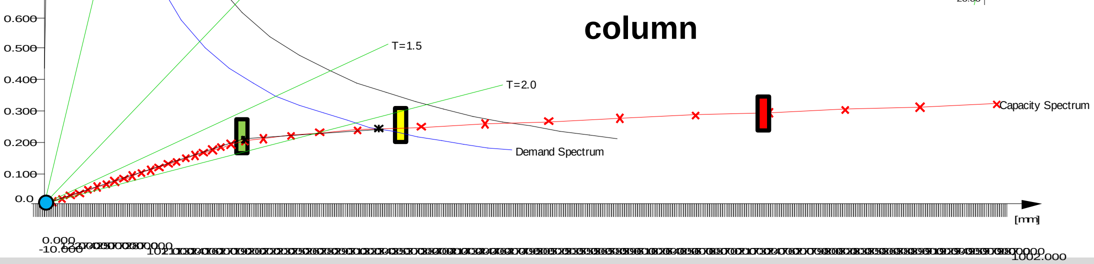
SOFiSTiK

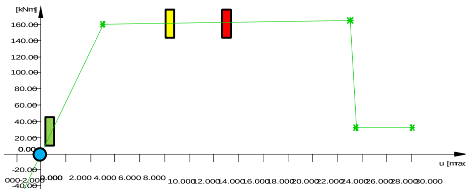
Capacity and demand spectra



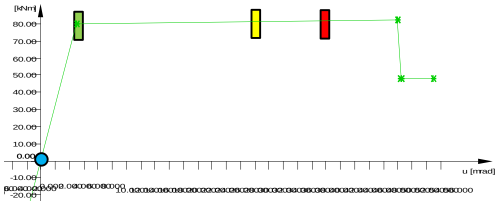
LC 01





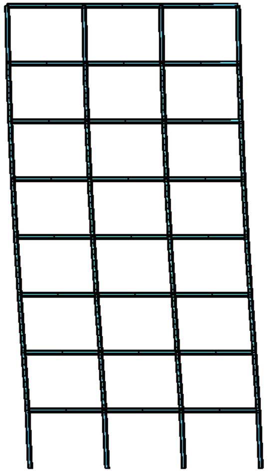


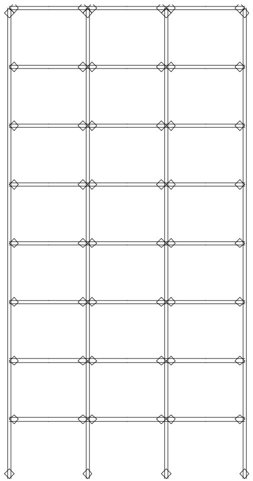
column

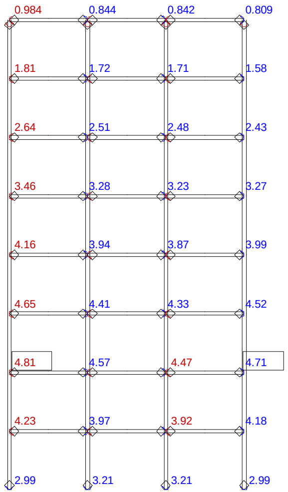


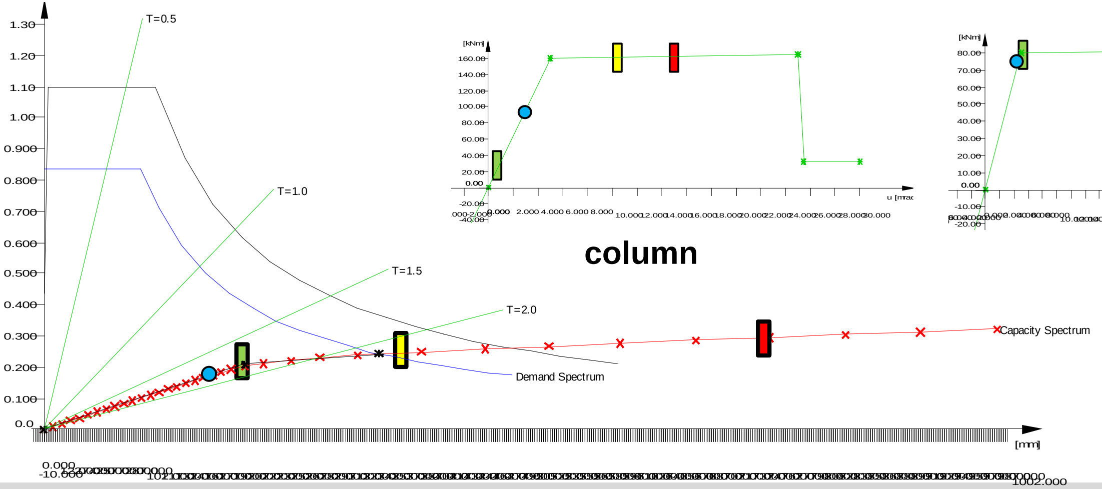
beam

LC 19

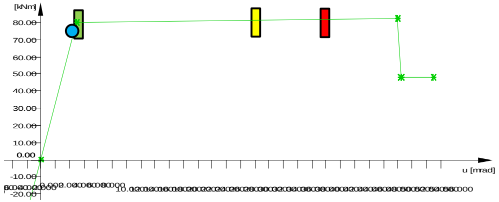






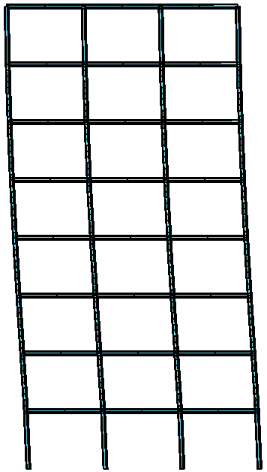


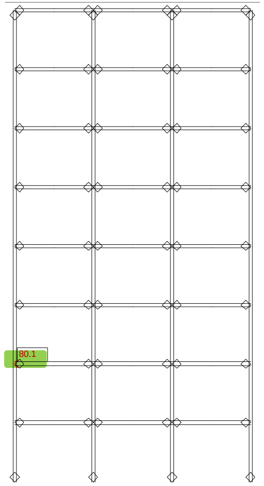
column

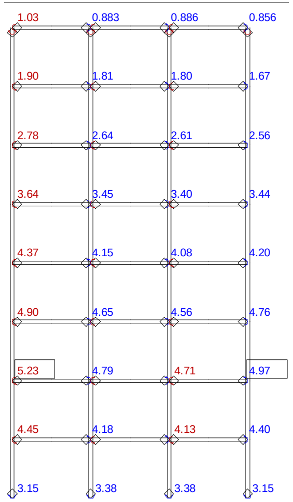


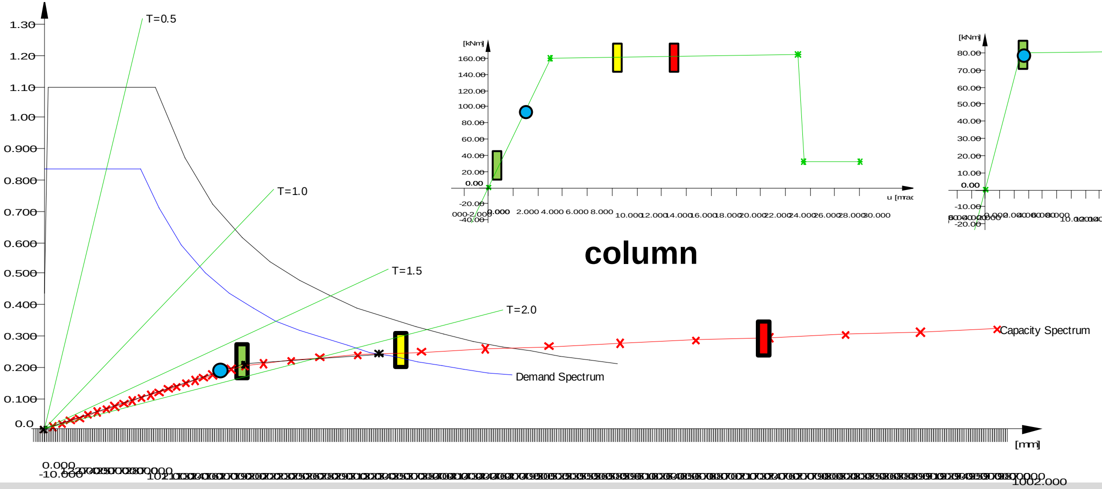
beam

LC 20

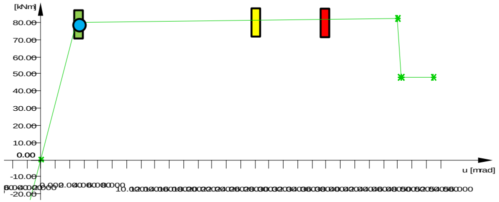






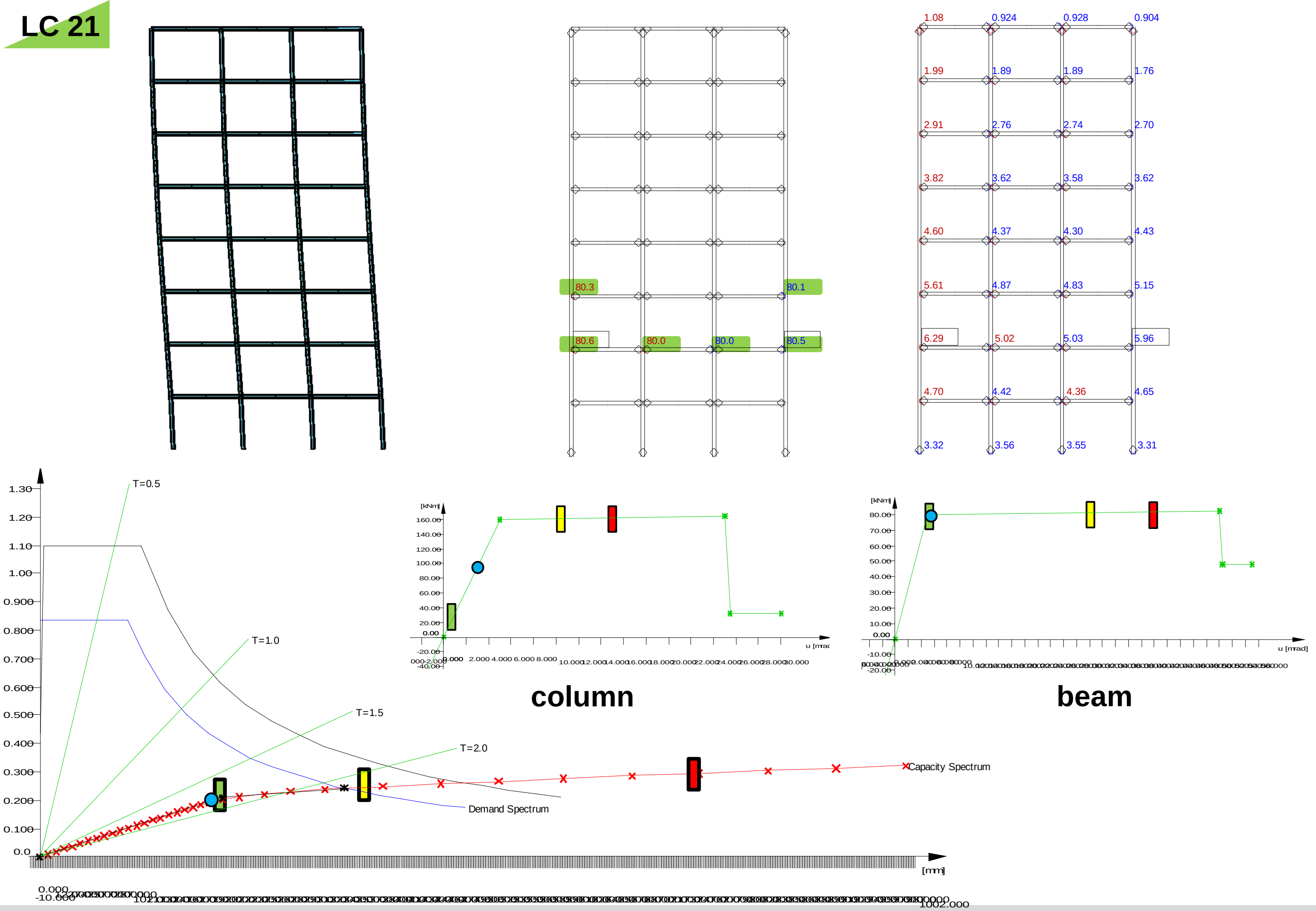


column



beam

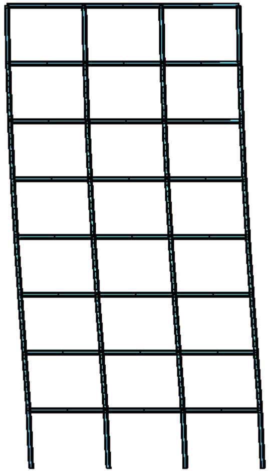
LC 21

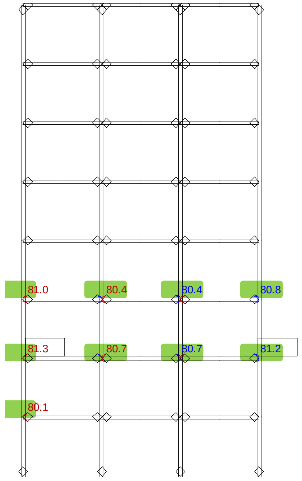


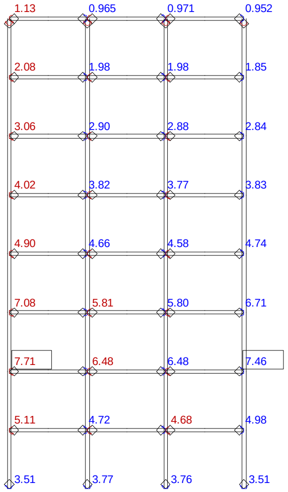
column

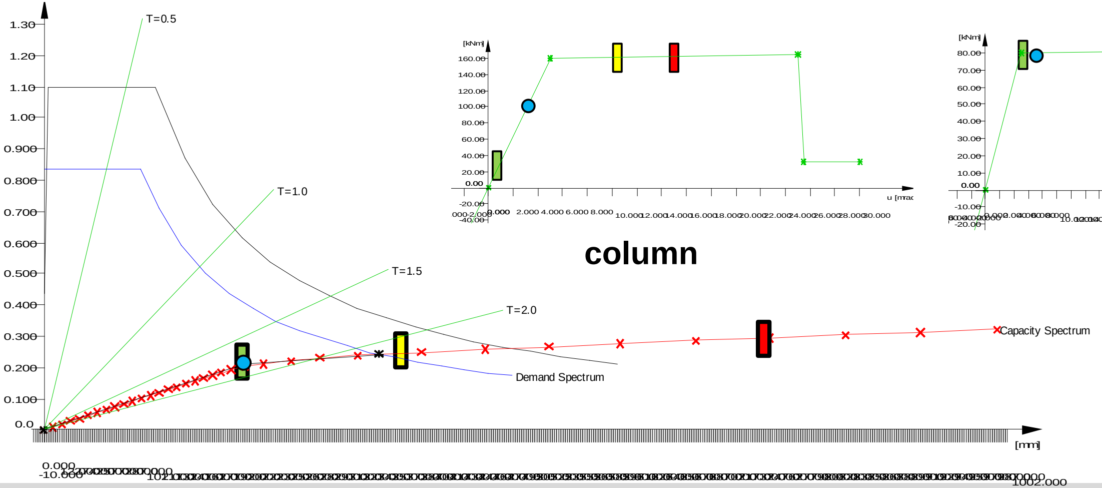
beam

LC 22

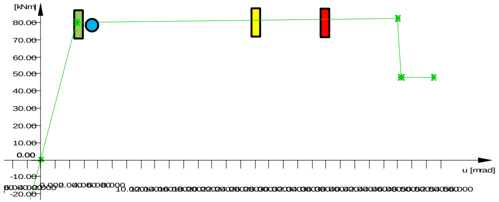






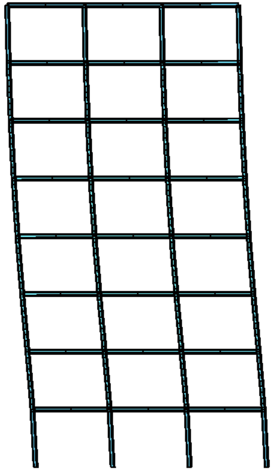


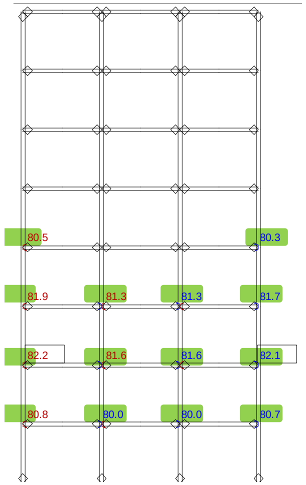
column

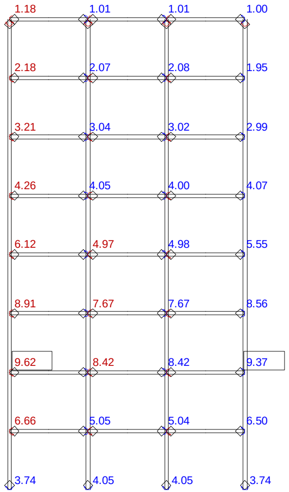


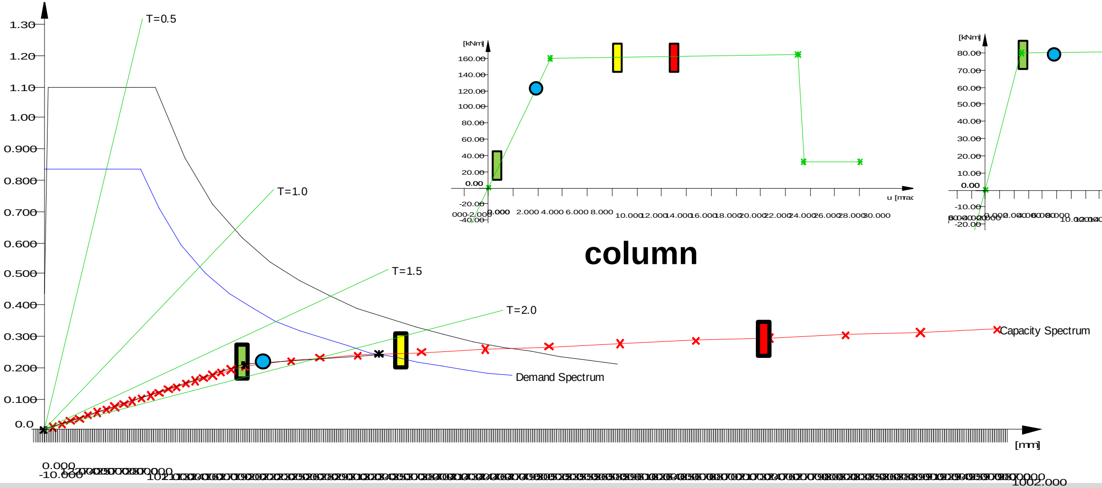
beam

LC 23

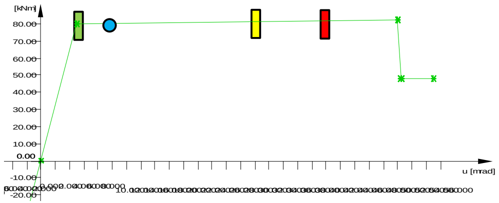






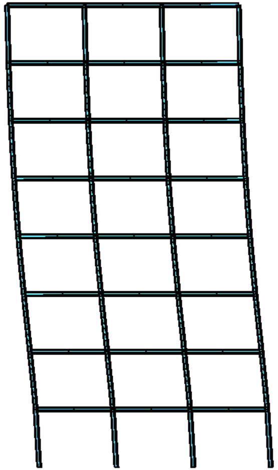


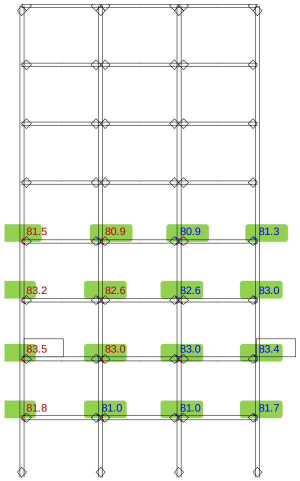
column

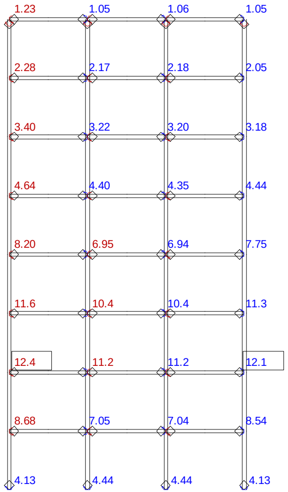


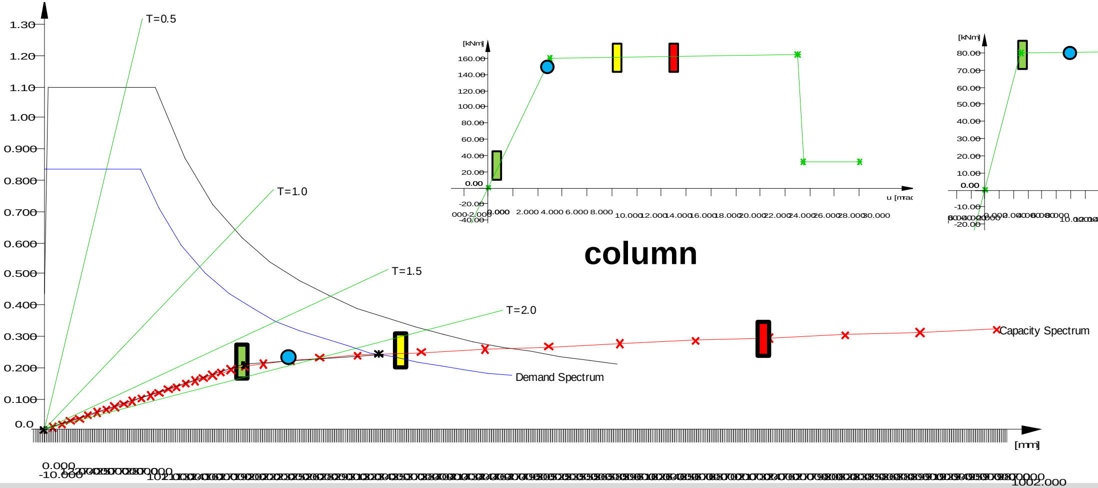
beam

LC 24

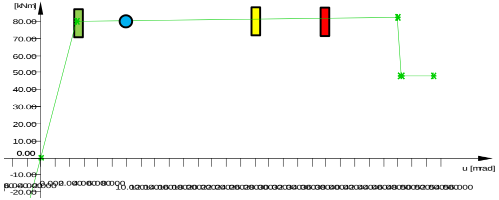






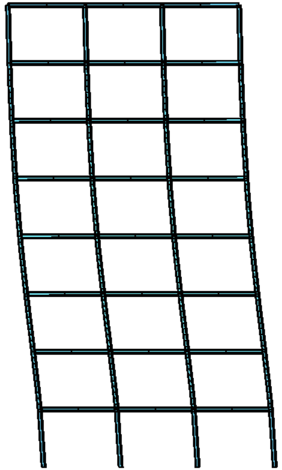


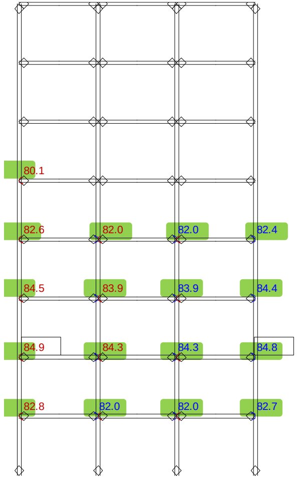
column

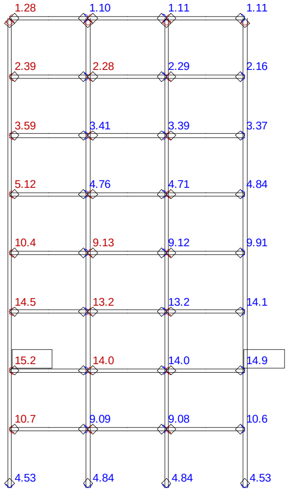


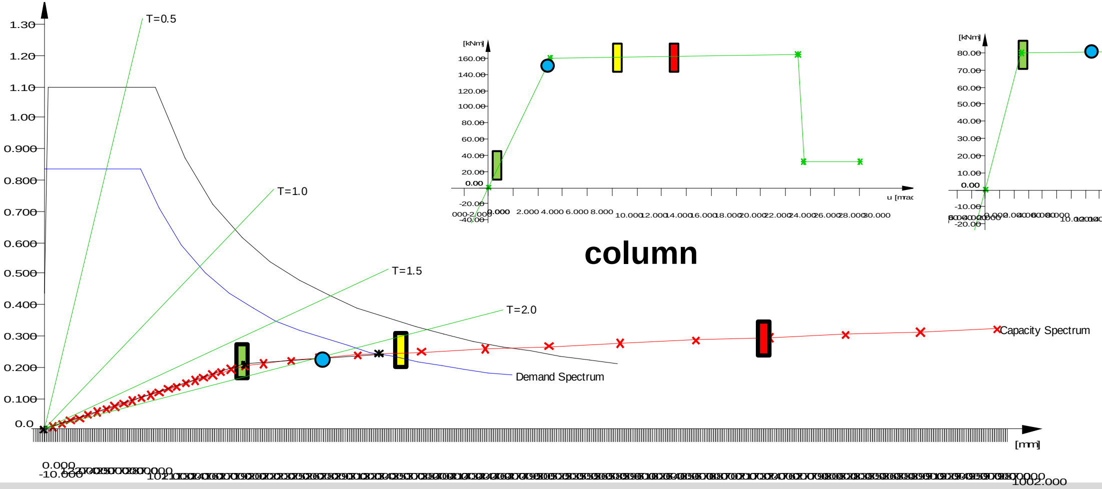
beam

LC 25









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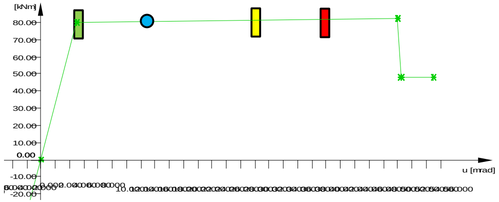
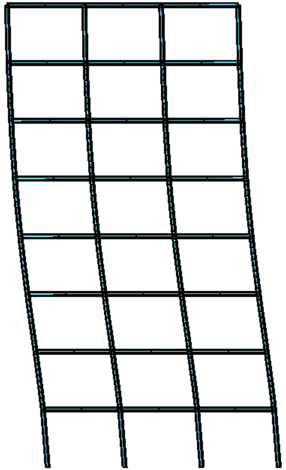
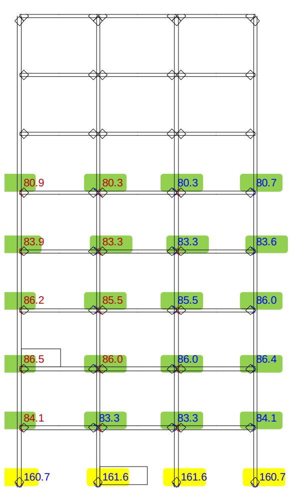
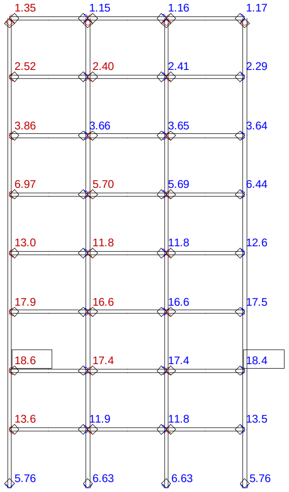


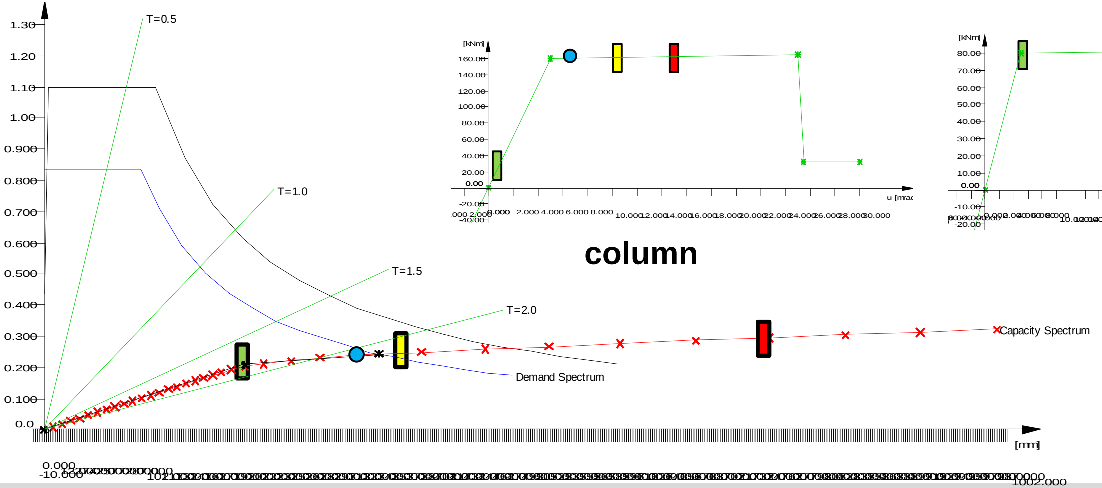
beam

LC 26









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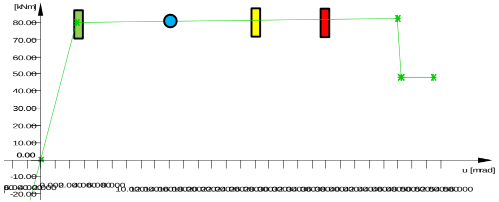
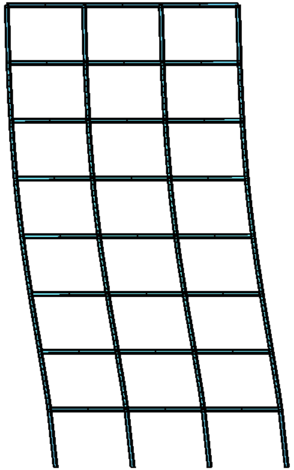
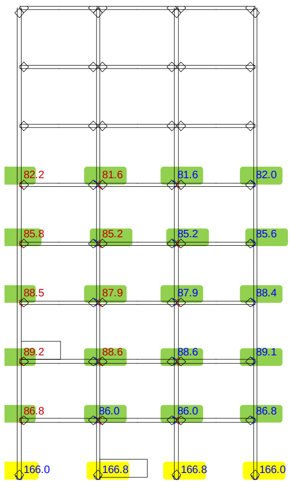
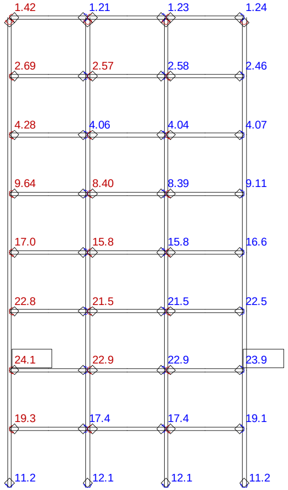


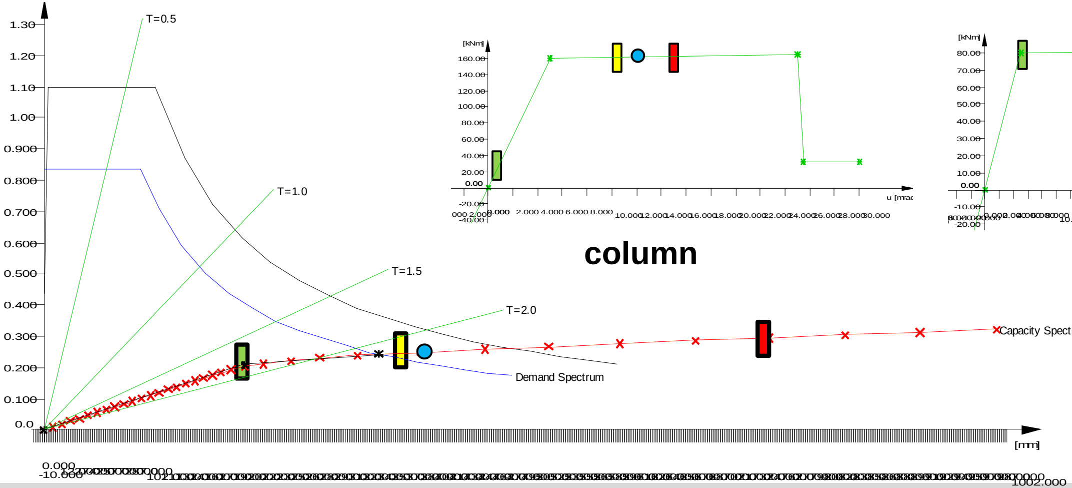
beam

LC 27

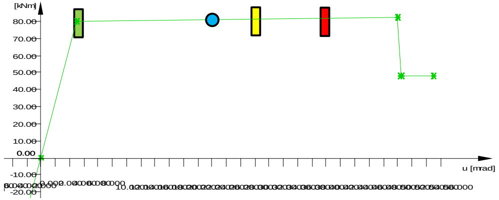






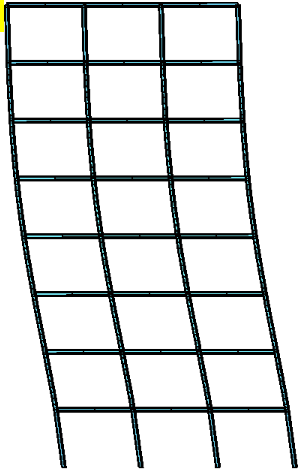


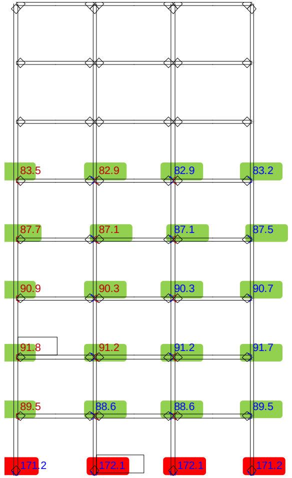
column

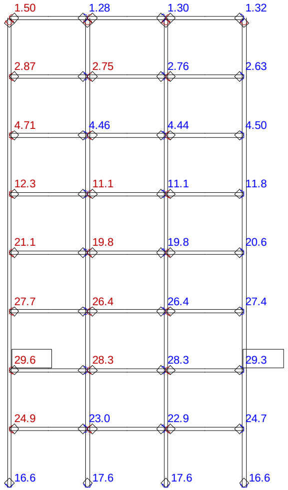


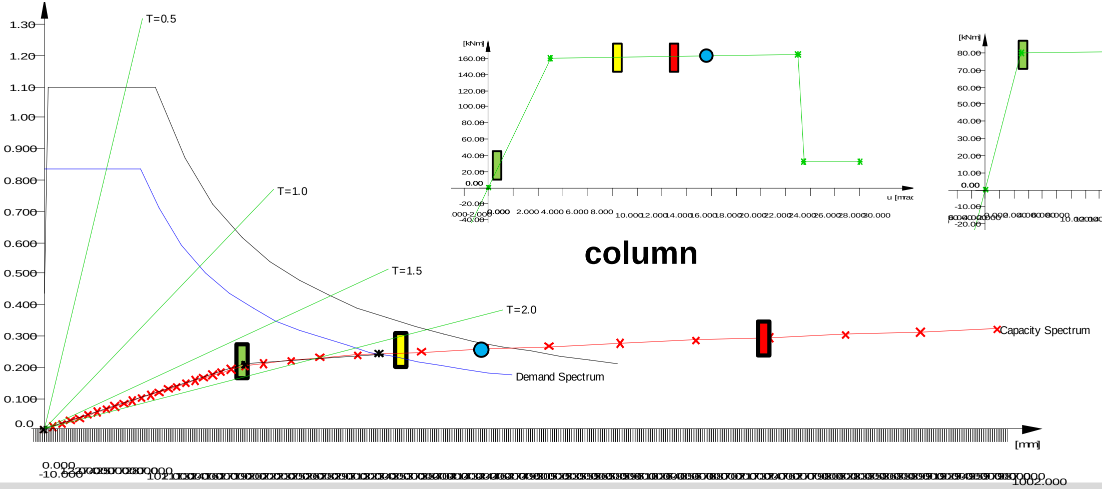
beam

LC 28









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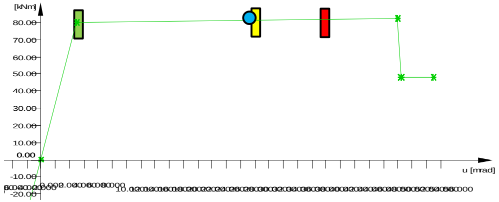
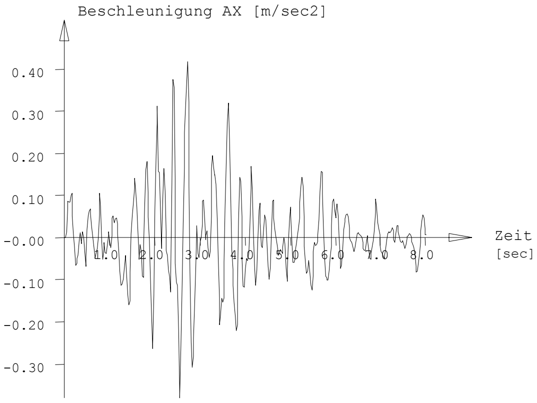


beam

Transient Analysis

- Many (!) artificial or measured earthquake response

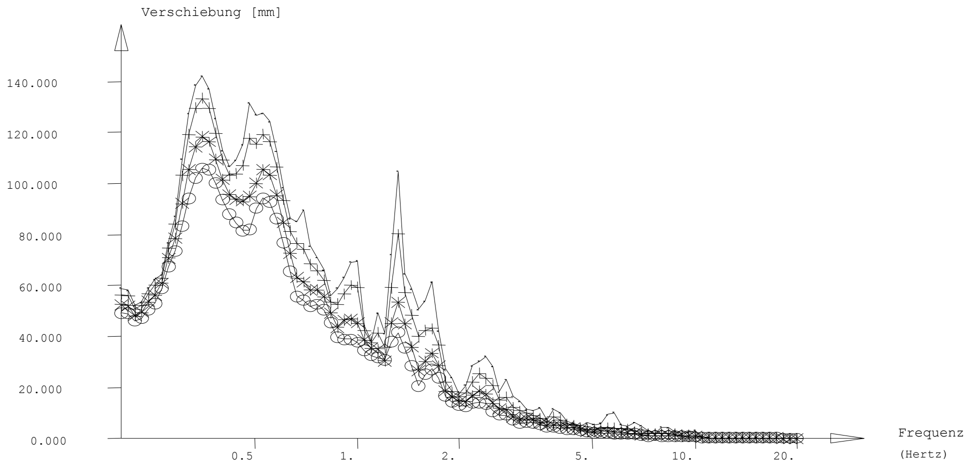


Number of Collapses	Likelihood for Various P[C|MCER] Values				
	0.05	0.10	0.15	0.20	0.30
0 of 11	93%	74%	51%	30%	7%
1 of 11	7%	23%	36%	38%	21%
2 of 11	0%	3%	11%	22%	29%
3 of 11	0%	0%	2%	8%	24%
4 of 11	0%	0%	0%	2%	13%
5 of 11	0%	0%	0%	0%	5%

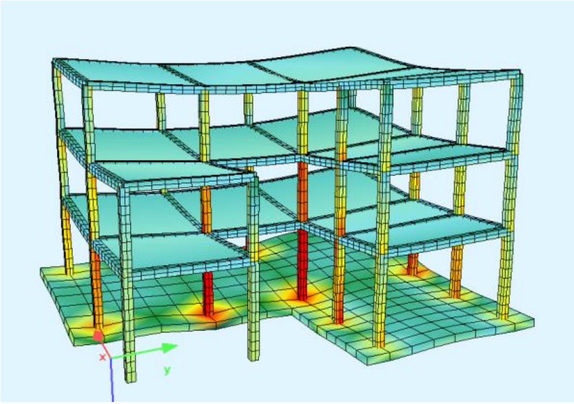
- Non linear time integration
- Averaging the results ?

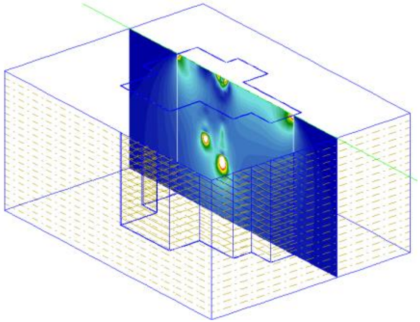
Floor response spectra (Deckenantwortspektrum)

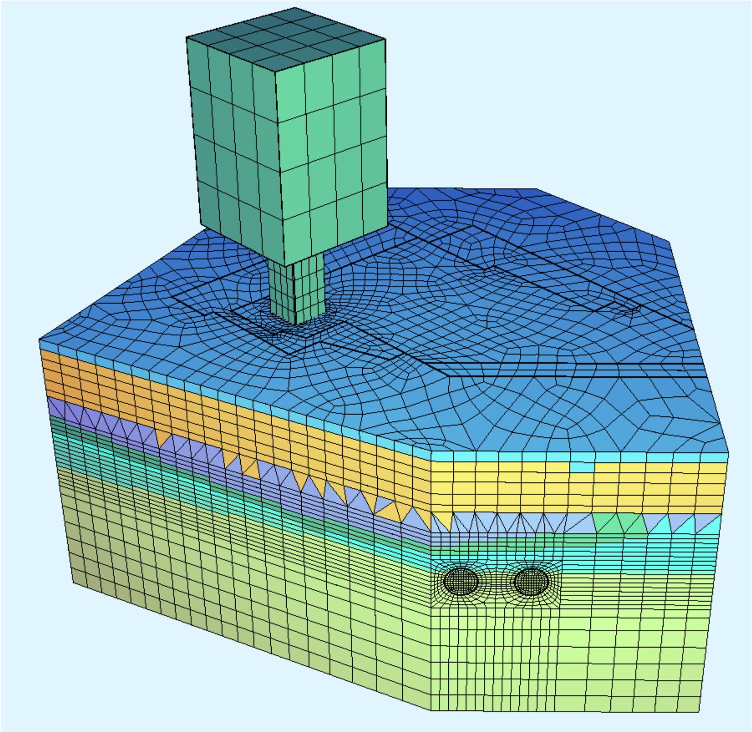
- If we have the accelerations at a given point in the structure we may then create from those values a response spectra for that point, to be applied for other components
- This is a replacement of the Transfer-Function of an Analysis in the Frequency domain



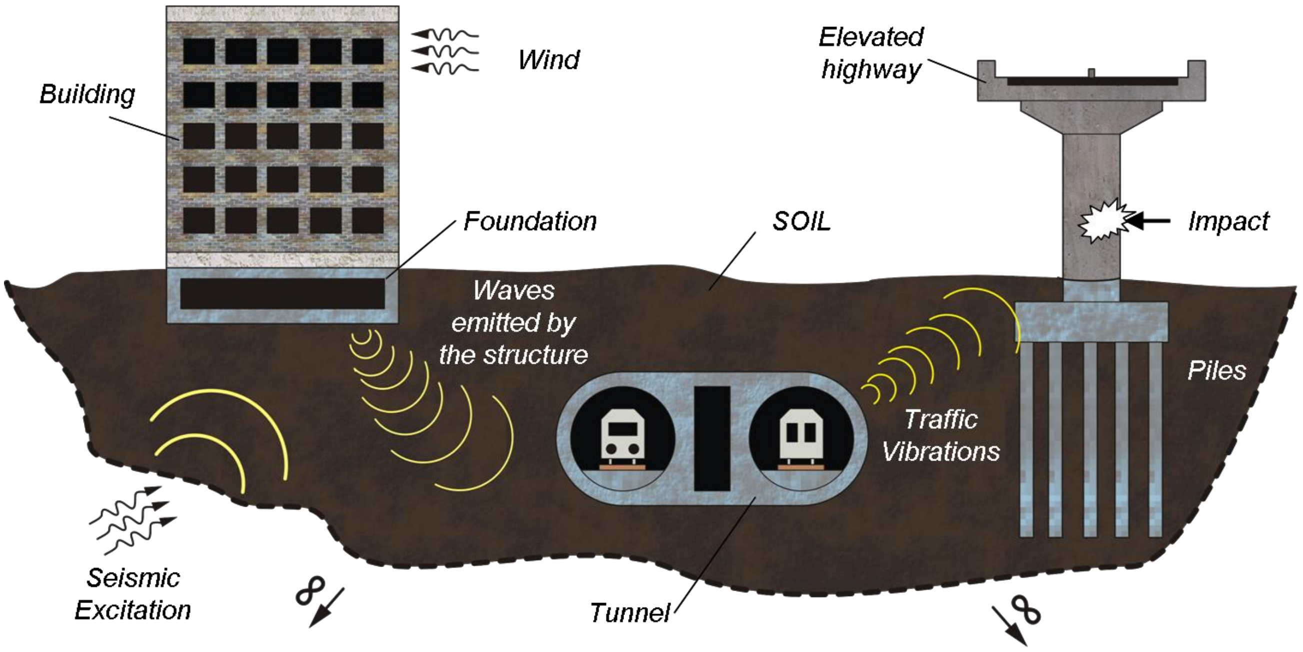
Static Soil Structure Interaction







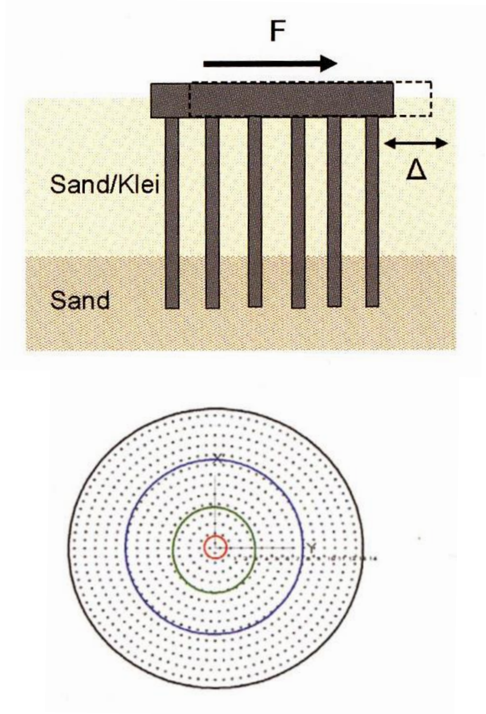
Soil-Structure-Interaction

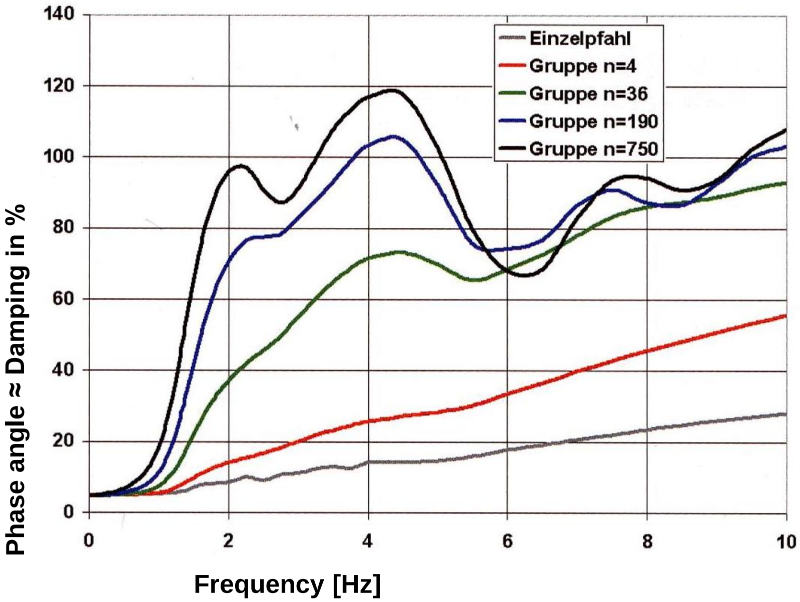


Why do we need SSI ?

- Elastic stiffness of soil has an impact on the structure
- Dynamic stiffness (mass + damping) of soil has an impact on the structure
- Favorable effects (e.g. damping)
- Unfavorable effects (e.g. change of frequency)
- Main applications
  - Earthquake
  - Dynamic loading (e.g. wind energy systems)
  - All kinds of base vibrations

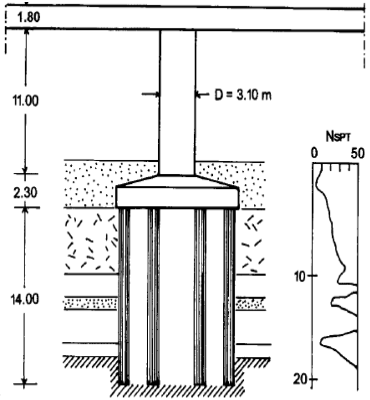
Damping of Foundation





**H. Sadegh-Azar et al., Bautechnik, 86, 2009*

Hanshin Expressway in Kobe



Without SSI*

T_{fixed} = 0.84s
ξ_{fixed} = 5 %
μ_{fixed} = 3

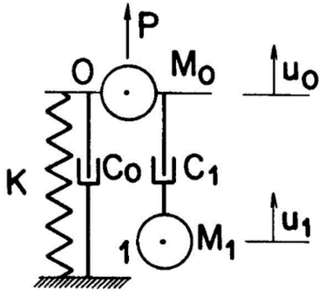
$\mu_{capacity}$ = 2 - 3

With SSI*

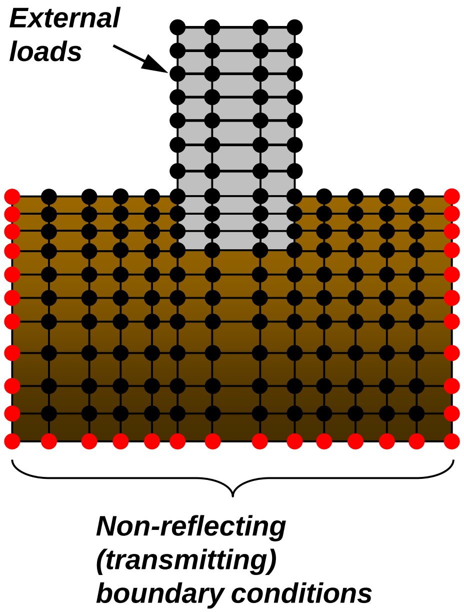
T_{SSI} = 1.04s
ξ_{SSI} = 9.8 %
μ_{SSI} = 4.7

* G. Mylonakis et al. (2006), EESD

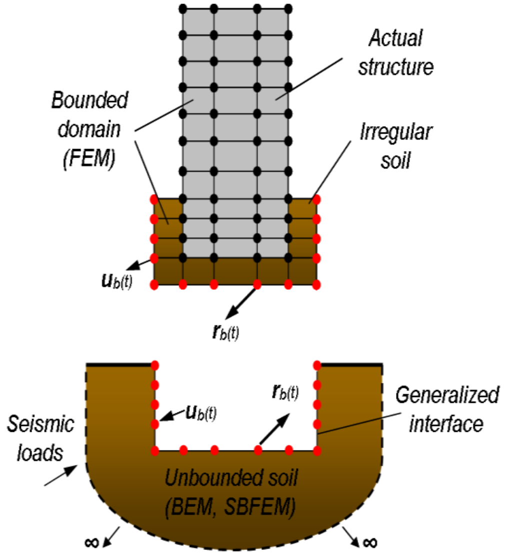
The range of possible methods for SSI



simplified



direct method



substructure

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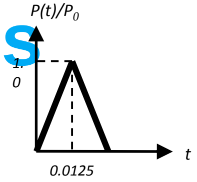
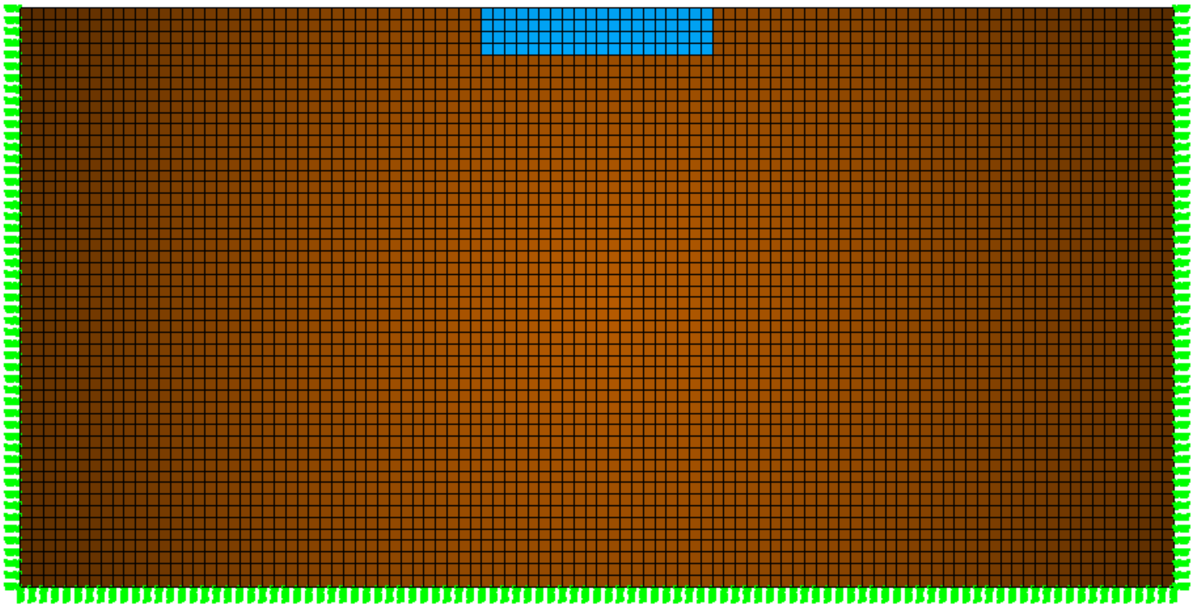
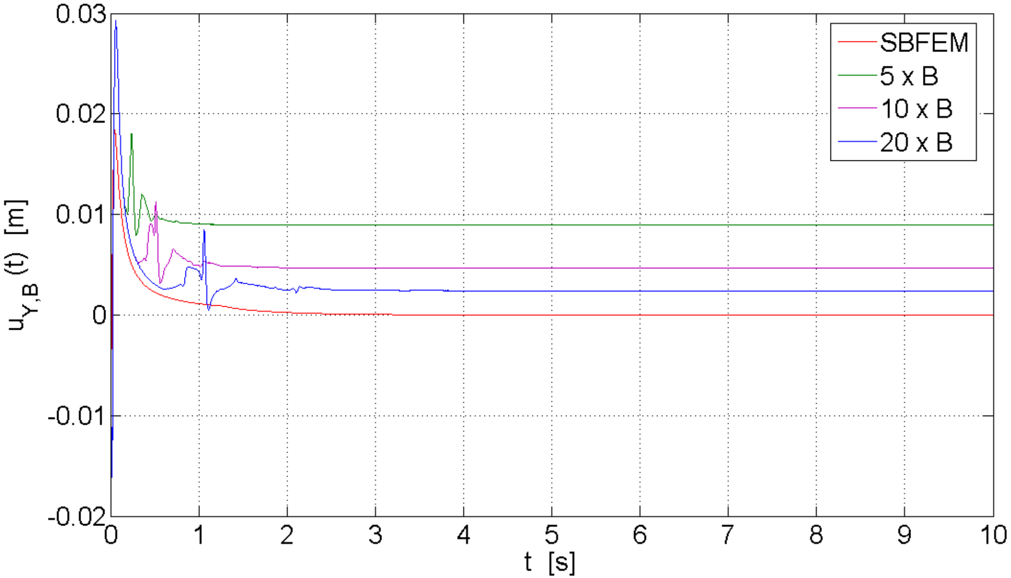
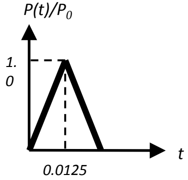
Non reflective Boundary Conditions





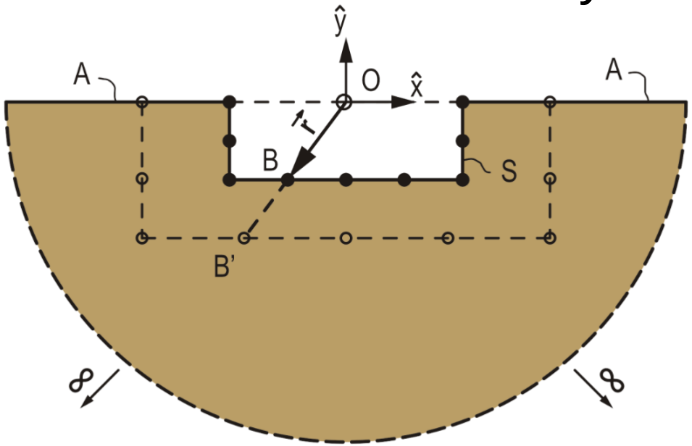
Effect of Boundary Conditions

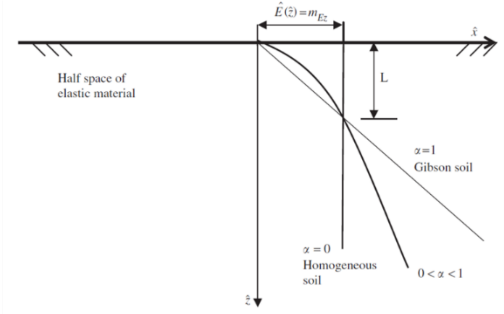




Substructure Methods

- Boundary Elements (most general solution)
- Thin Layer Elements
- SBFEM = Scaled Boundary Finite Elements



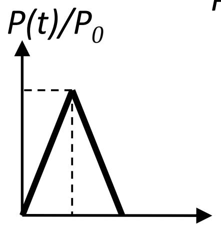


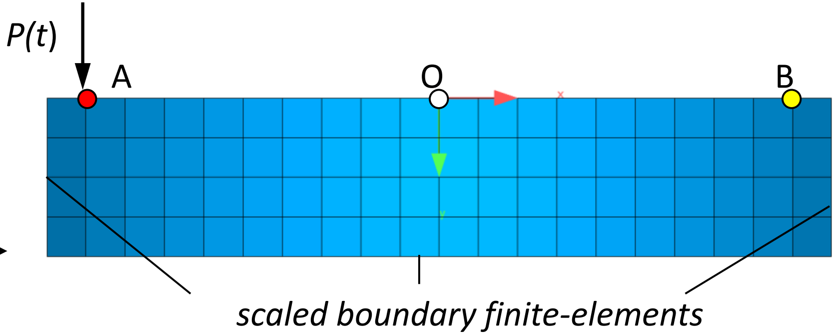
$$\mathbf{r_b}(t) = \int_0^t \mathbf{M_b^\infty}(t)\ddot{\mathbf{u}}(t-\tau)d\tau$$

Why SBFEM (J.P. Wolf / Song 1996)

- Best compromise between numerical effort and modelling possibilities
- Sound coupling to existing FEM code
- Solution in 2D and in 3D possible
- Drawbacks
  - Fully populated matrices of interface
  - Convolution Integral
  - Numerical instabilities
- Total effort may become quite large

Example 1

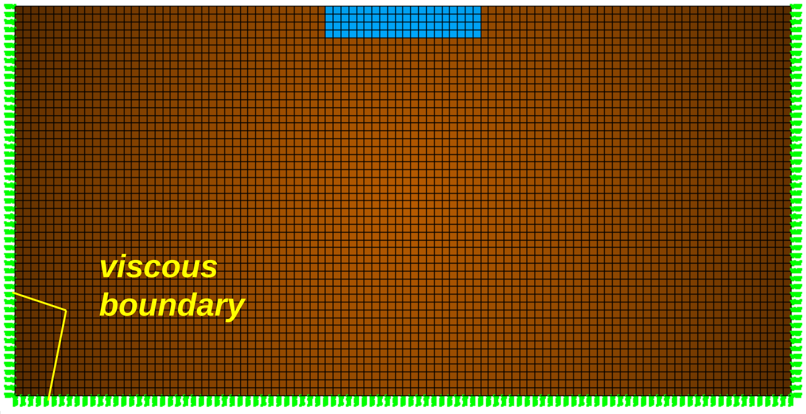




E = 200 GPa

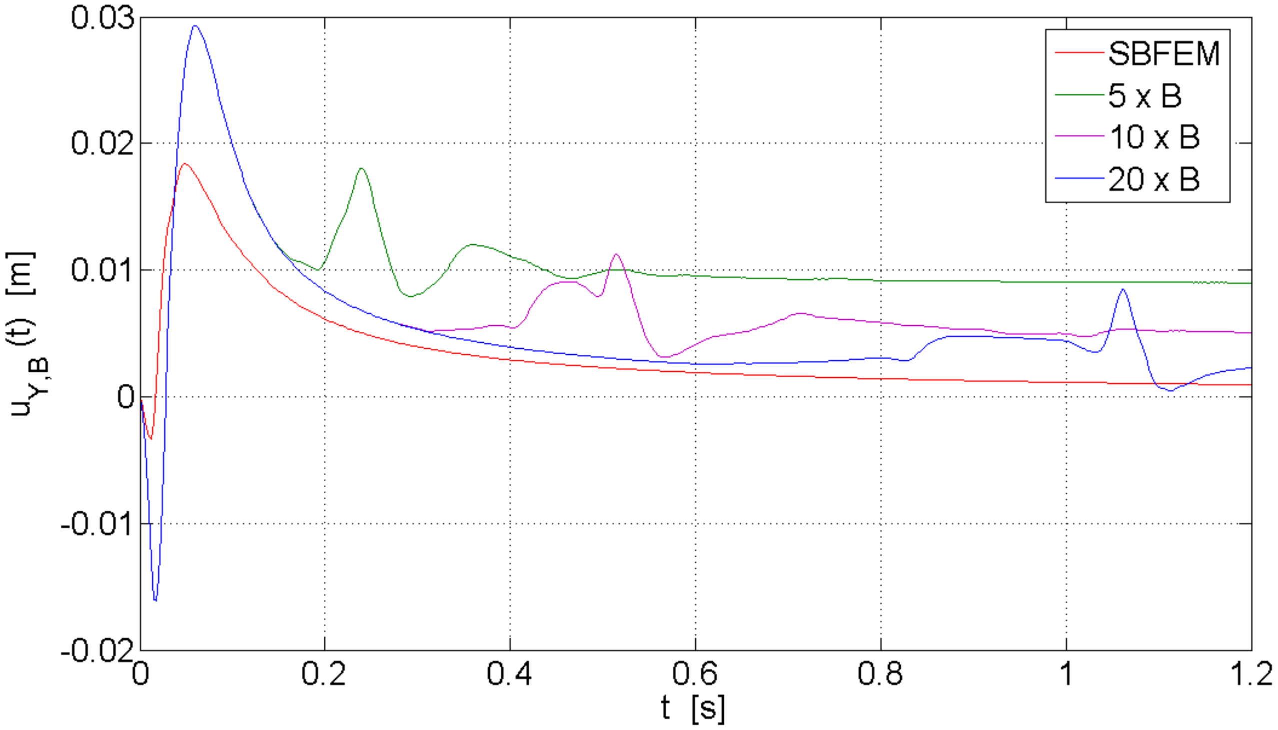
ν = 0.30

γ = 0.0

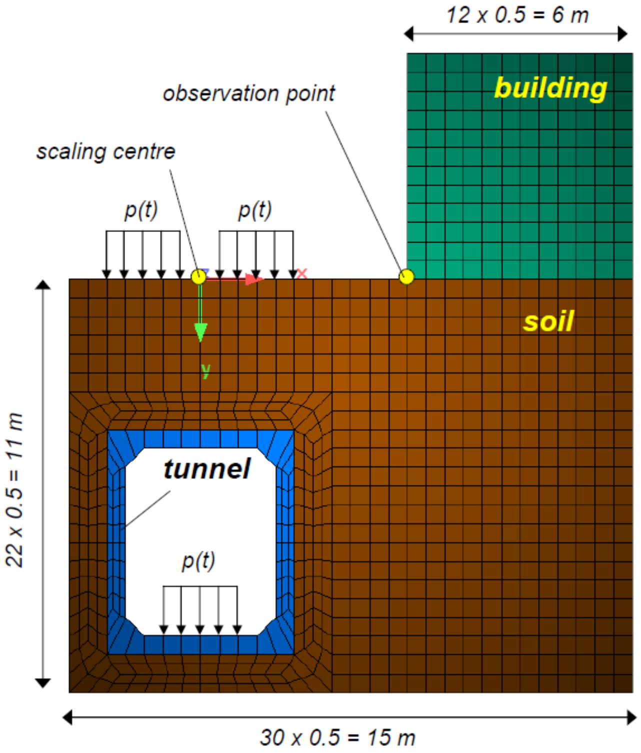


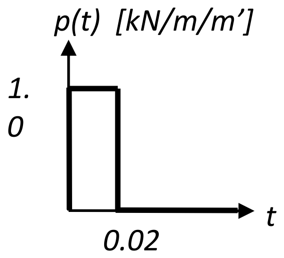
c_s = 100 m/s
ν = 0.25
ρ = 2.0 t/m^3

Results for Example 1



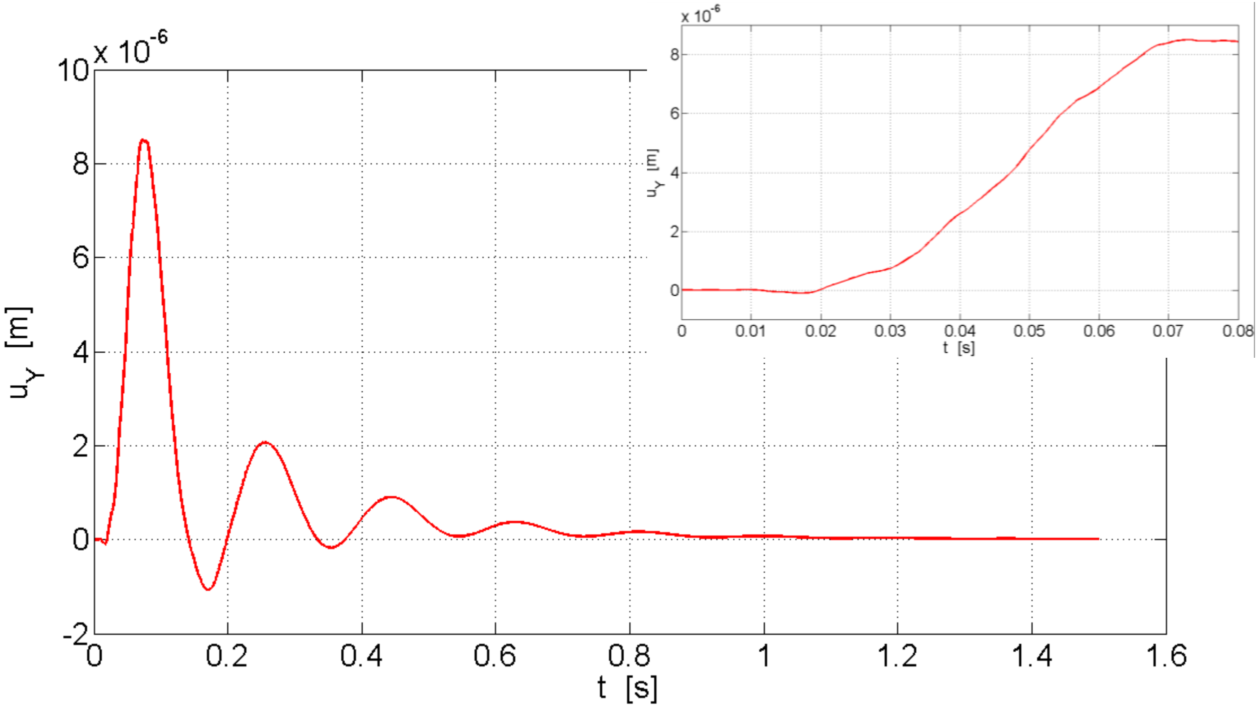
Example 2



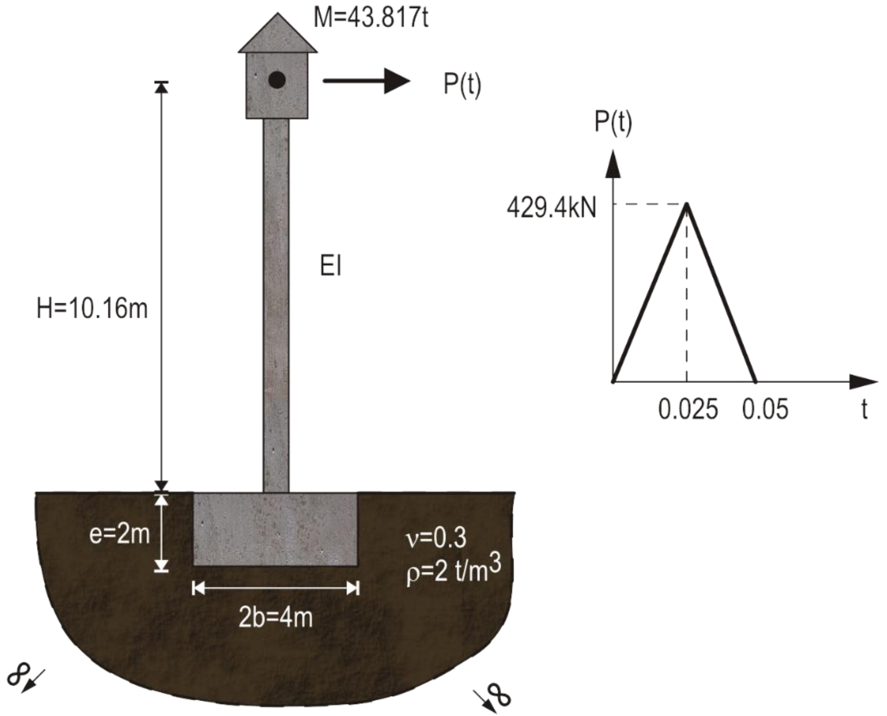


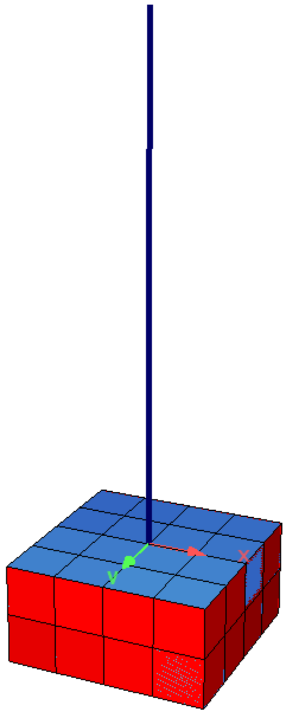
- tunnel: *E= 30 GPa, ν=0.33, ρ=2.0 t/m³*
- building: *E= 3 GPa, ν=0.30, ρ=2.0 t/m³*
- soil: *E=266 MPa, ν=0.33, ρ=2.0 t/m³*

Results Example 2

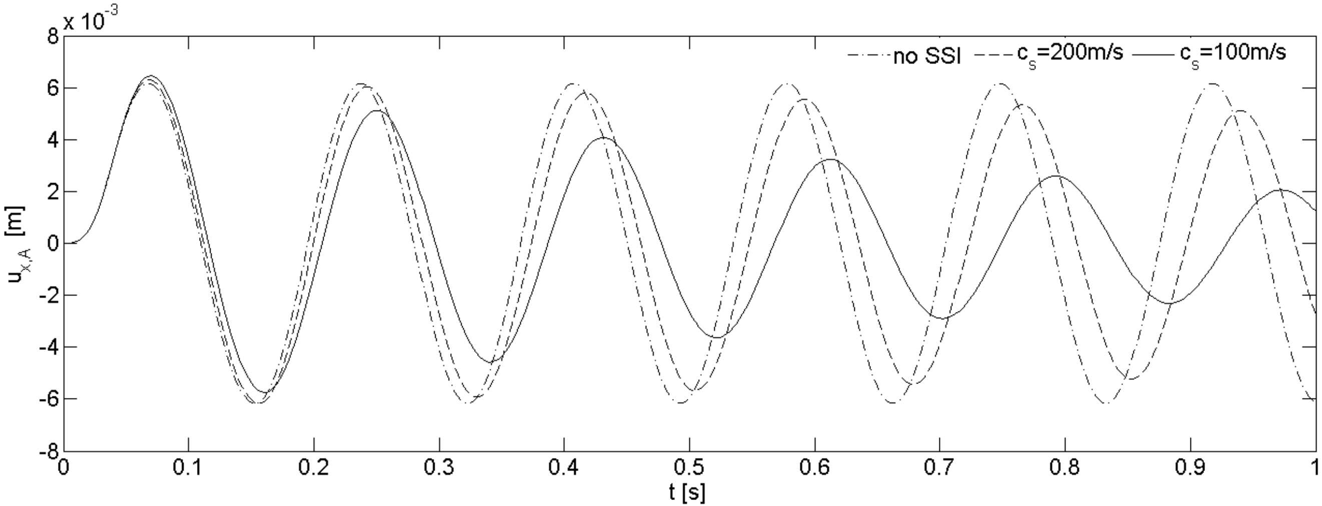


Example 3

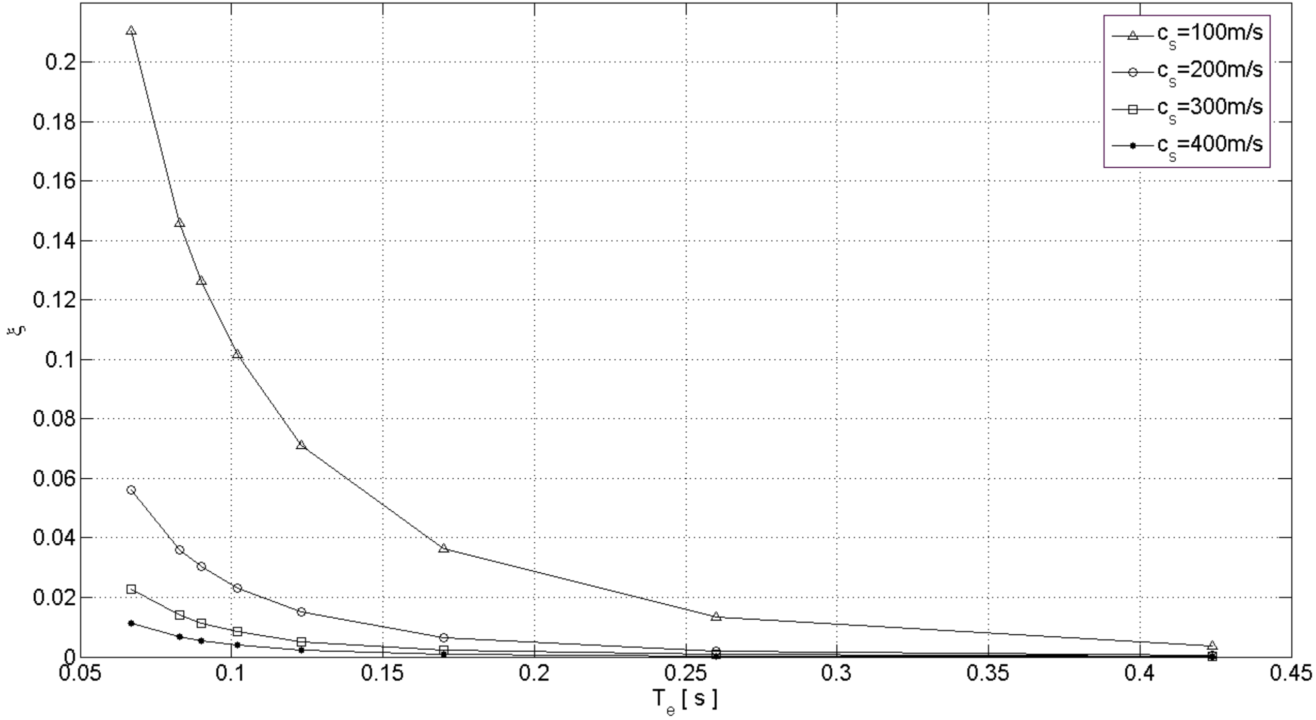




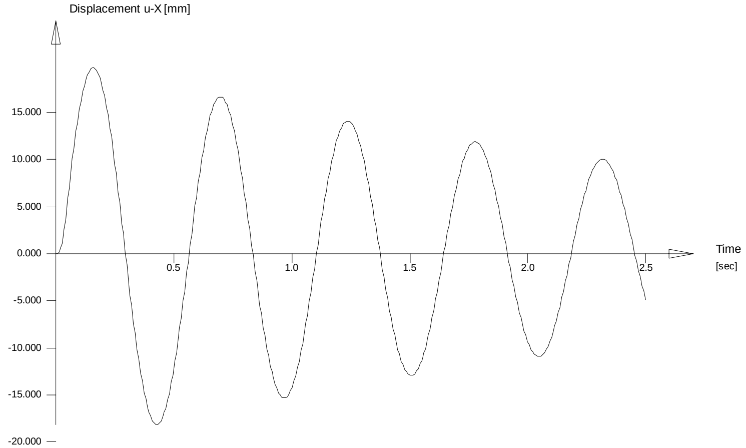
Results of Example 3



Influence of Soil Stiffness



Comparison with a spring system



- Qualitatively similar
- Amplitudes higher, frequency lower
- Expected, as on the surface, not imbedded

SBFEM - Solution in the time domain

$$\begin{bmatrix} \mathbf{M}_{ss} & \mathbf{M}_{sb} \\ \mathbf{M}_{bs} & \mathbf{M}_{bb} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_s(t) \\ \ddot{\mathbf{u}}_b(t) \end{Bmatrix} + \begin{bmatrix} \mathbf{C}_{ss} & \mathbf{C}_{sb} \\ \mathbf{C}_{bs} & \mathbf{C}_{bb} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}}_s(t) \\ \dot{\mathbf{u}}_b(t) \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{ss} & \mathbf{K}_{sb} \\ \mathbf{K}_{bs} & \mathbf{K}_{bb} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_s(t) \\ \mathbf{u}_b(t) \end{Bmatrix}$$

$$= \begin{Bmatrix} \mathbf{p}_s(t) \\ \mathbf{p}_b(t) \end{Bmatrix} - \begin{Bmatrix} \mathbf{0} \\ \mathbf{r}_b(t) \end{Bmatrix}$$

$$\mathbf{r}_b(t) = \int_0^t \mathbf{M}_b^\infty(t)\ddot{\mathbf{u}}(t-\tau)\,d\tau$$

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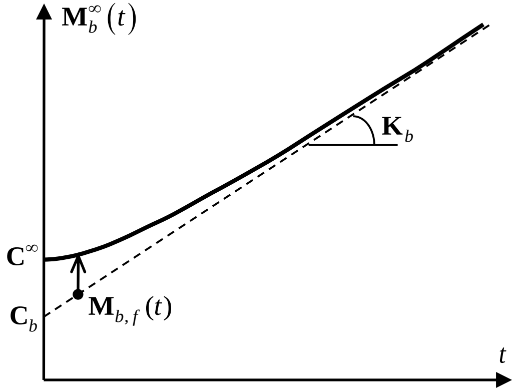
Convolution Integral

$$\mathbf{r}_b(t) = \int_0^t \mathbf{M}_b^{\infty}(t)\ddot{\mathbf{u}}(t-\tau)\,d\tau$$



- For" larger" values of t, the unit impulse matrix grows unlimited
- Thus the noisy influence of early times becomes numerically dominant
- Computers with limited precision can not handle that

Discretized “Integration by parts”

$$\mathbf{r}_i = \sum_{j=1}^{i} \mathbf{M}_{i-j+1}^{\infty}\left(\dot{\mathbf{u}}_j - \dot{\mathbf{u}}_{j-1}\right) = \dot{\mathbf{u}}_i \mathbf{M}_1^{\infty} + \sum_{j=2}^{i-1} \dot{\mathbf{u}}_j \left(\mathbf{M}_{i-j+1}^{\infty} - \mathbf{M}_{i-j}^{\infty}\right) - \dot{\mathbf{u}}_0 \mathbf{M}_i^{\infty}$$

- Based on the classical discretization scheme for a constant value of M within one time step.
- We can drop the last term, and we can split the sum numerically for a given limiting step number n

$$\sum_{j=2}^{i-1} \dot{\mathbf{u}}_j \left(\mathbf{M}_{i-j+1}^{\infty} - \mathbf{M}_{i-j}^{\infty}\right) \approx \sum_{j=2}^{n-1} \dot{\mathbf{u}}_j \left(\mathbf{M}_{i-j+1}^{\infty} - \mathbf{M}_{i-j}^{\infty}\right) + \Delta \mathbf{M}_n^{\infty} \sum_{j=n}^{i-1} \dot{\mathbf{u}}_j$$

Integration by parts

$$\int_0^t \mathbf{M}_b^\infty(t)\ddot{\mathbf{u}}(t-\tau)\,d\tau =$$

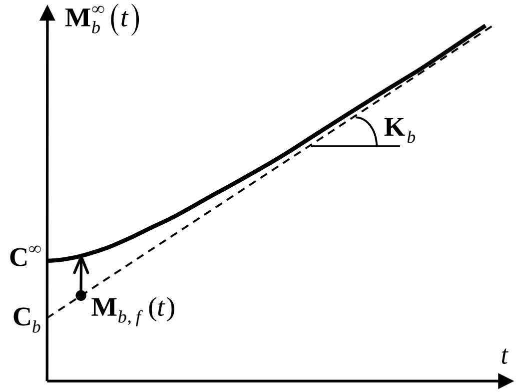
$$\mathbf{M}_b^\infty(t)\dot{\mathbf{u}}(0)-\mathbf{M}_b^\infty(0)\dot{\mathbf{u}}(t)-\int_0^t \frac{d\mathbf{M}_b^\infty(t)}{dt}\dot{\mathbf{u}}(t-\tau)\,d\tau$$

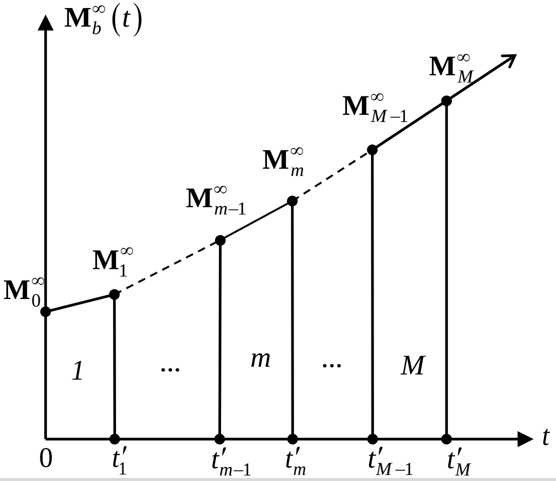
- As we start with velocity zero the first part will vanish
- As the derivative of the unit impulse matrix M converges to a constant value we may simplify the last remaining integral term

Simplified Representation of the M-Matrix

- The Unit Impulse Matrix can be decomposed into three parts constant, linear and a part which tends to zero for late times:

$$\mathbf{M}_b^{\infty}(t) = \mathbf{C}_b H(t) + \mathbf{K}_b t H(t) + \mathbf{M}_{b,f}(t)$$





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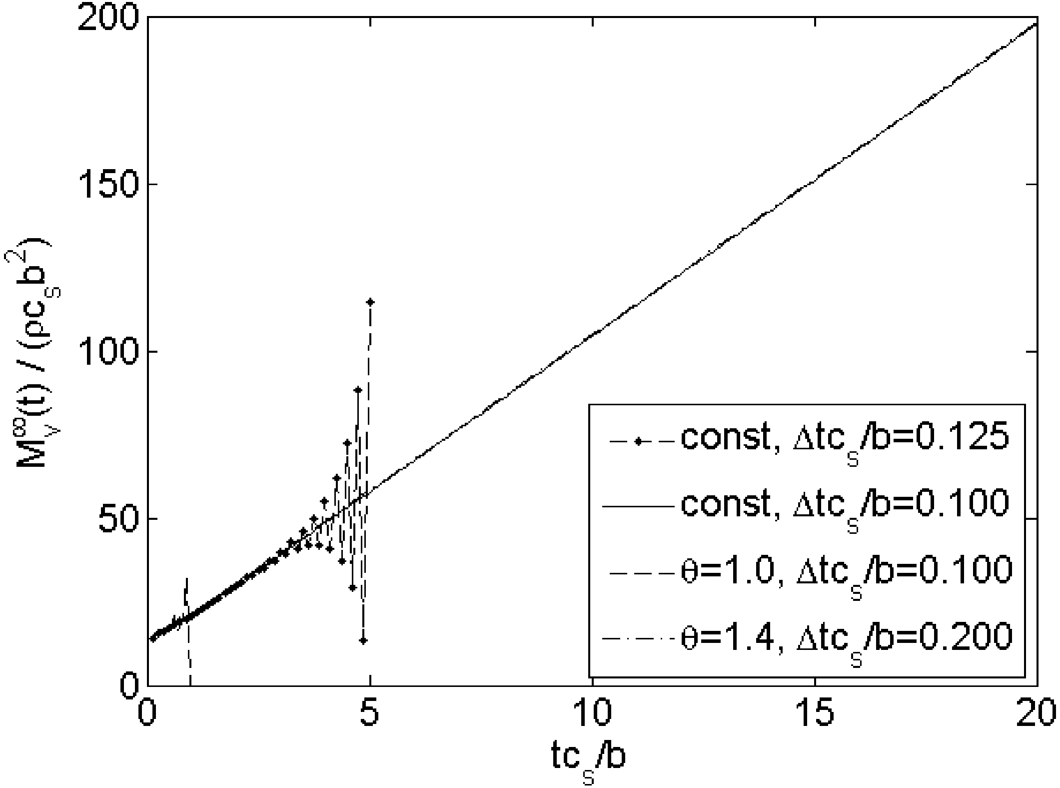
Stabilisation of the Integration Scheme

- Classical integration with a constant value of M is only unconditional stable. The time step has to be selected below a certain stability criteria.
- There are well known techniques for time dependant problems using a linear variation of the acceleration and stabilizing the solution with an extrapolation parameter (0.5 < Θ < 2.0).

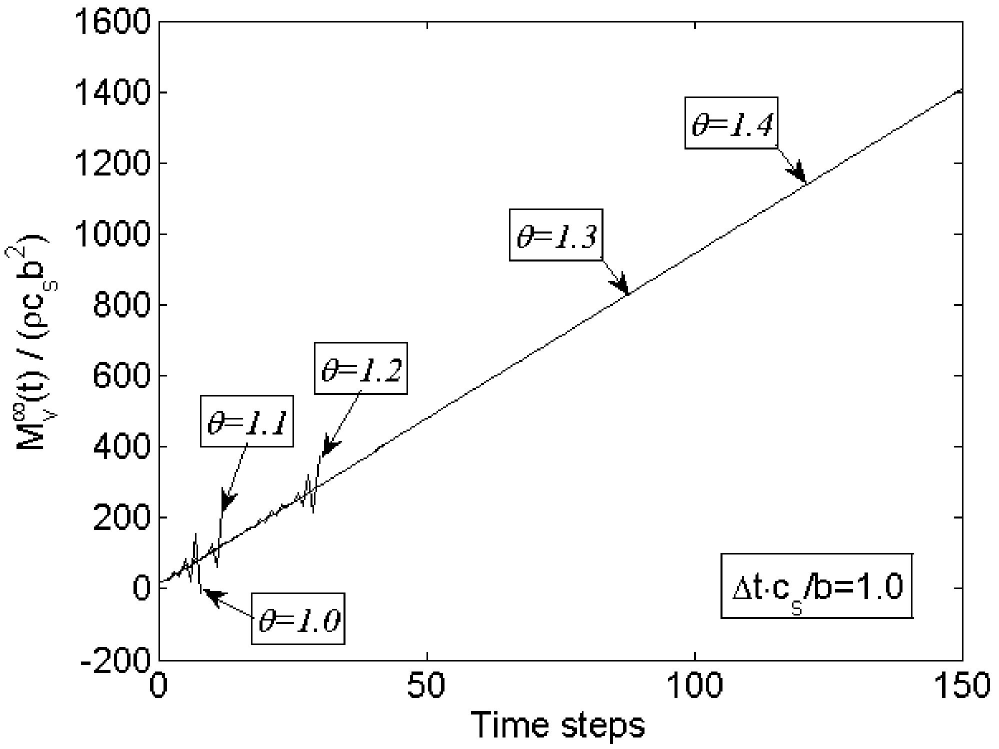
$$\mathbf{m}_m^\infty = (\theta - 1)\theta^{-1}\mathbf{m}_{m-1}^\infty + \theta^{-1}\overline{\mathbf{m}}_m^\infty$$

- More formulas in the paper!

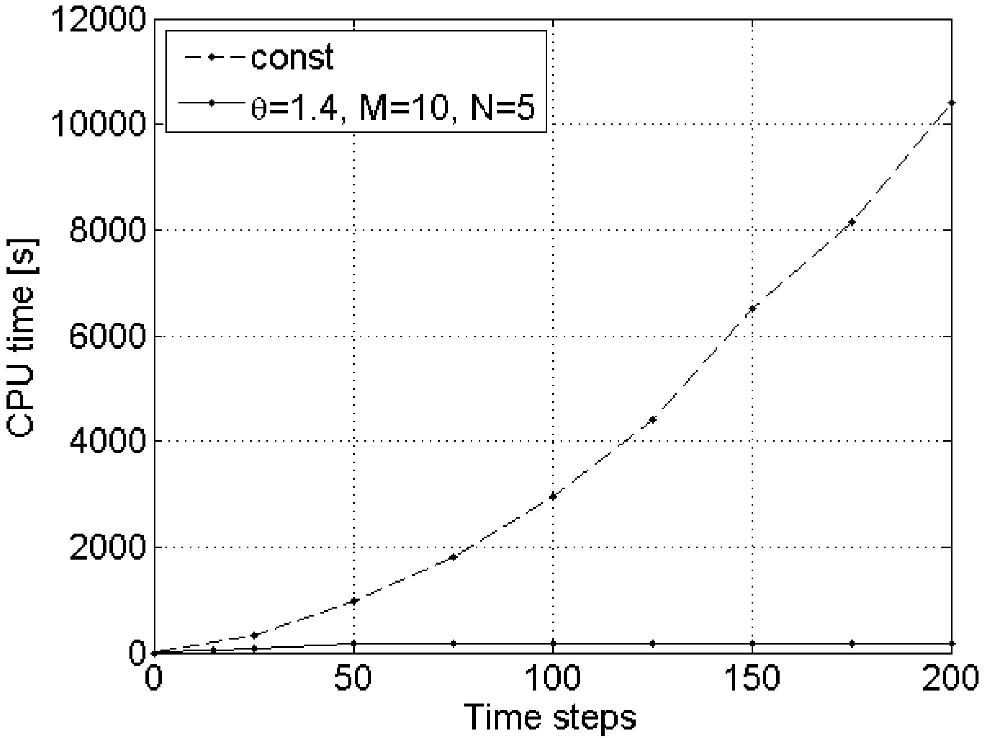
Stabilisation of the Integration Scheme



Stabilisation of the Integration Scheme



CPU Time



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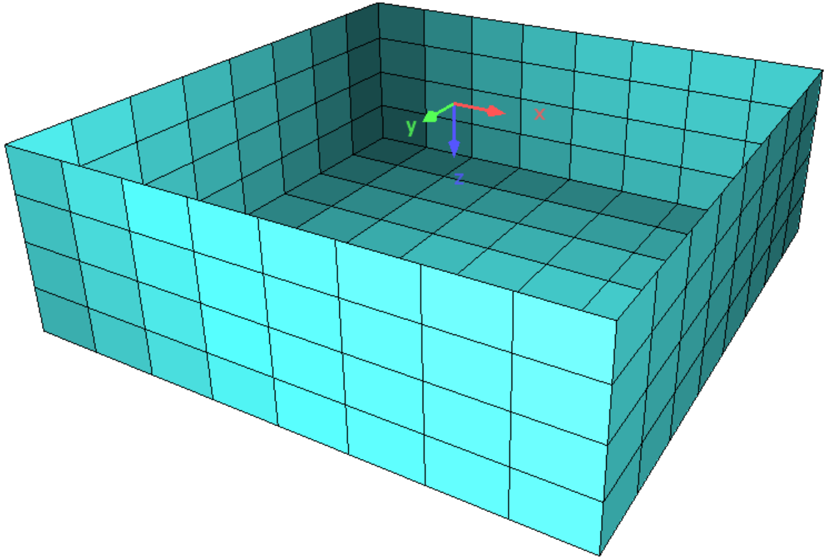
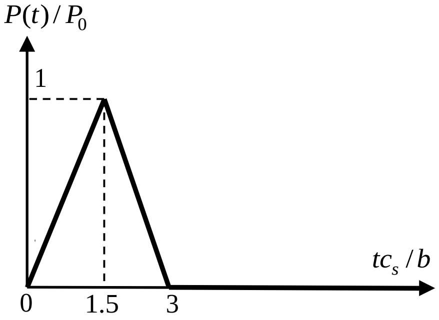
Example:
rigid embedded foundation





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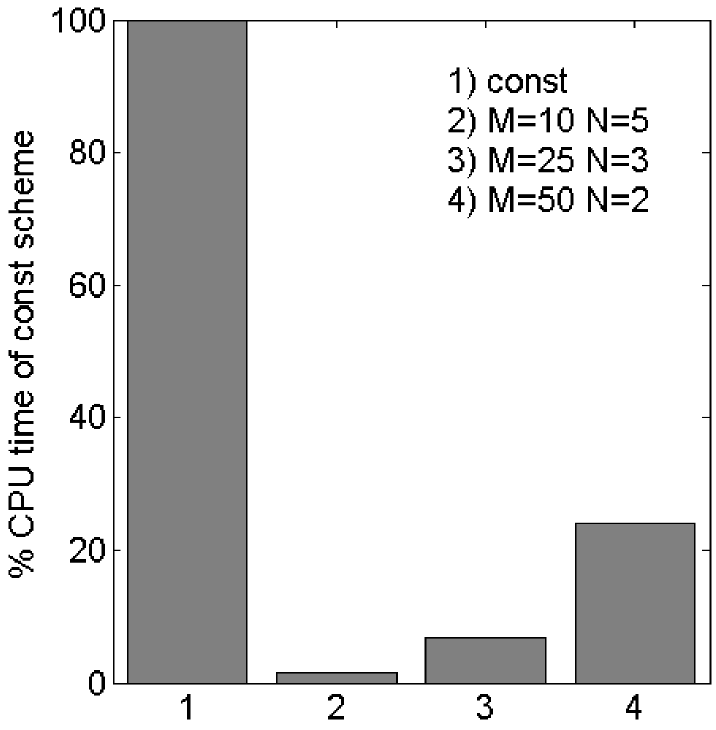
Required CPU Time

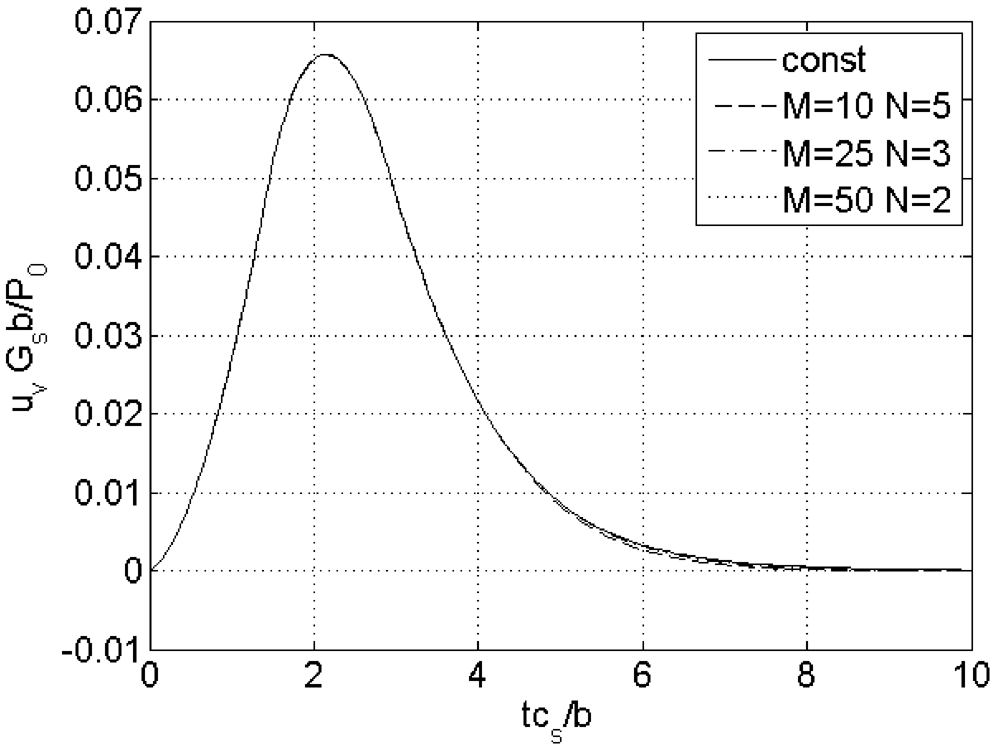
Δt‘ = N·Δt

M = Number of intervals of size Δt‘ for Unit Impulse Matrix

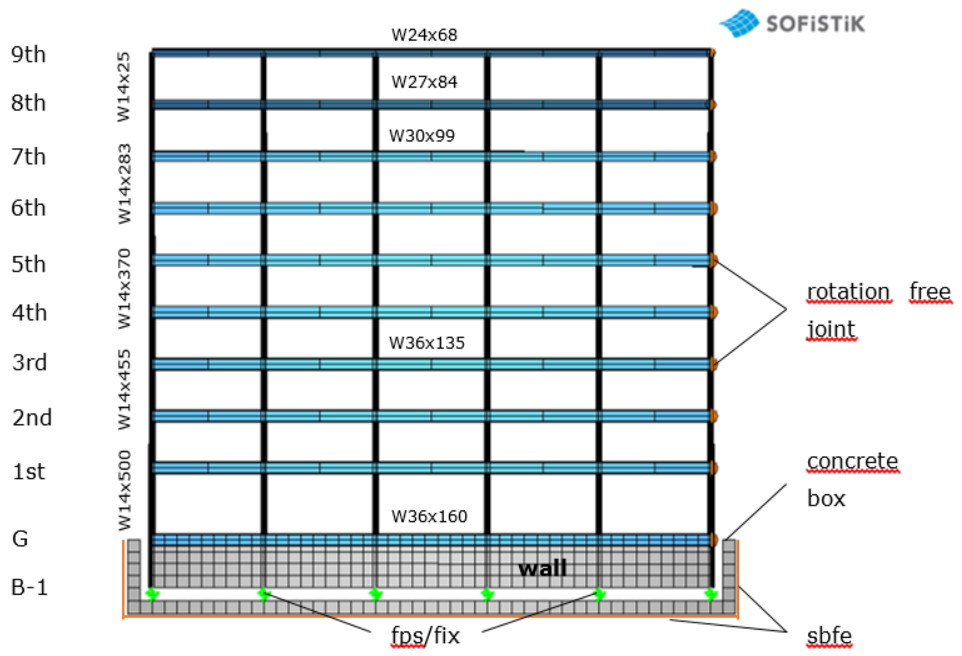


SOFiSTiK

Vertical displacement



Non linear Analysis for a Frame



Conclusion I

- Improved evaluation of the acceleration unit-impulse response matrix by a new integration scheme based on the piece-wise linear approximation within time step size and an extrapolation parameter
- truncation time after which the acceleration unit-impulse response matrix is linearized.
- Soil-structure interaction force vector represented by the convolution integral is evaluated using a new and very efficient scheme based on the integration by parts.

Conclusion II

- The combination of these enhancements leads to a very significant reduction of computational effort and linear dependency with respect to the number of time steps.
- No instabilities have been observed
- So we call it a high performance SBFEM method.