Computational Mechanics
Civil Engineering
Extended Problems - Seepage

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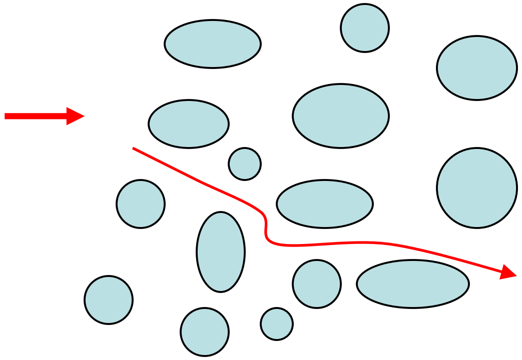
Groundwater Seepage

- Soil is composed by
  - Rather stiff soil particles (gravel, sand, clay)
  - Voids between particles filled with air
  - Voids between particles filled with water
- If we have a different pressure on two sides of a soil there will be a movement of air and water through the soil

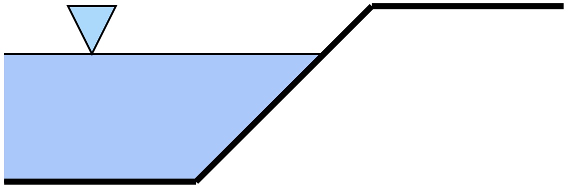


Hydraulic Head

- Energy expressed in height = Energy/($\rho \cdot g$)
  - Potential Energy h
  - Pressure Energy p/γ
  - Kinetic Energy $\frac{1}{2} \cdot v^2/g$
  - Acceleration Energy (Stream height)
- For groundwater seepage only the first two terms are important:

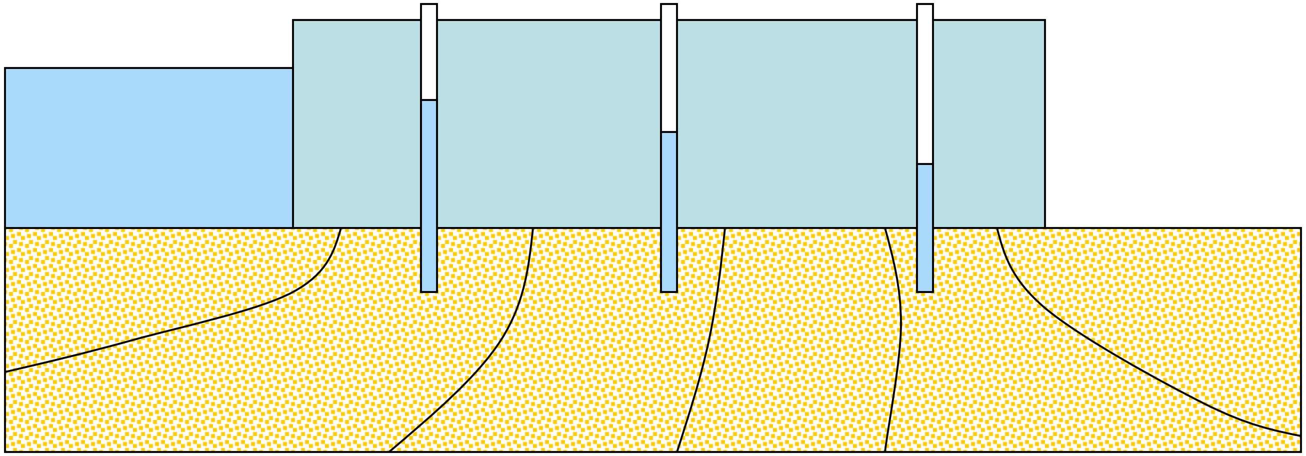
$$H = h + p/\gamma$$

- Thus for a water side of a dam we have a constant hydraulic head



Remark

- **The hydraulic or piezometric head can be observed in nature as water table in a bore hole:**



Forces on Fluid Particles

- A pressure gradient causes forces on the fluid particles.
- In a free fluid the pressure gradient is in equilibrium with the weight of the fluid.
- In other cases we have resultant forces.
- Some forces are introduced by the surface tension of the fluid
- If the fluid moves we have shear forces from the viscosity of the fluid at the walls. We have then a kind of tube flow.

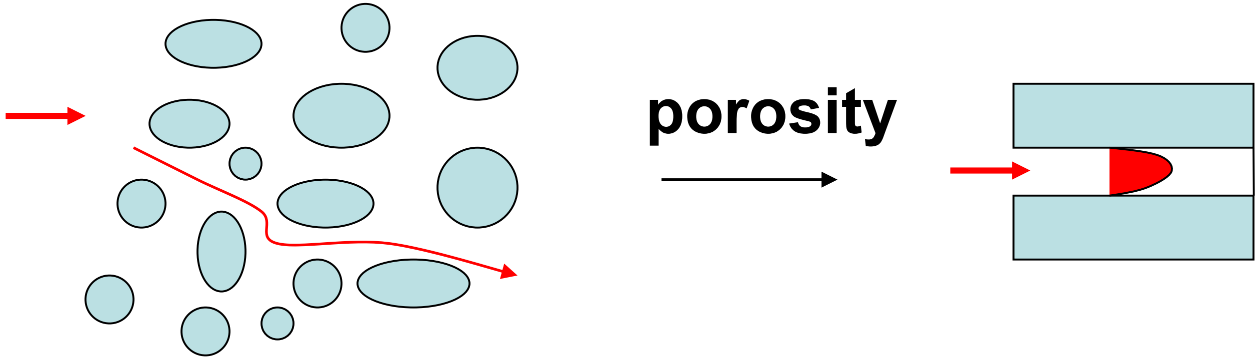
Definitions / Terms

- **There is the void ratio e defined as ratio of void volume to solid volume (deutsch: Porenzahl)**

$$e = \frac{V_v}{V_s} = \frac{n}{1-n}$$

- **And the porosity n defined as ratio of void volume to total volume (deutsch: Porenanteil)**

$$n = \frac{e}{(1+e)}$$



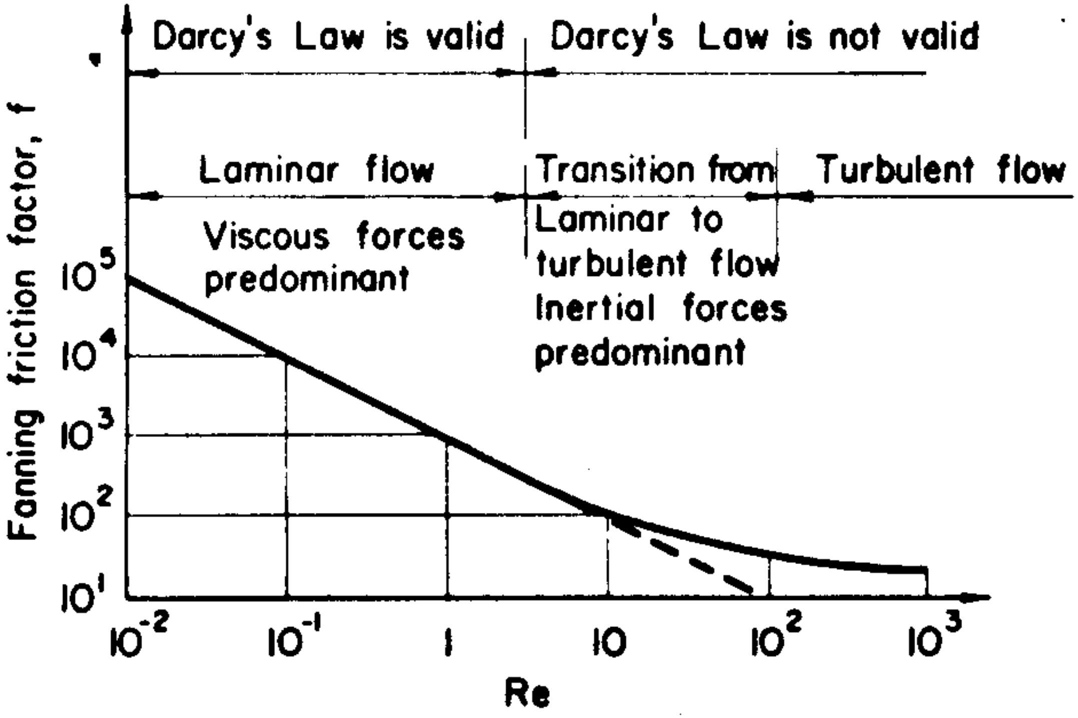
porosity

Velocities

- We have two velocities
  - A velocity describing the mass balance i.e. the quantity is velocity times sectional area = discharge velocity (Filtergeschwindigkeit)
  - A real velocity describing the travelling time of a fluid particle which is important for tracer experiments, the transport of contaminations and the size of water protection areas. (Abstandsgeschwindigkeit)
  - The relation between those two values is depending on the porosity.
- If the soil is compacted we have to expect a strong influence on the velocities

Resistance

- **For the small velocities and small diameters the Reynolds number is very small we may expect a laminar flow.**



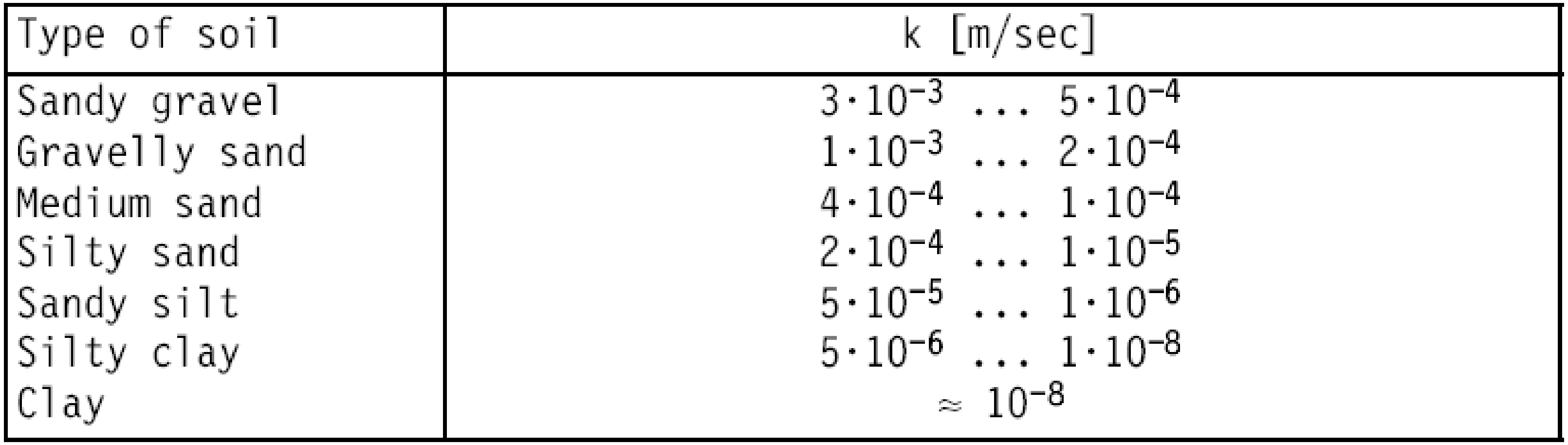
Darcy‘s Law

$$\begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} = \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix} \cdot grad\ H$$

- The velocities are linear dependant on the gradient
- The permeability (conductivity) is a symmetric Tensor, i.e. it is transformed like a tensor
- Kezdi has defined a relation for the dependence of the permeability on the void ratio e

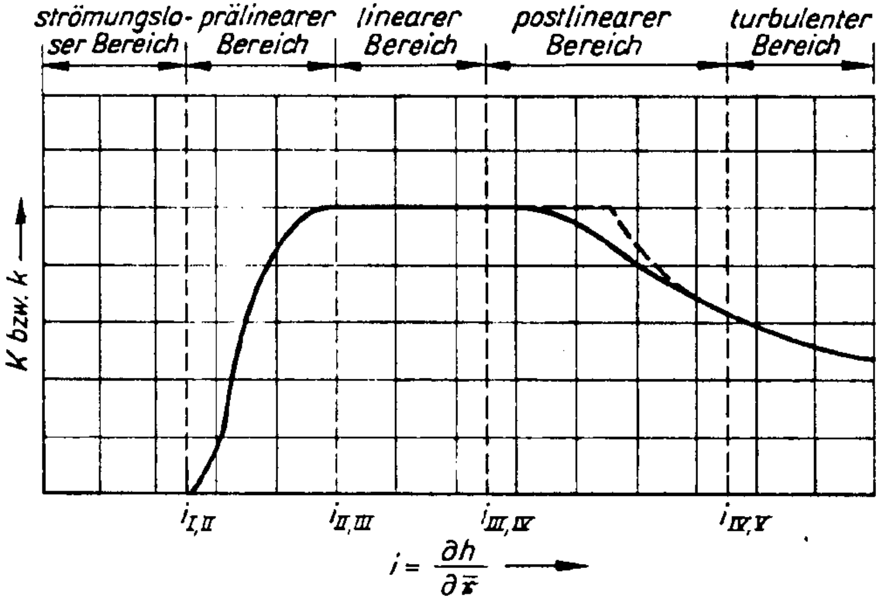
$$\frac{k_1}{k_2} = \frac{e_1^3}{e_2^3} \frac{(1+e_2)}{(1+e_1)}$$

Typical values for k



Type of soil	k [m/sec]
Sandy gravel	$3 \cdot 10^{-3}$ ... $5 \cdot 10^{-4}$
Gravelly sand	$1 \cdot 10^{-3}$ ... $2 \cdot 10^{-4}$
Medium sand	$4 \cdot 10^{-4}$ ... $1 \cdot 10^{-4}$
Silty sand	$2 \cdot 10^{-4}$ ... $1 \cdot 10^{-5}$
Sandy silt	$5 \cdot 10^{-5}$ ... $1 \cdot 10^{-6}$
Silty clay	$5 \cdot 10^{-6}$ ... $1 \cdot 10^{-8}$
Clay	$\approx 10^{-8}$

Nonlaminar flow



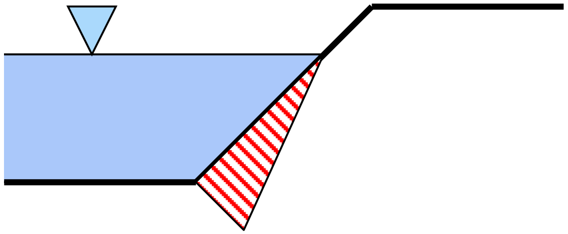
- For very small diameters the surface tension is to strong and we have no movement at all or a plastic flow according to Bingham‘s Law.
- Turbulent Flow according to Forchheimer

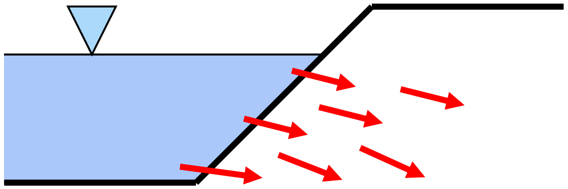
$$\text{grad } H = a \cdot u + b \cdot u^2$$

- There are some theoretical estimates, typical values are according to Forchheimer

$$0.03 < a < 1.5 \quad \text{and} \quad 0.8 < b < 240$$

Important Hint





- If we calculate the forces on a dam we have either
  - » An impermeable cover of the dam
  - » A seepage through the dam
- In the first case we have the full water pressure on the cover
- In the second case we have instead (not additionally !) the gradient of the fluid pressure acting on the soil grains which is calculated from
  - – The gradient of the potential - the buoyancy effects

$$f = grad\ p = \gamma \cdot grad\ H - \gamma \cdot grad\ z$$

Tube and Channel flow

- Similar to a truss element in structural analysis we may include tubes and channels. The basic relation is for the loss of Potential dependant on the flow velocity u, the hydraulic radius R_h and the length L:

$$\Delta h = \lambda \cdot \frac{L \cdot u^2}{R_h \cdot 8 \cdot g}$$

- For laminar flow Re<2320 we have

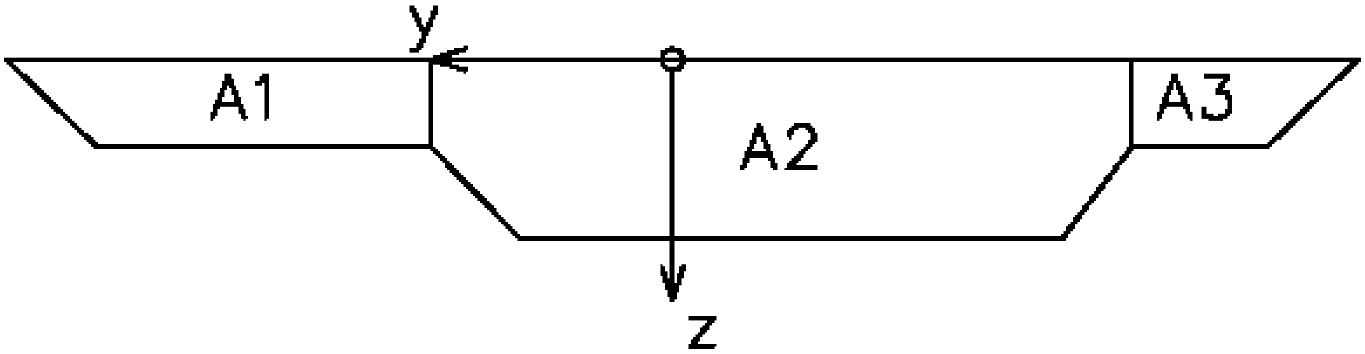
$$\lambda = \frac{64}{\mathrm{Re}} = \frac{16 \cdot \nu}{R_h \cdot u}$$

- For Re > 2320 we have the law from Prandtl-Colebrook

$$\frac{1}{\sqrt{\lambda}} = -2\lg\left[\frac{2.51}{\sqrt{\lambda} \cdot \mathrm{Re}} + \frac{k}{14.84 \cdot R_h}\right]$$

Channel Flow

- **The flow is subdivided for different hydraulic sections (river, coast etc.)**

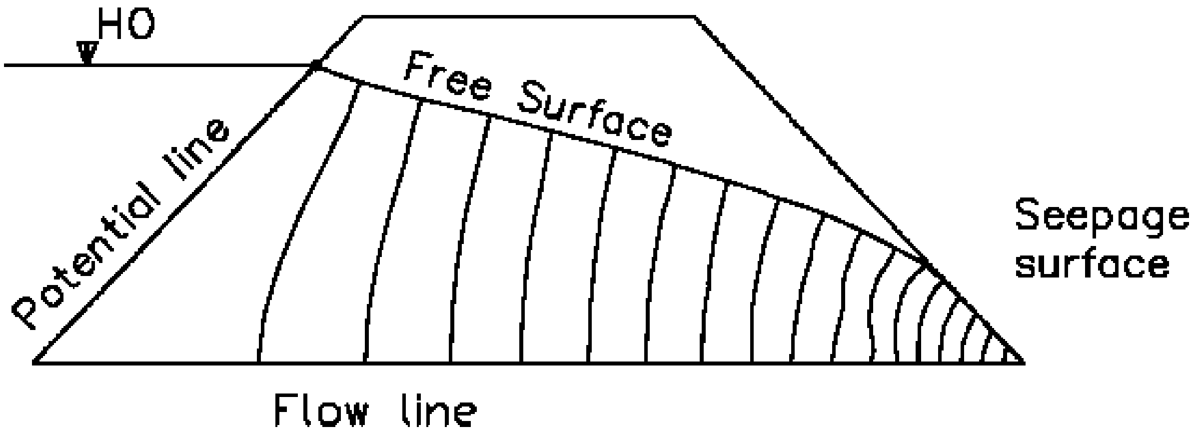


Storage coefficient

$$S = \gamma_w \left\lfloor (1-n)/K_s + n/K_w \right\rfloor + \int_{\Gamma s} n \, d\Gamma$$

- The storage capacity is defined as the volume of fluid to be stored within a control volume if the pressure increases. We have four components in increasing order
  - » The compressibility of the soil itself
  - » The compressibility of the fluid itself
  - » The compressibility of the contained air
  - » The raise of the water table increasing the saturation (moisture content) of the soil
- For unconfined free surface flow the last term is dominant
- For the transition to a confined flow the numerics are quite difficult !

Boundary Conditions



- **Potential line** $H = H_0$
- **Flow line** $v^{t\cdot} \cdot n = q$
- **Free Surface** $p = 0$ and $v^{t\cdot} \cdot n = 0$
- **Seepage surface** $p = 0$ and $v^{t\cdot} \cdot n > 0$

Free Surface

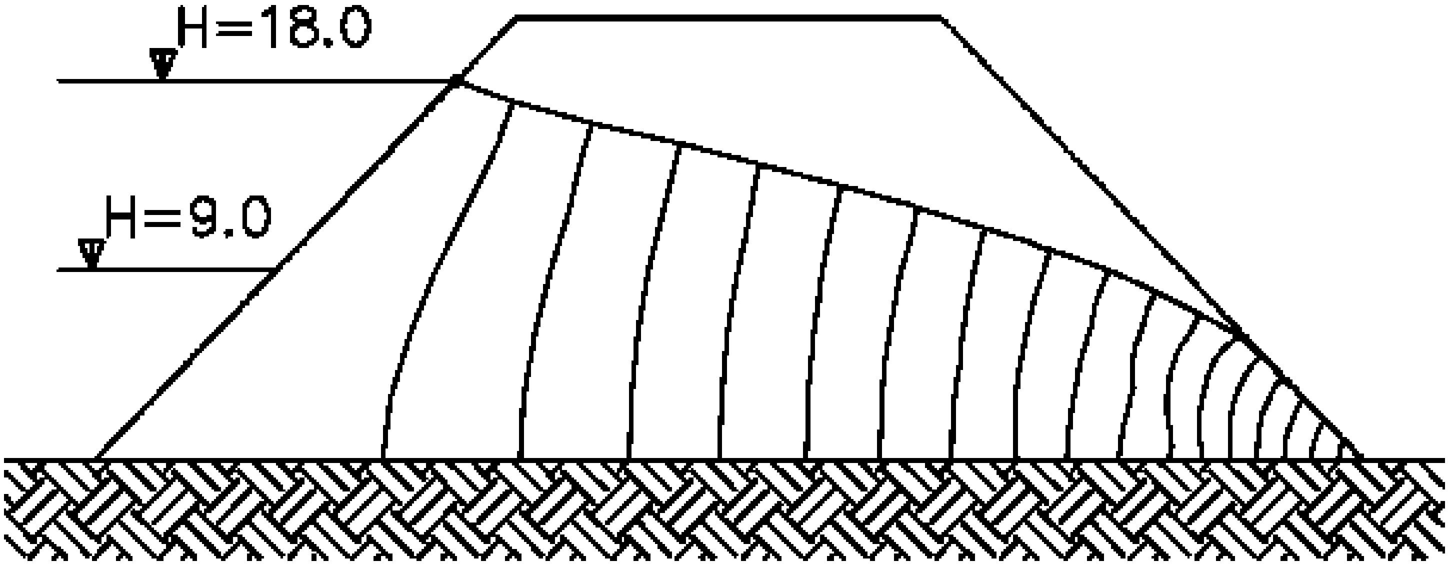
- We have a boundary where the location is not known in advance
- If we restrict the integration of the functional only to the region limited by an assumed boundary we obtain the natural boundary condition

$$v^{t\cdot}\cdot n = 0$$

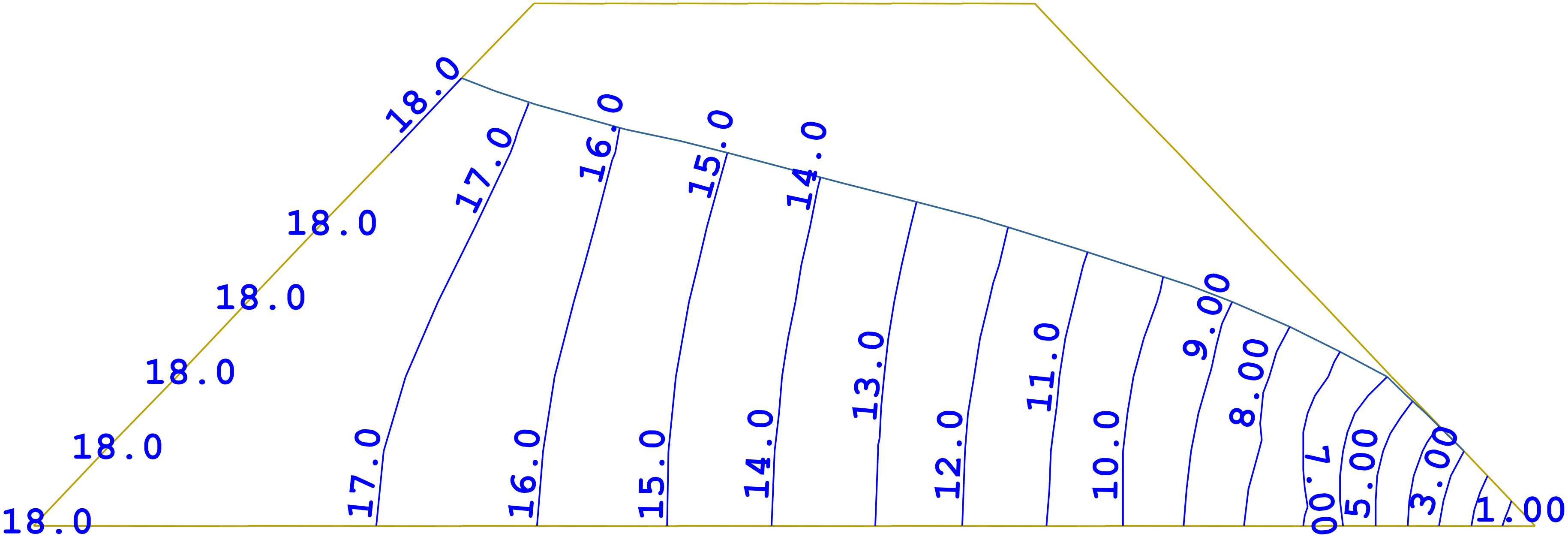
- Solution I - Adaptive meshes
  The mesh is adopted to the flow to ensure p=0 at the boundary, which is a really difficult problem and not state of the art !
- Solution II - Selective Integration
  Based on a given potential we restrict the integration of the functional only to the region with positive pressures, which is rather easy !

Example

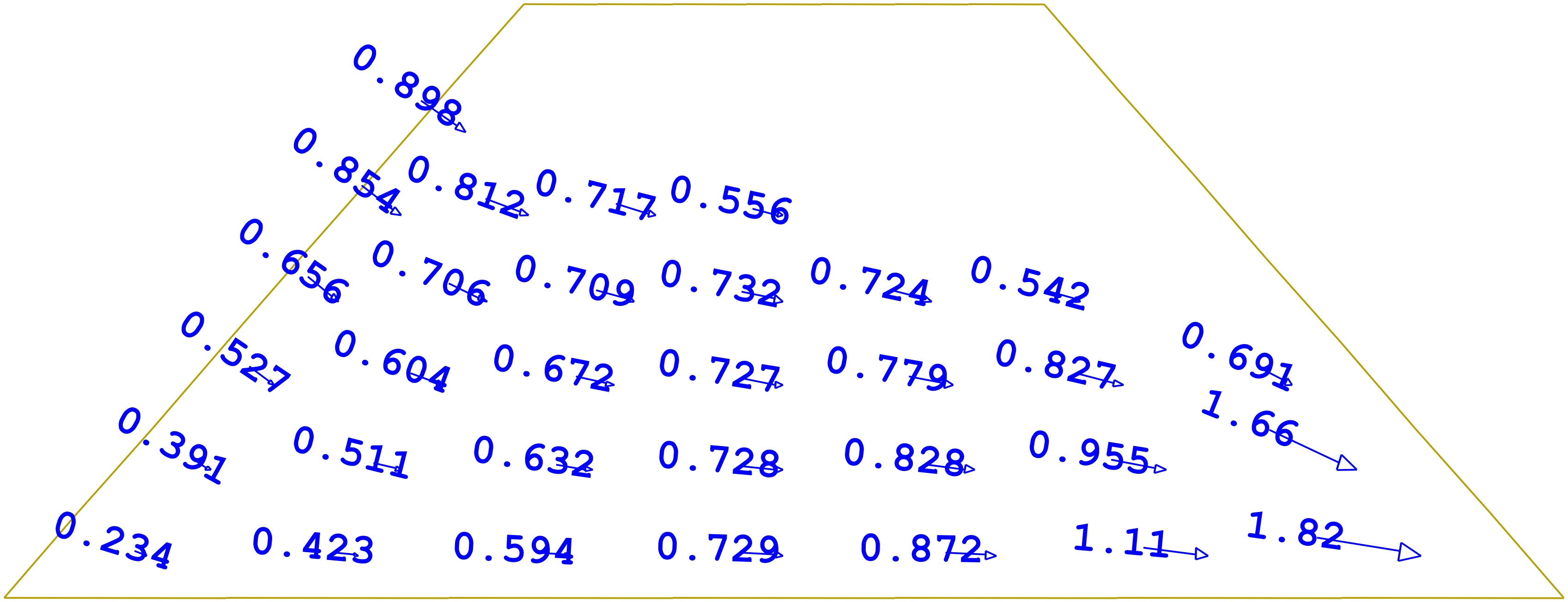
- **A homogeneous Dam**



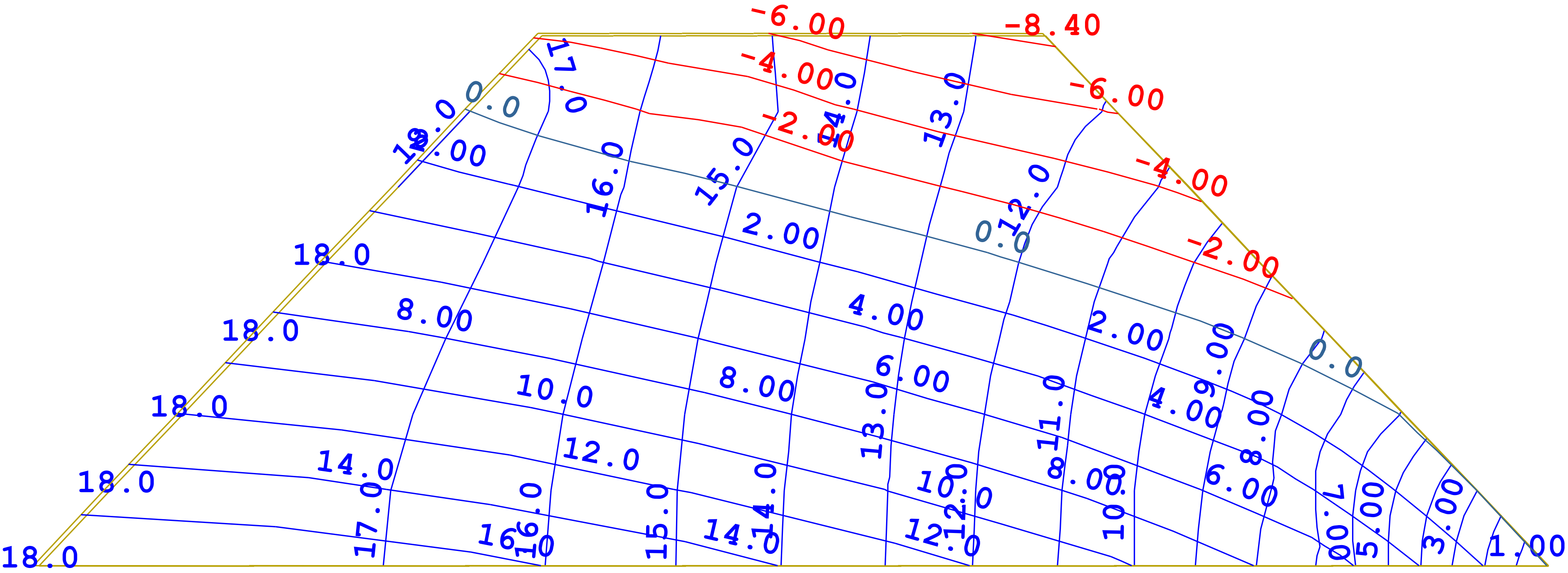
The free surface flow



Flow quantities in elements



And some details



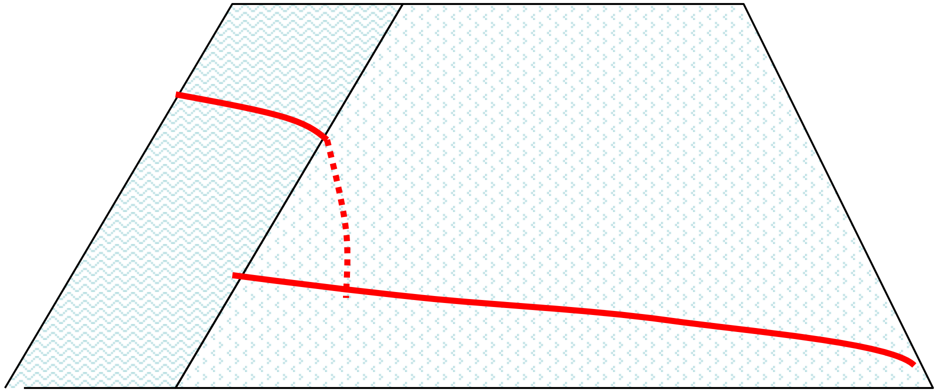
Yeah, some problems

- It is better to reduce the permeability for the negative pressure by a factor of 0.00001 to avoid numerical problems and to get a reasonable value for H there to allow a better post processing
- When numerical integration is used, we should avoid a sudden deactivation by a weighted reduction factor:

$$red = \frac{1}{2}\left[1 + \frac{\sum N_i p_i}{\sum N_i \left|p\right|_i}\right]$$

And a really severe problem

- If the free surface is very steep the numerics become strange



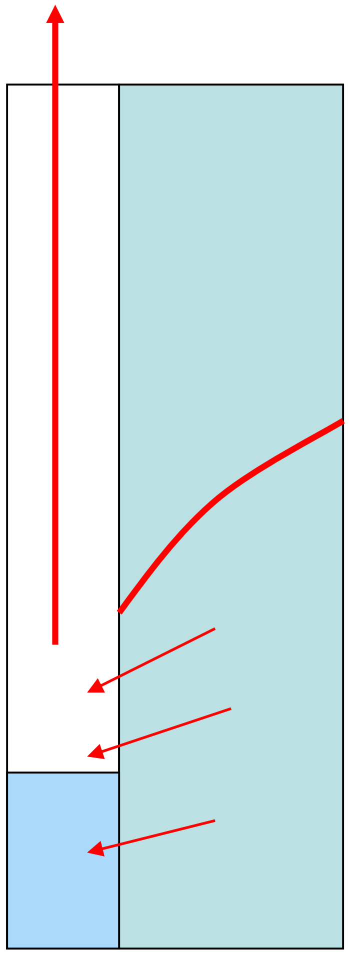
- Beside numerical damping only a partially saturated flow may be a solution

Seepage surface

- Apply the boundary condition H=z
- Control the fluid quantity in all nodes from the residual
- If we have an inflow, drop the boundary condition, but allow for a later reattachment
- The free surface can connect only at a distinct node into the seepage surface

The problem of a well

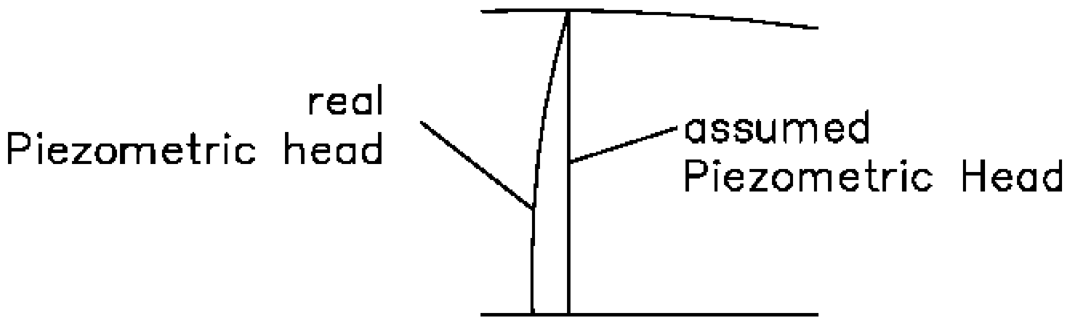
- The well has a pump with a given maximum quantity
- If the pump is strong enough the well becomes empty and the maximum flow is obtained with a complete seepage surface
- If the seepage surface becomes to small the quantity calculated from the potential difference can not be taken out
- If the pump is not strong enough the well fills partly with water until the inflow is equal to the outflow
- An iteration is always needed, where the integral flow from a set of nodes is used.



Dupuit Hypothesis

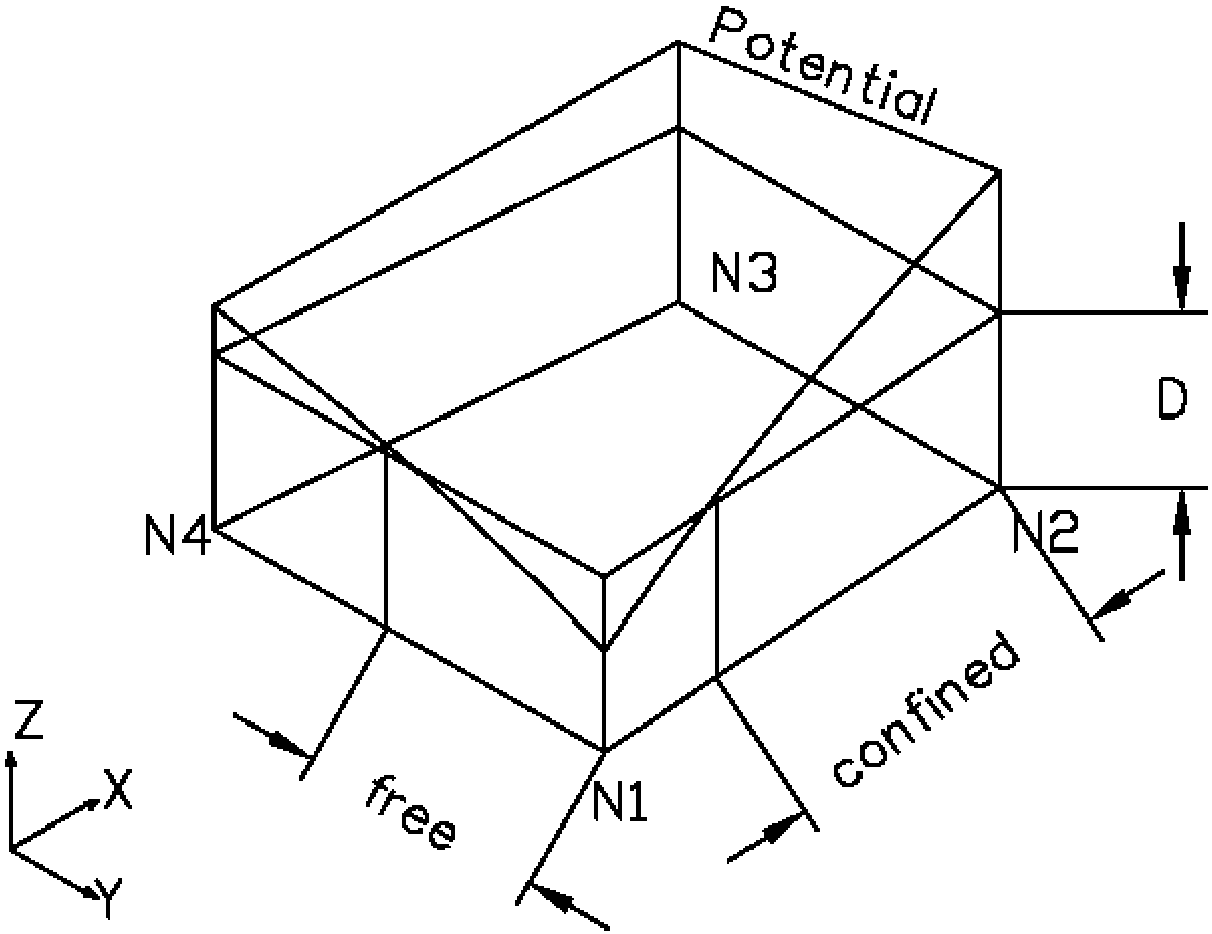
- For large models with rather small vertical dimensions it is possible to reduce the computational effort by the Dupuit assumption:

  The potential is constant along the height of the aquifer.



- The model is not necessarily flat, but it is rather 2D

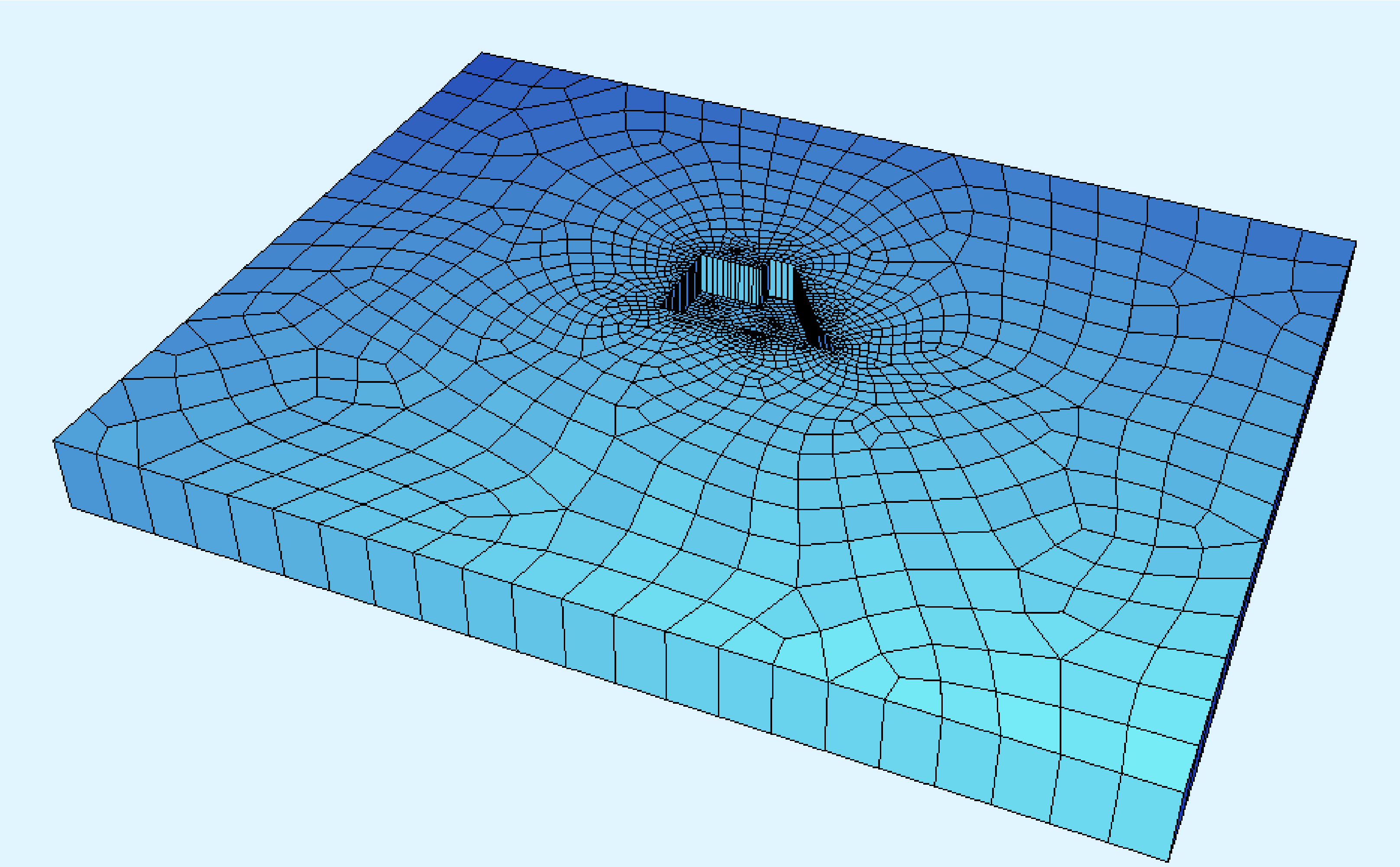
Extended 2D QUAD element



Typical applications

- Large hydrological models to predict the ground water behaviour, tuned on measurements to predict changes in the water flow
    - Global balance
    - Level of Groundwater
    - Drink water protection
    - Contaminations including diffusions
- Dams and barrages loaded by seepage
- Pits or constructions submerged in groundwater

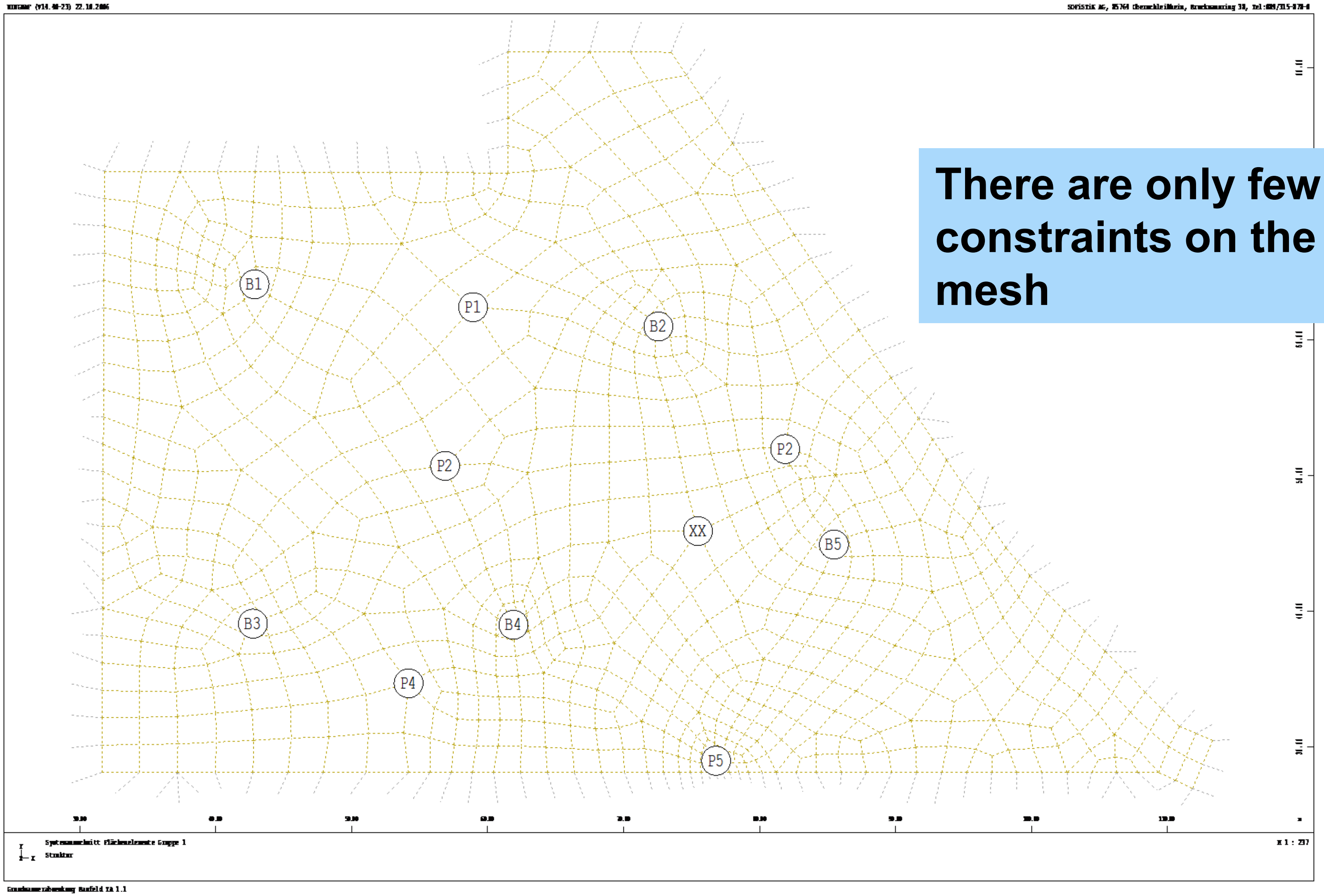
Groundwater lowering



Possible Questions

- How much water (How many pumps) has to be taken out to achieve a certain effect or what is the effect if we pump a certain quantity.
- Do we want to get to get the pit completely dry (not allowed in general)
- Or if we install a tight sole of the pit (e.g. HDI), what is the remaining pressure
- What is the effect on the pressures on the retaining walls

Mesh within pit

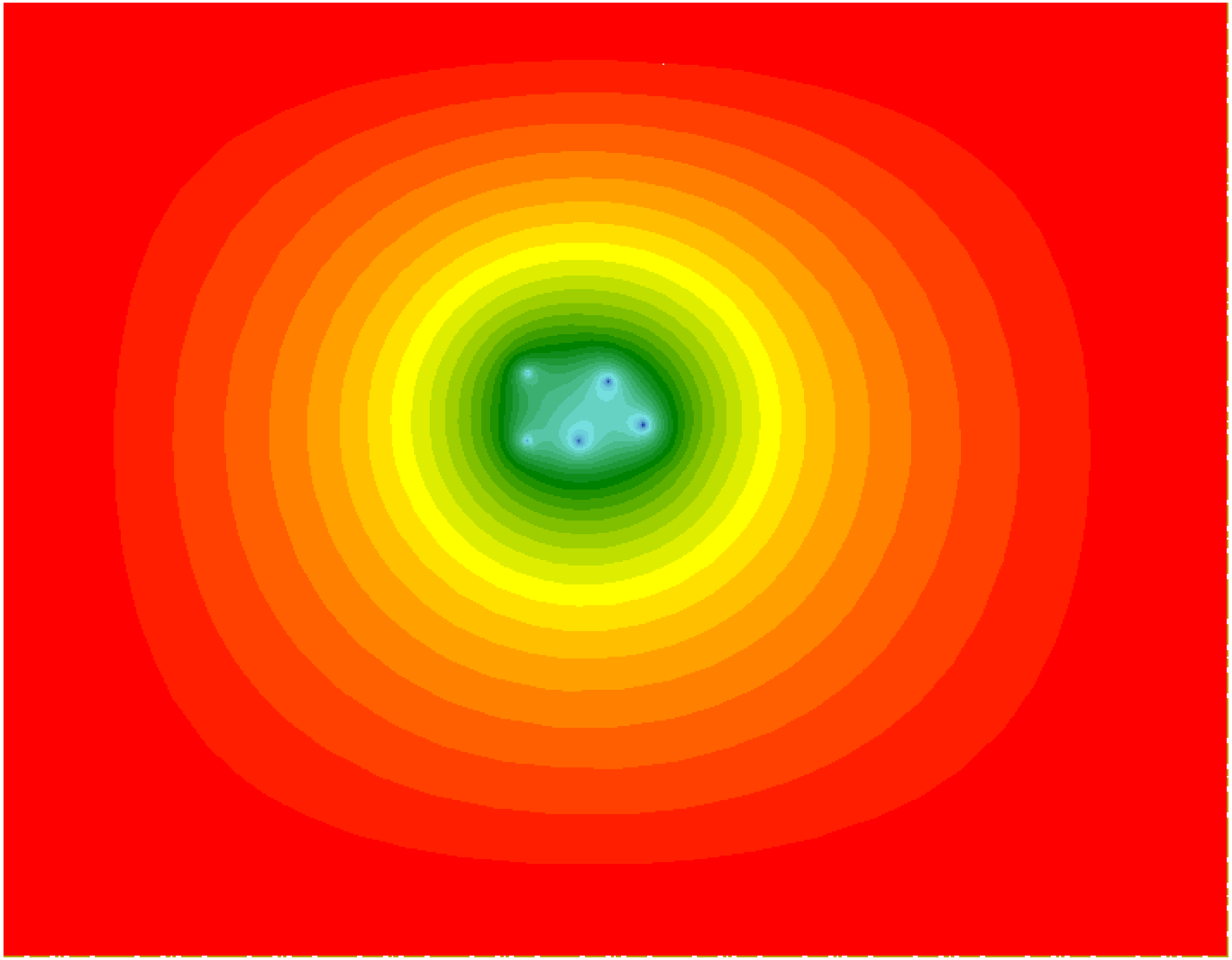


There are only few constraints on the mesh

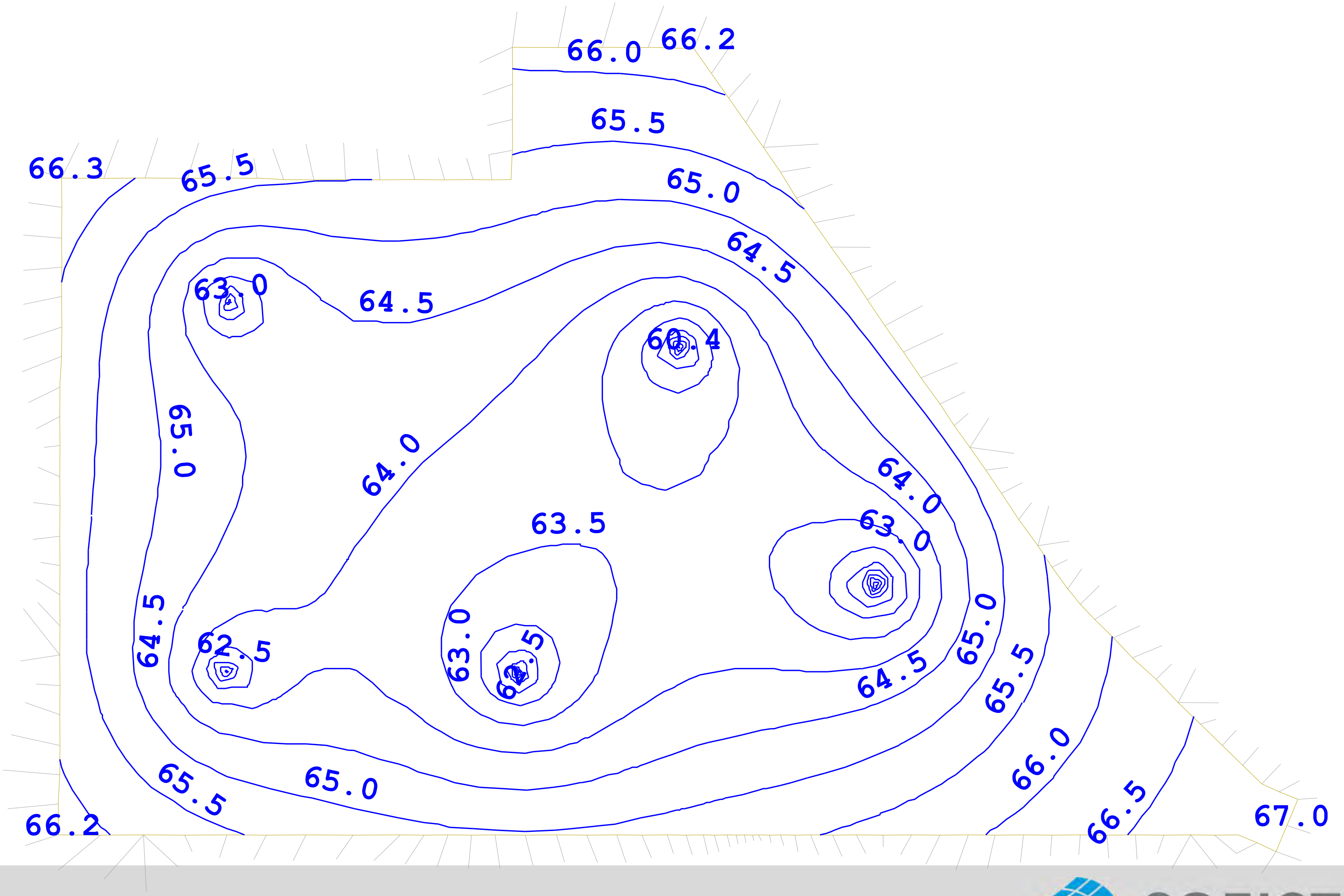
Possibilities / Problems

- We have to apply a fixed value of the potential at the outer boundaries. There are estimates how far we have to go, but the principle problem is that we have to assume a value somehow.
- For the interior of the pit we may treat the area as a seepage surface, if the hydraulic gradient is less than the weight of the soil (otherwise we have a hydraulic soil failure „Hydraulischer Grundbruch“)
- We may prescribe a water level in each well, which leads to a “well” defined problem
- Or we may prescribe a certain quantity at the wells and may have the problem that if the gradient is to large, the whole aquifer will become dry and the solution will collapse.
- In both cases we might need to iterate a little bit or use a special switching well-boundary condition

Piezometric Head



Piezometric Heads



The well boundary condition

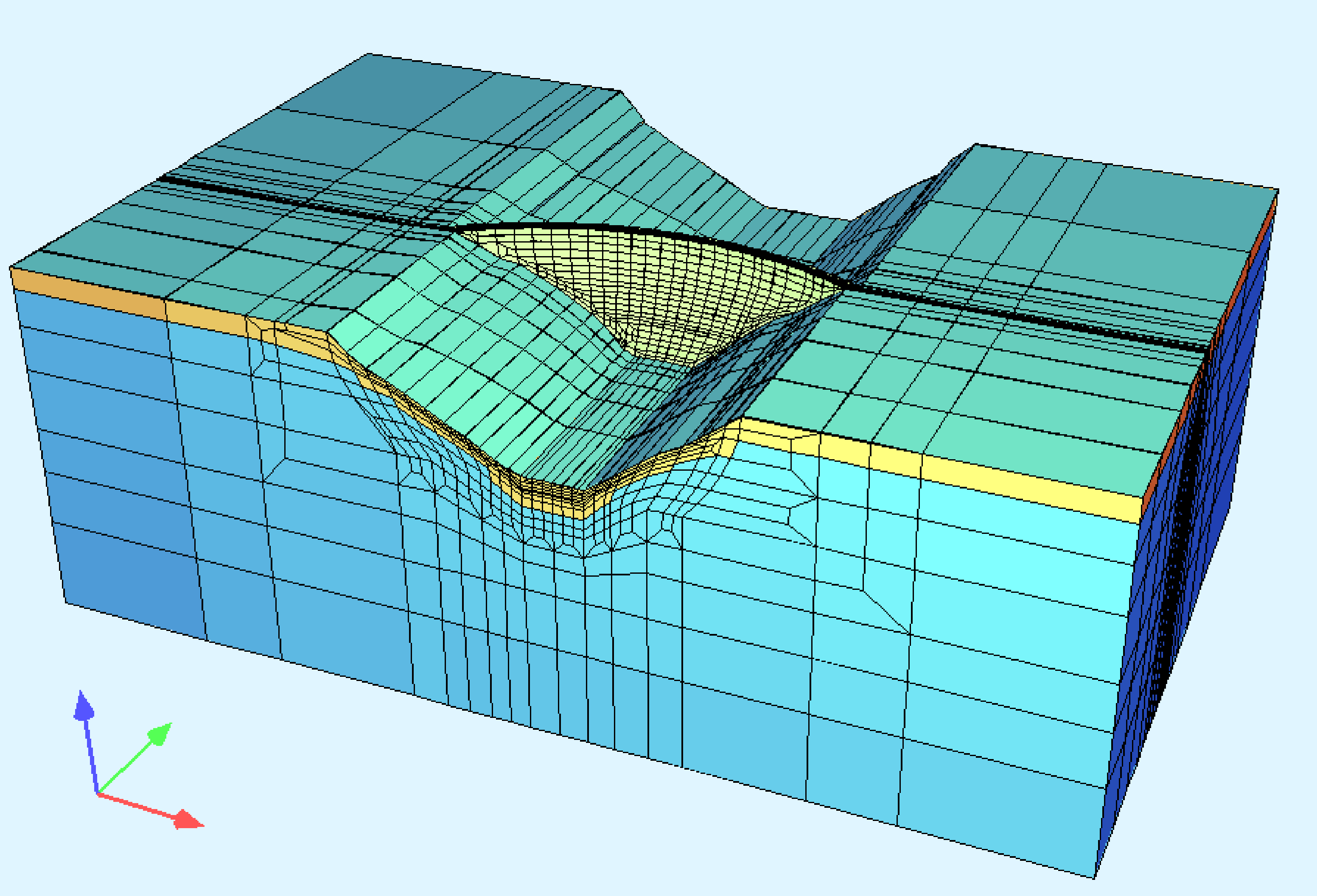
- **Boundary Conditions:**

```
WELL   1   0    21     21      0    210.000 [l/min]
WELL   1   0    22     22      0    250.000 [l/min]
WELL   1   0    23     23      0    200.000 [l/min]
WELL   1   0    24     24      0    230.000 [l/min]
WELL   1   0    25     25      0    300.000 [l/min]
```

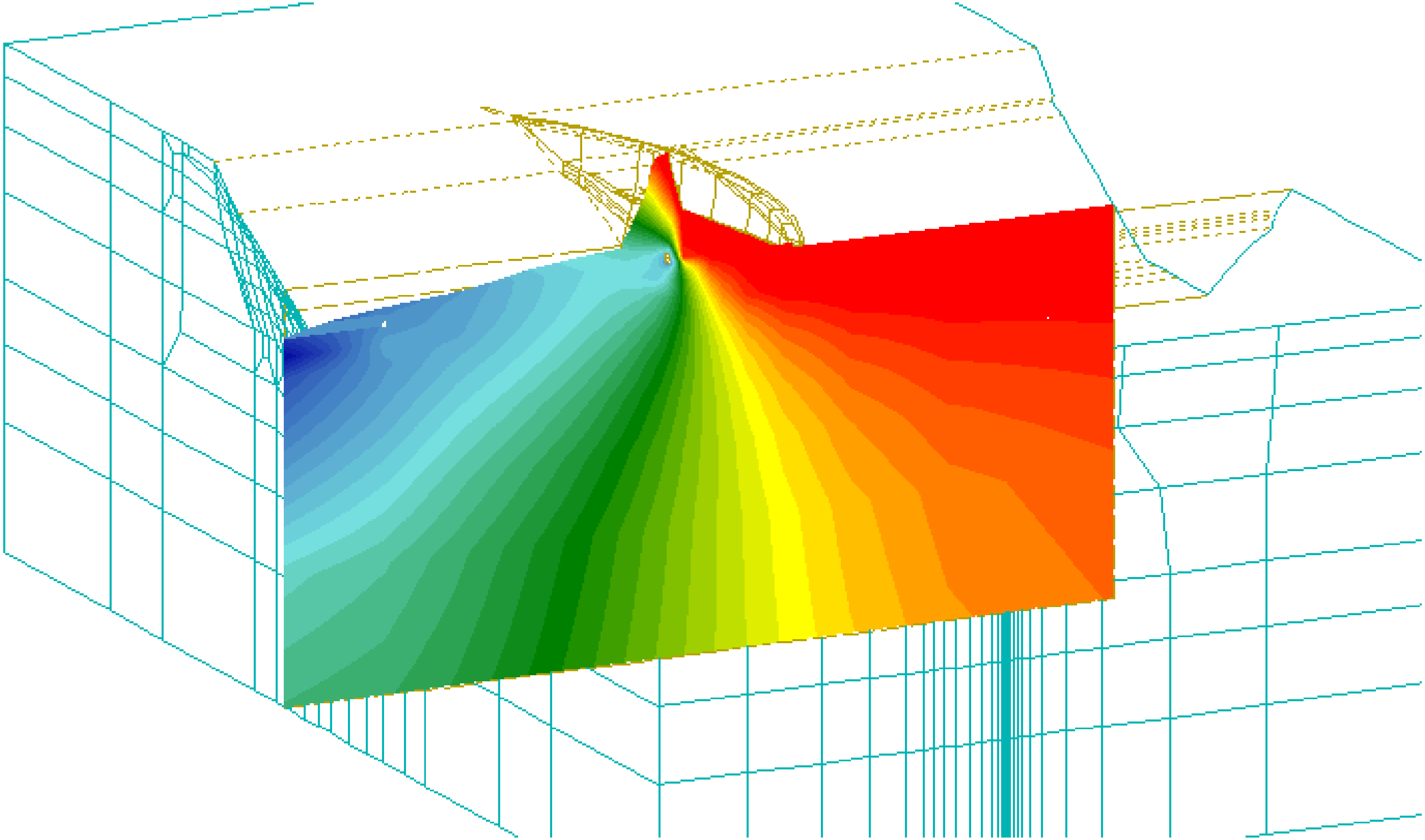
- **Values after 30 Iterations:**

```
WELL    1(    21) HEAD   54.134 M, CHARGE      215.454 [l/min]
WELL    1(    22) HEAD   49.687 M, CHARGE      246.362 [l/min]
WELL    1(    23) HEAD   53.169 M, CHARGE      197.987 [l/min]
WELL    1(    24) HEAD   48.616 M, CHARGE      225.870 [l/min]
WELL    1(    25) HEAD   47.613 M, CHARGE      302.152 [l/min]
```

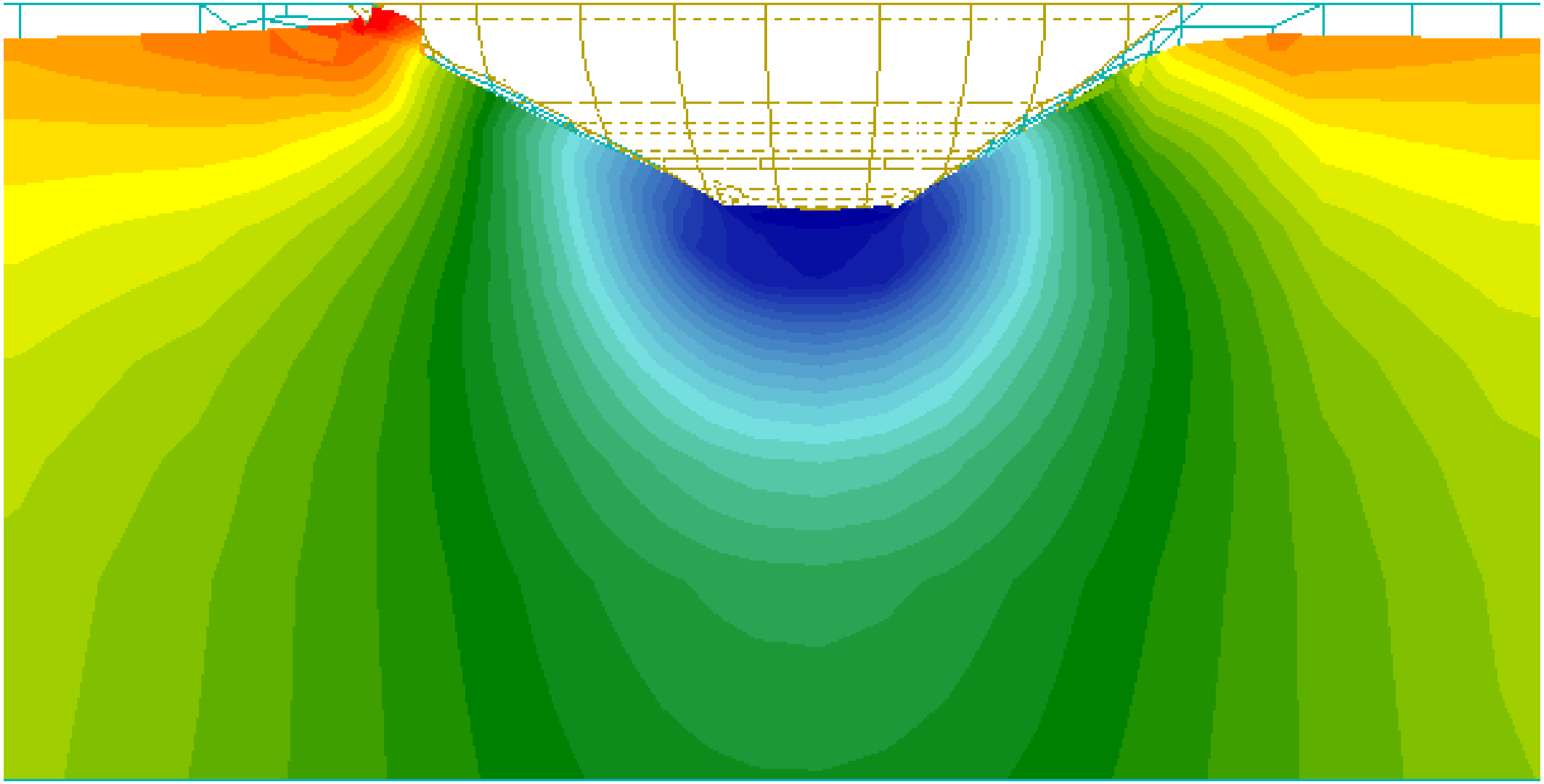
A 3D example



Cut along the valley



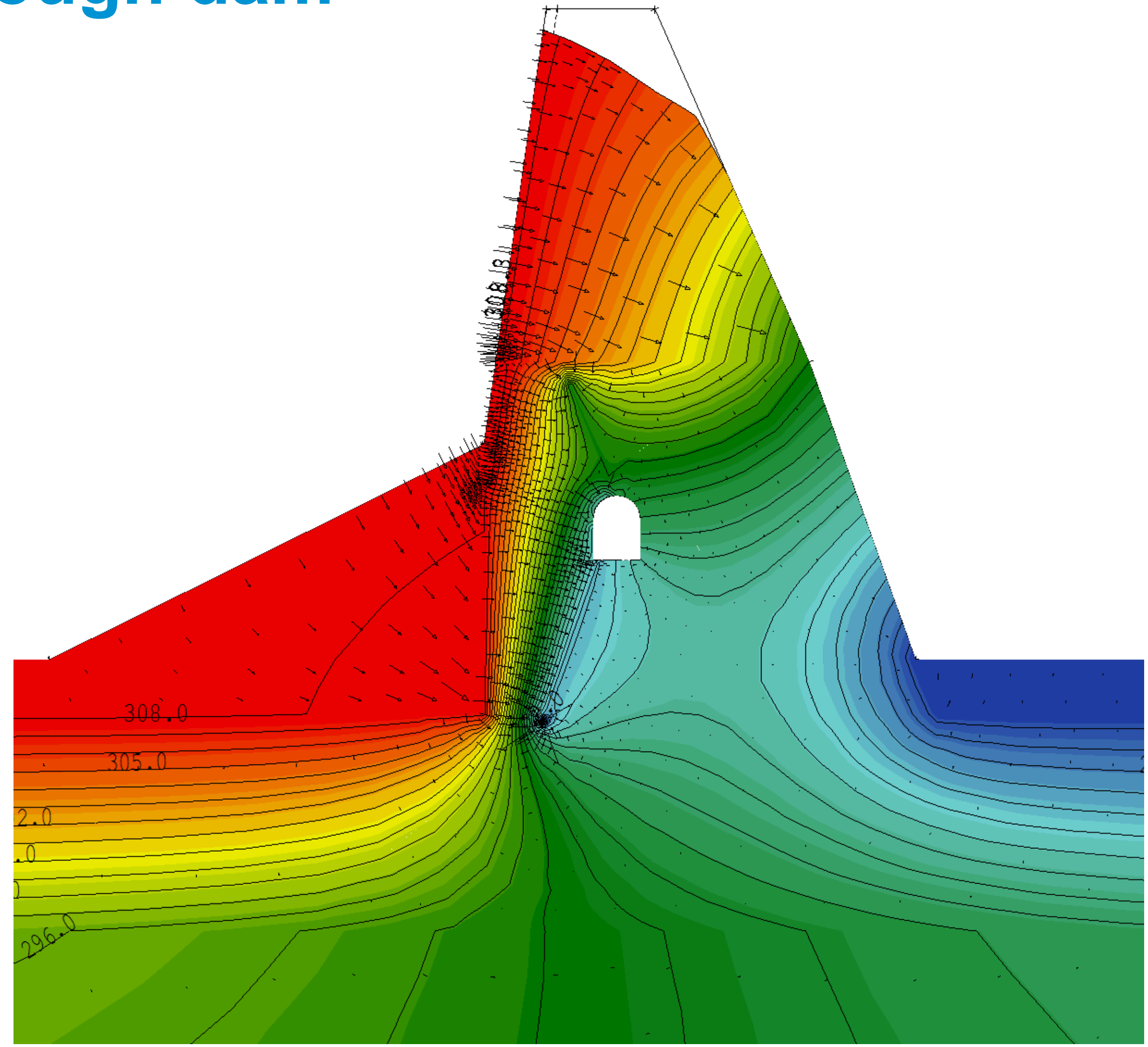
Cut across the valley



Problem of a 2D barrage

- **Stability of a barrage with a height of 32,50 m and a width of 21 m.**
- **Load cases to be considered:**
  - » **Seepage**
  - » **Temperature**
  - » **Ice pressure**
  - » **Earthquake**
- **On the water side there is an additional brickwork and a so called „Intze-Keil“ to ensure water tightness**
- **Non linear material for rock and dam**

Seepage through dam



Stresses including seepage

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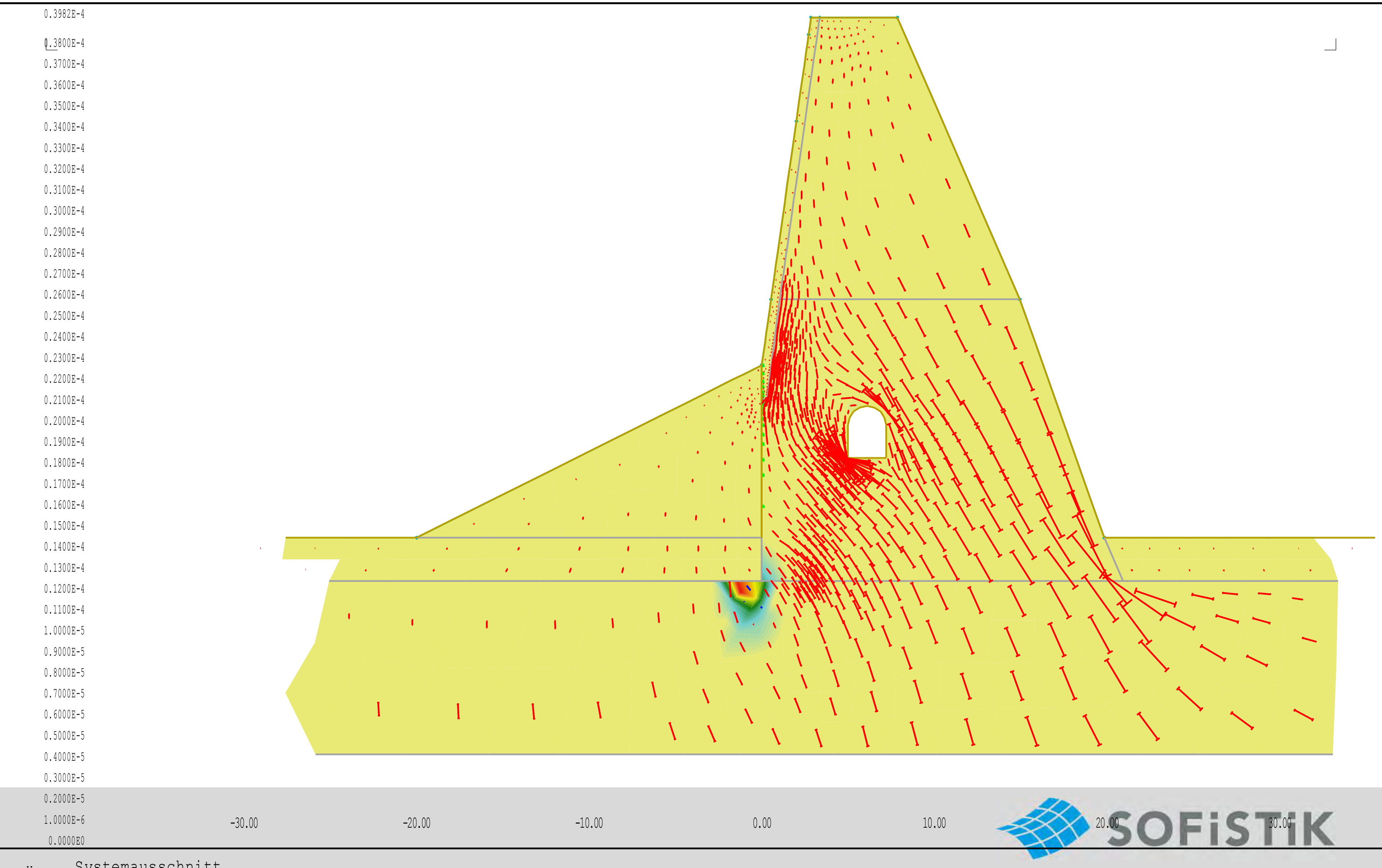
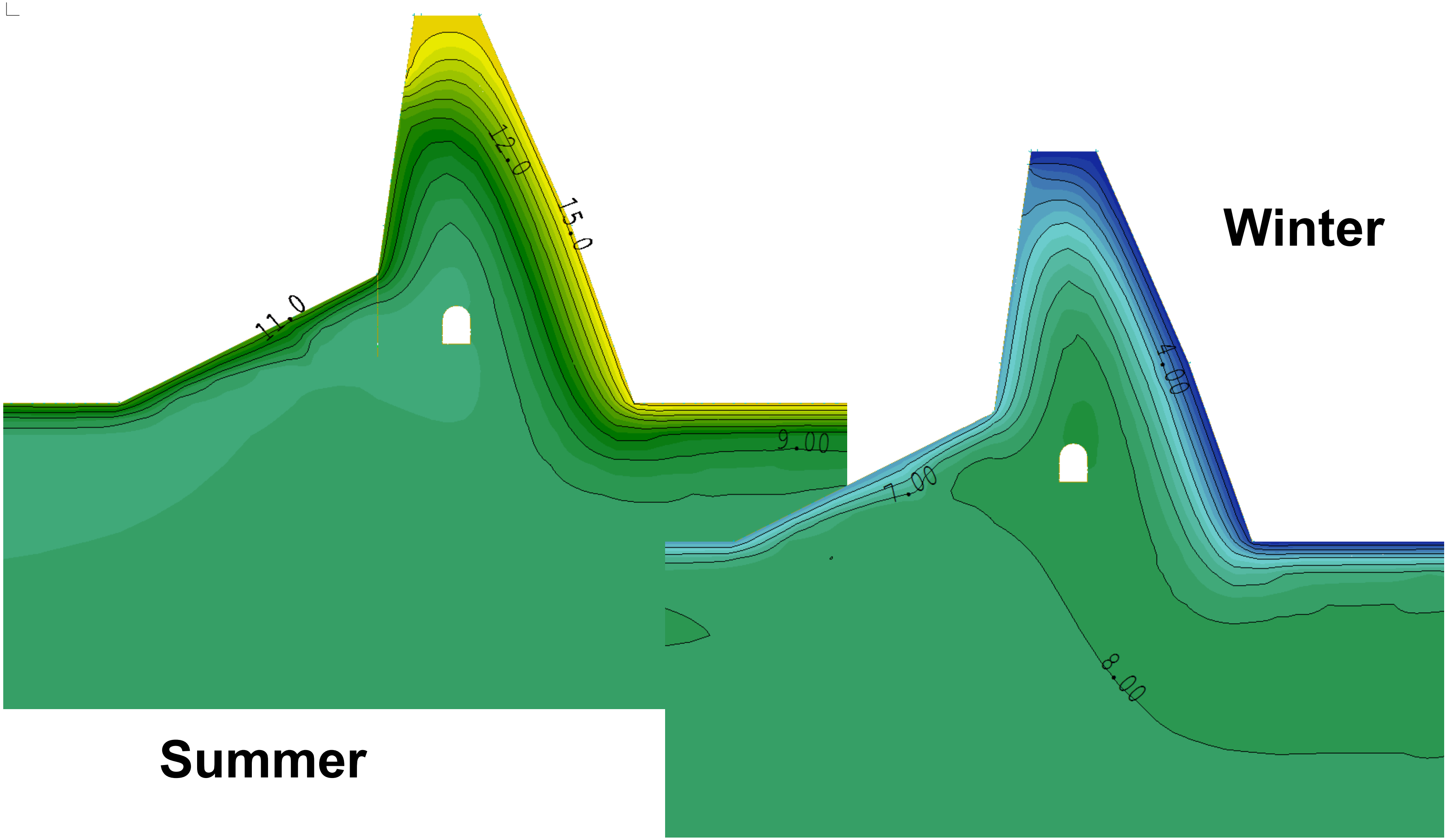


Systemausschnitt

Transient Temperatures

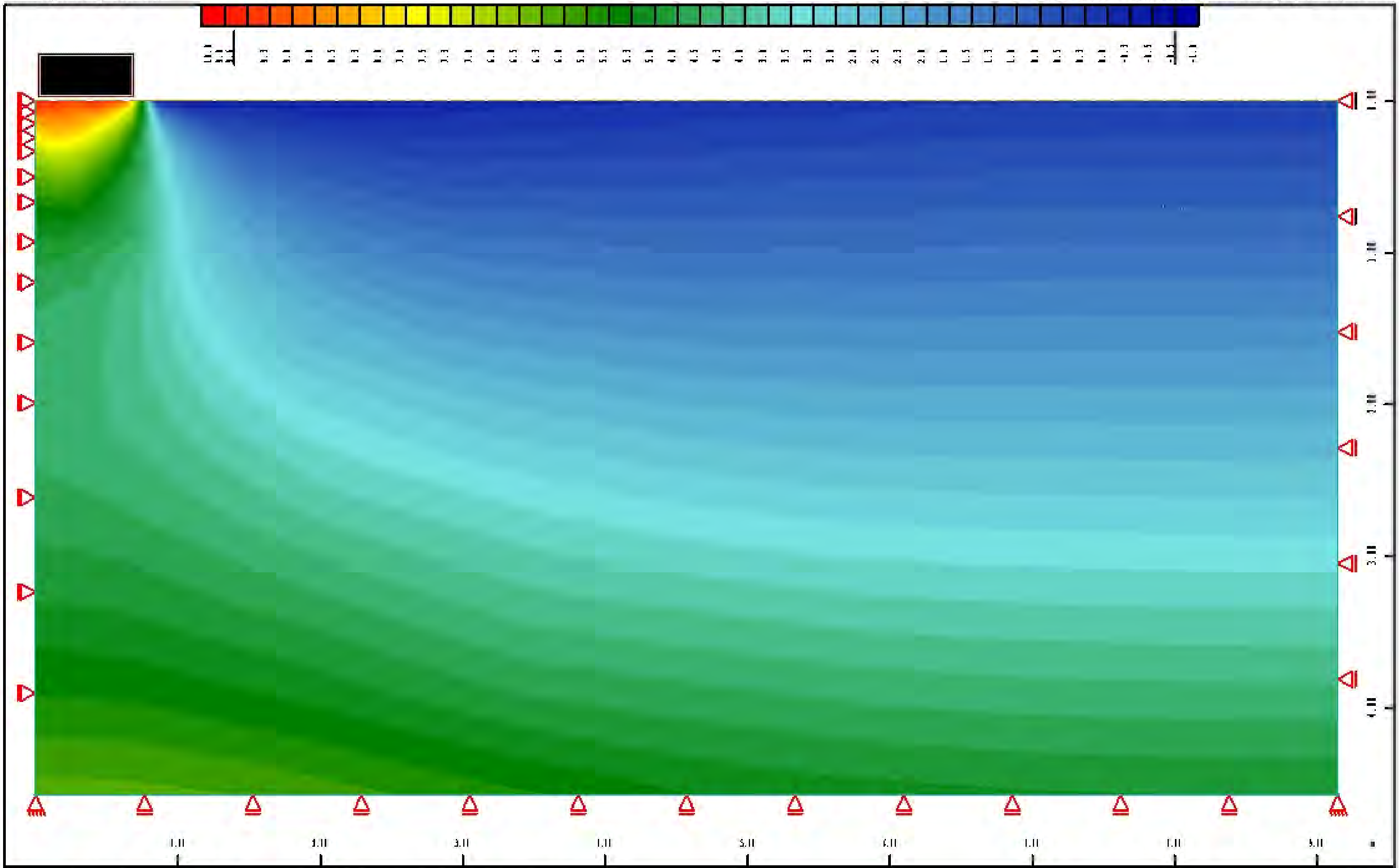


Consolidation

- Clays will carry their load initially by an increase of the pore water pressure, the effective stress between the soil particles is reduced and they may loose their contact. The strength, especially the friction is reduced.

$$\sigma' = \sigma - p_u$$

- The pressure gradient will then create a seepage flow
- Reduction of pore water pressure will reactivate the strength of the soil
- The permeability will be reduced by the compression of the soil
- Thus
  - » Most critical state is immediately after loading
  - » Very complex problem, hardly analysed by FE-Methods and few Software is available



Druckhöhe in Knoten ⇕ , Lastfall 12 Zeitpunkt 0.00 sec, von -0.520 bis 10.0 Stufen 0.250 m

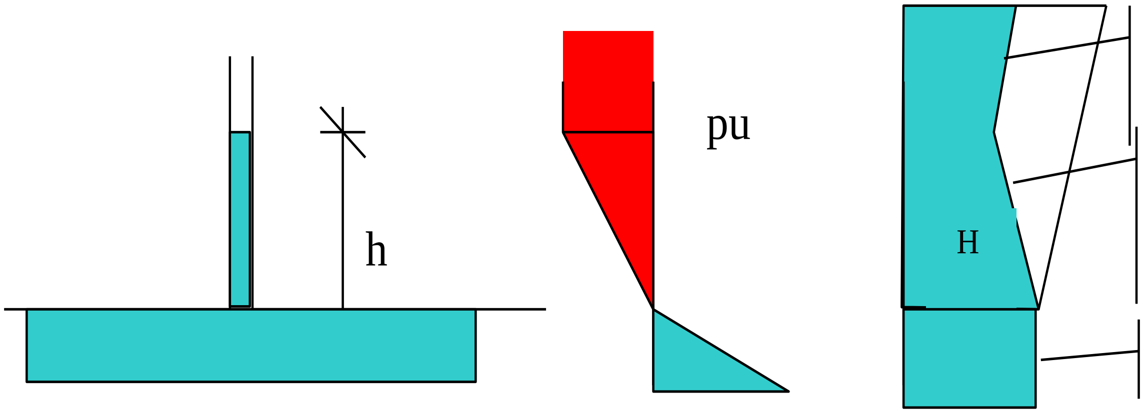
M 1 : 35

Multiphase Flow

- There are many problems with more than one liquid
  - Oil / Water Mixtures
  - Gas / Water Mixtures
  - Unsaturated Soils
- If we have multiple fluids we may have also chemical reactions
- The best solution for those is to use a multiphysics software
- For unsaturated soils there are some possibilities within a classical seepage program however

Suction of Unsaturated Soils

- The surface tension will create a suction (negative pressure) allowing the water table to raise within small capillar tubes much higher than the free surface
- A dry soil has the negative capillar height as minimum pressure
- A wet soil has atmospheric pressure (p=0 at surface)



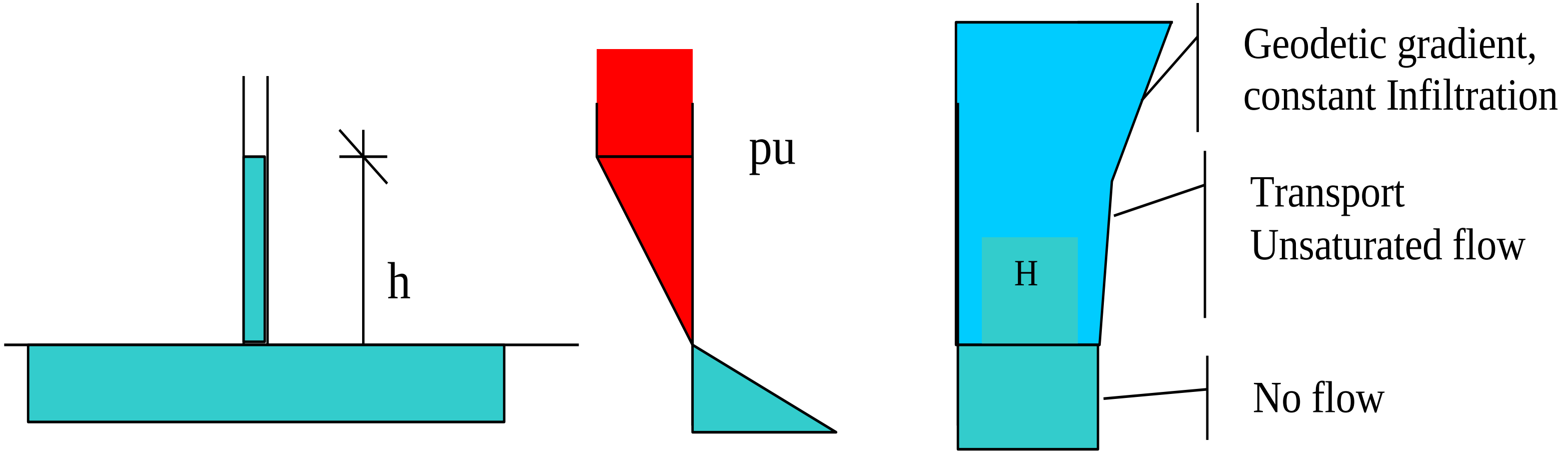
Geodetic gradient,
but no flow

Suction,
pressure gradient is
smaller than geodetic

No Flow

Infiltration through Unsaturated Soil

- If the equilibrium between height and suction is not yet reached the fluid moves upward
- If the equilibrium is reached and we have an additional infiltration from above the gradient will become downwards throughout the structure.

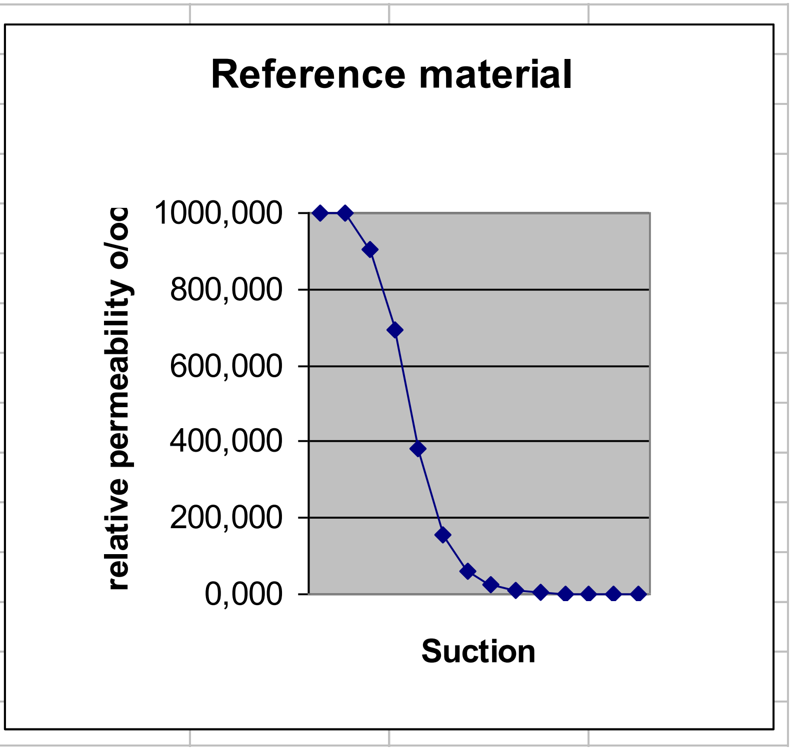


Pressure dependant Material

- **Material is defined as a table of permeability and water content depending on the pore water pressure:**

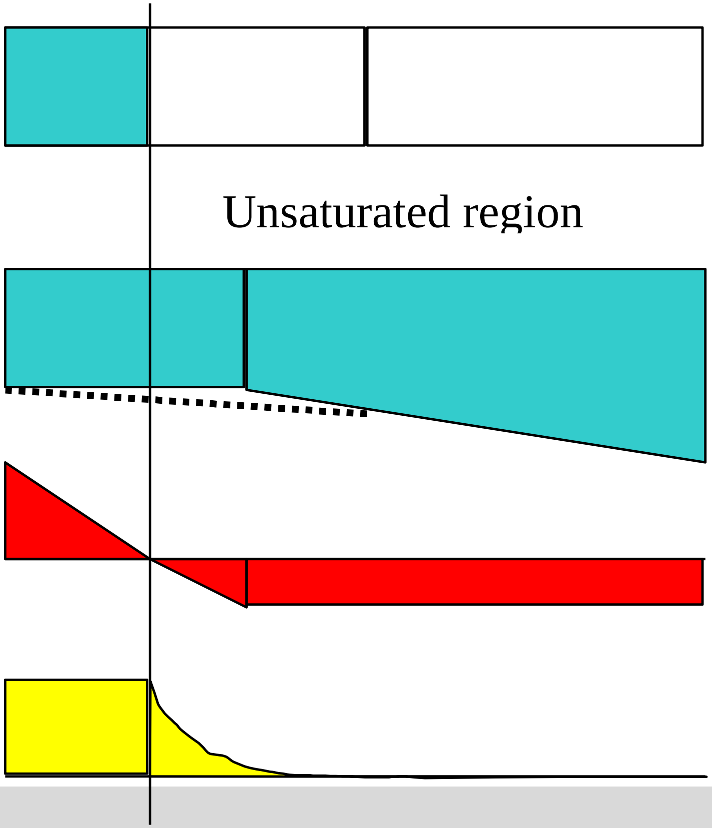
- **The storage coefficient S is given by the derivative of the water content curve**

pu [kPa]	rel. Perm
0,5	1000,000
1	998,000
2	905,000
2,5	692,000
3	383,000
3,5	156,000
4	58,170
4,5	23,610
5	11,090
6	3,587
7	1,633
10	0,402
20	0,066
40	0,019



Problems

- **The Element has a linear variation of the potential. So ist needs special provisions to model the bend in the potential distribution**



Element meshing

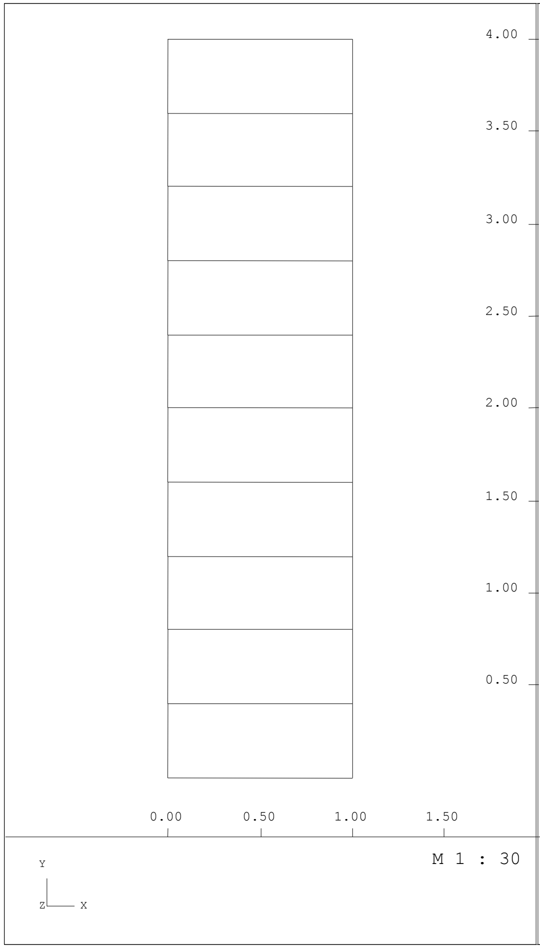
Unsaturated region

Potential
Required distribution /
Element Interpolation

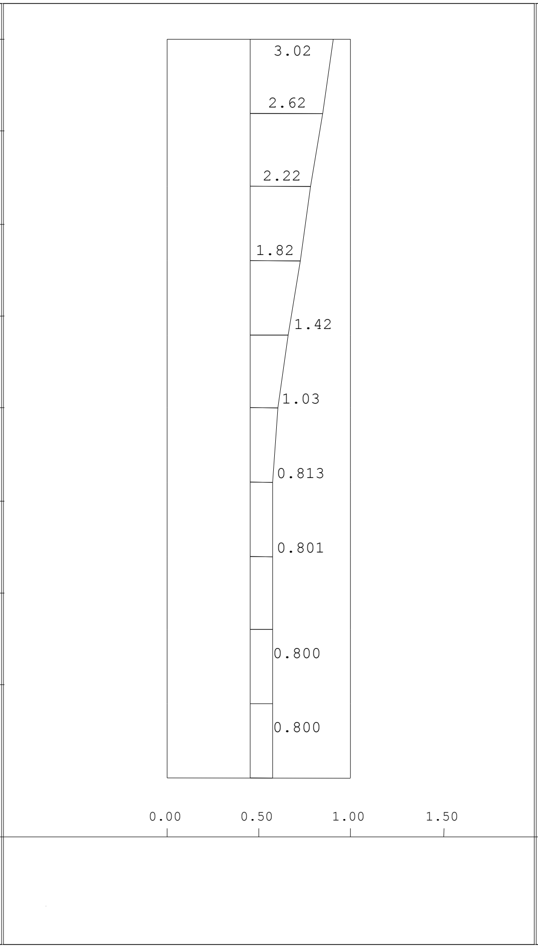
Pore water pressure

Permeability

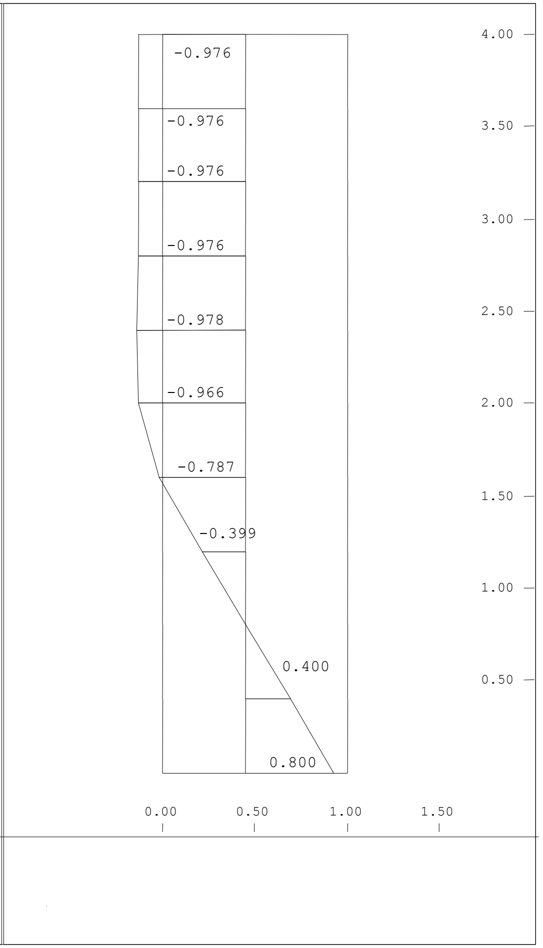
1D Testcase



Structure



Potential

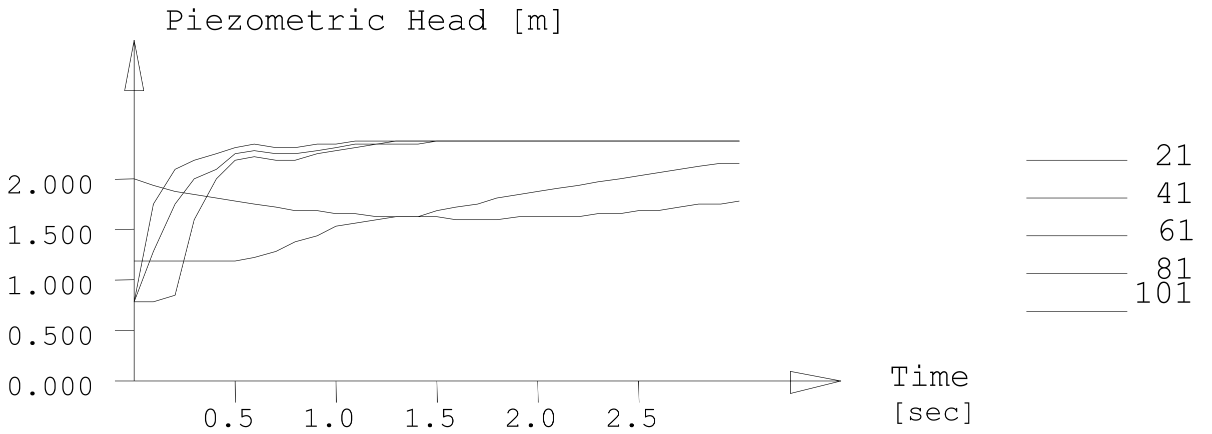


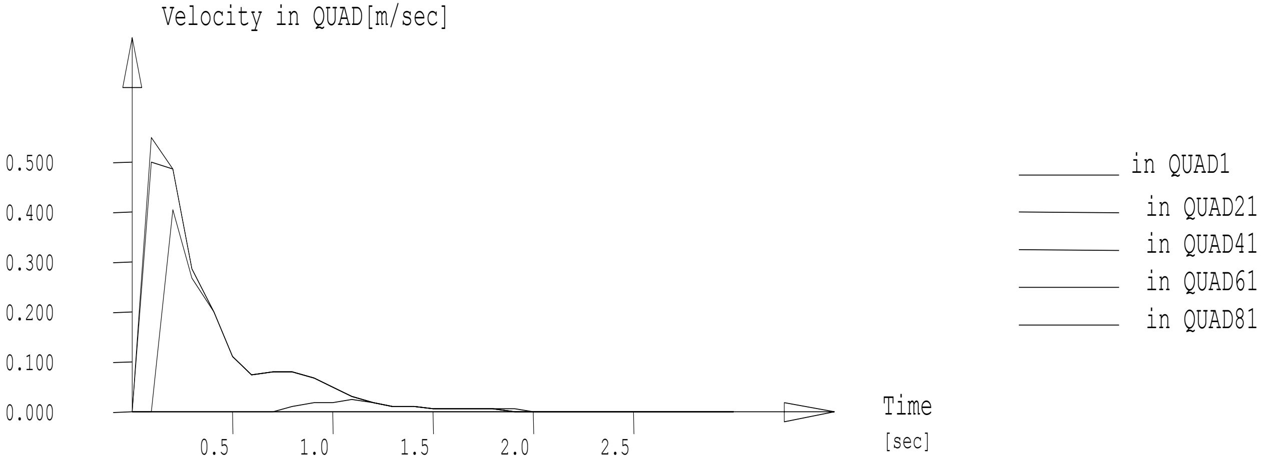
Pressure

Convergence

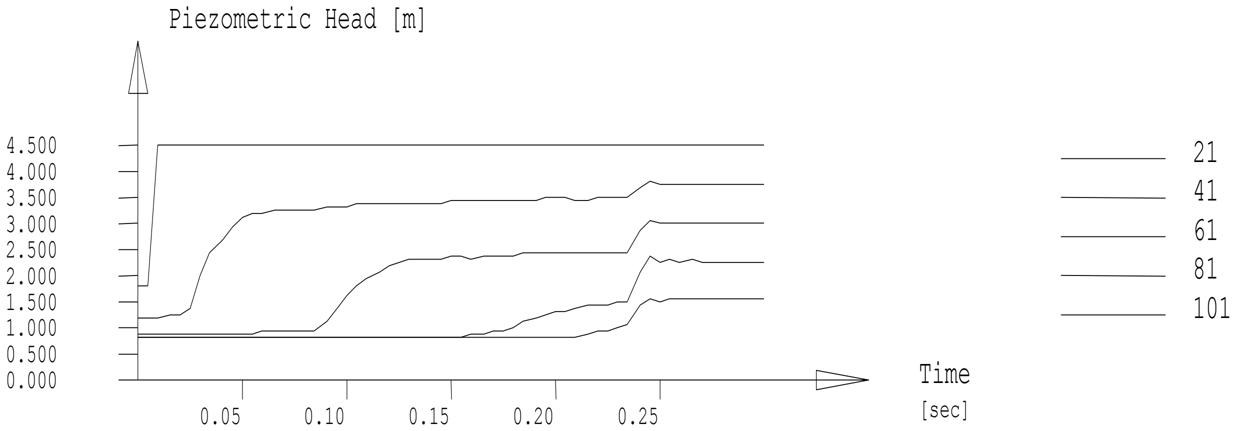
- Boundary Condition at upper edge: quantity
  - No Problem
- Boundary Condition at upper edge: Potential
  - Iteration behaviour is chaotic, small changes have strong effects.
  - The total set of tricks is needed to get convergence of the numerical procedure.

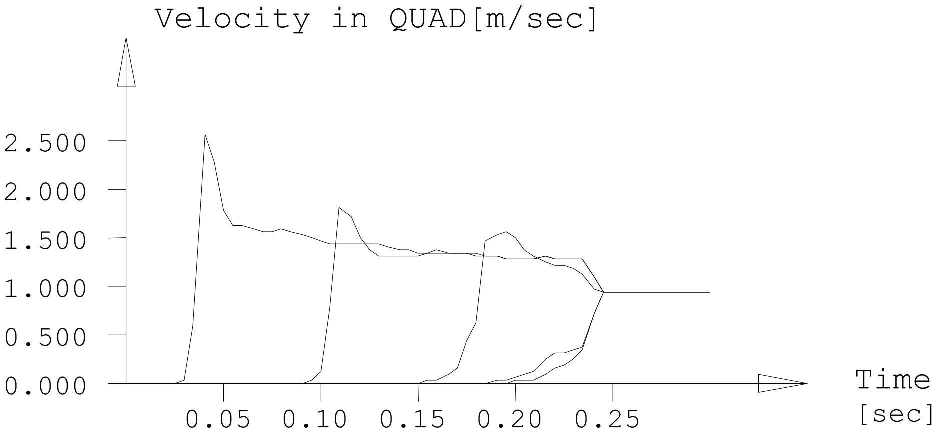
Transient saturation increase of lower pressure



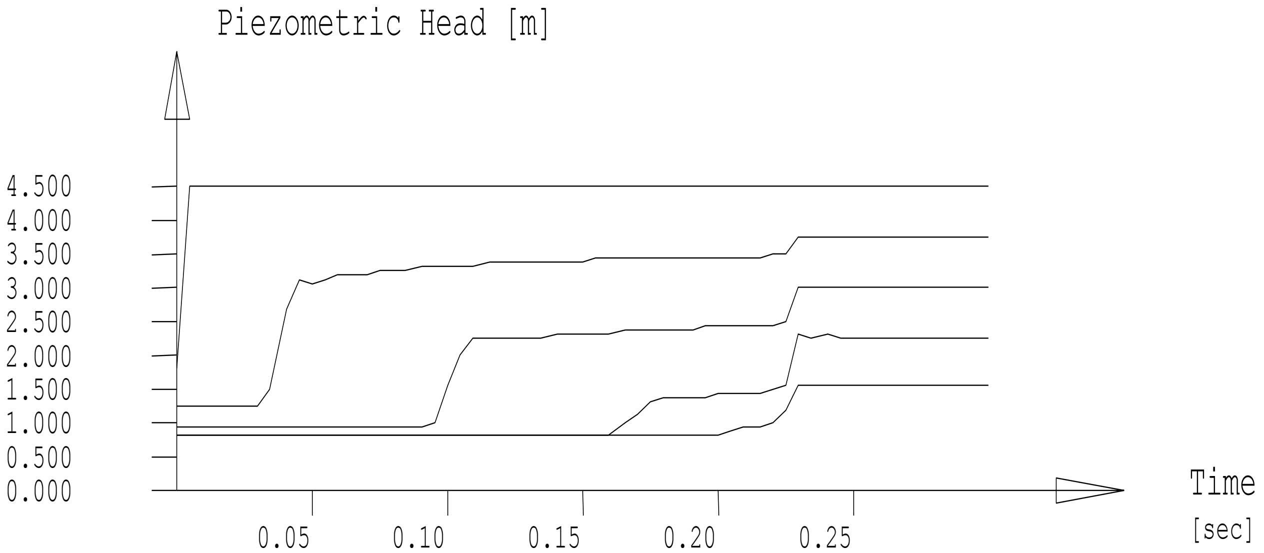


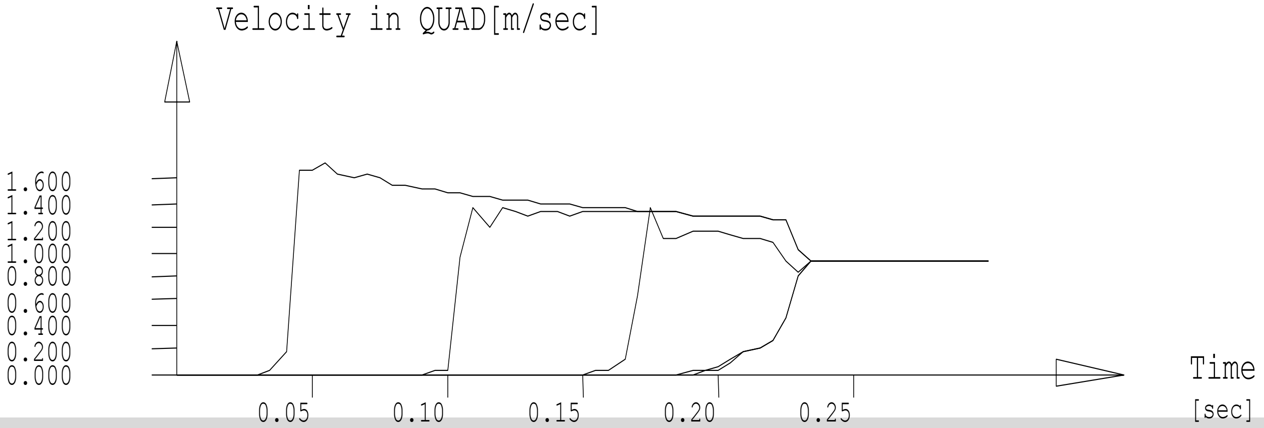
Transient
increase of upper pressure



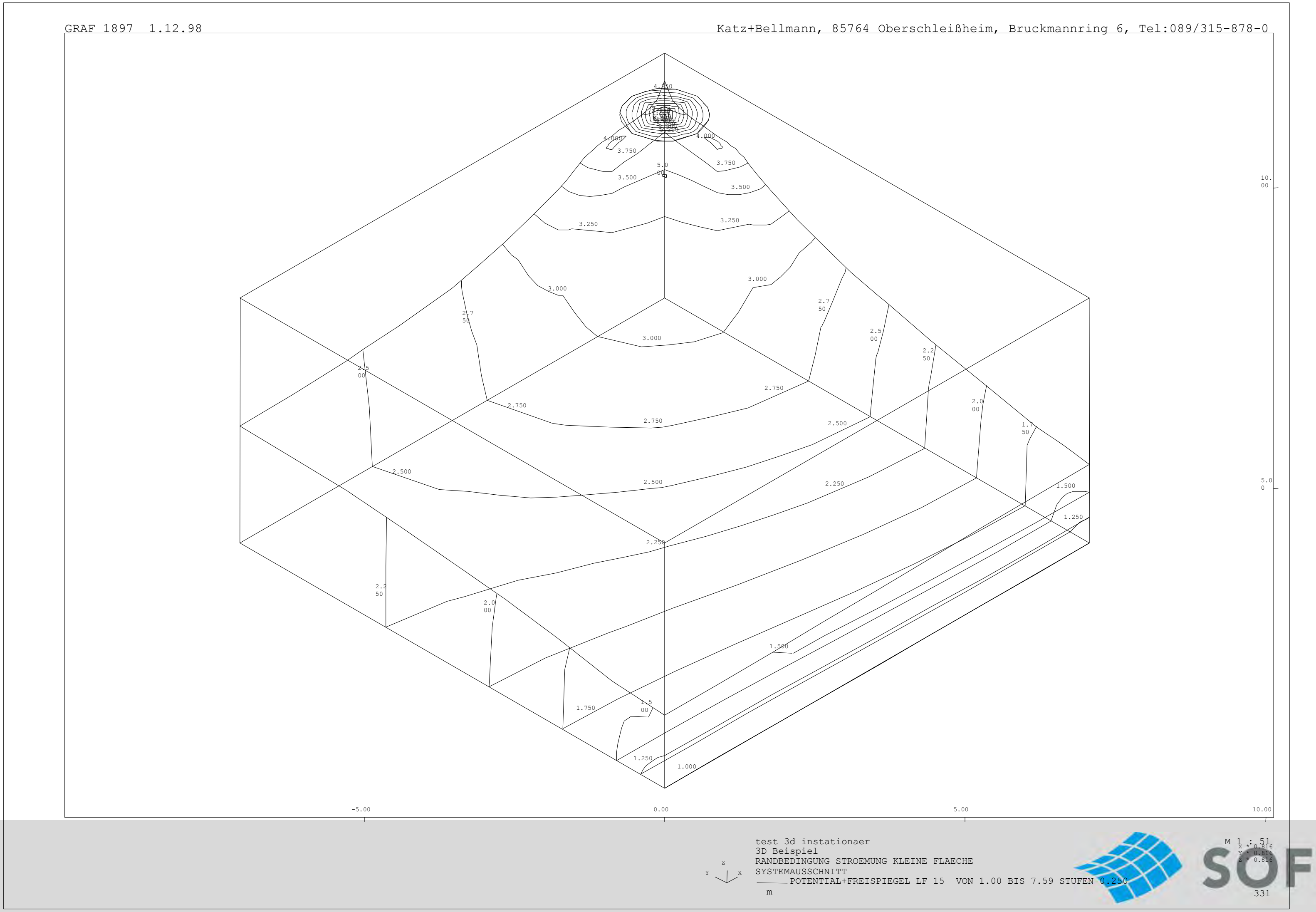


Refined Transient increase of upper pressure





3D Example



GRAF 1897 1.12.98

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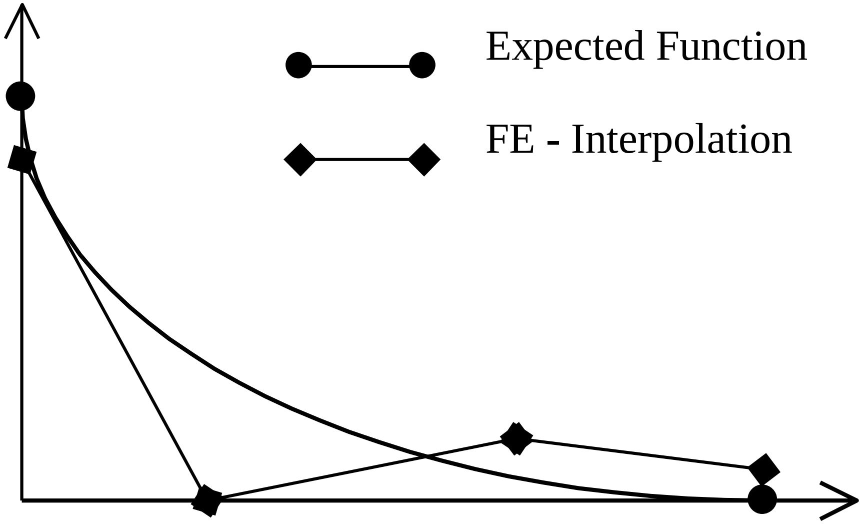
test 3d instationaer
3D Beispiel
RANDBEDINGUNG STROEMUNG KLEINE FLAECHE
SYSTEMAUSSCHNITT
POTENTIAL+FREISPIEGEL LF 15 VON 1.00 BIS 7.59 STUFEN 0.250
m

M 1 : 51
X * 0.816
Y * 0.816
Z * 0.816
331

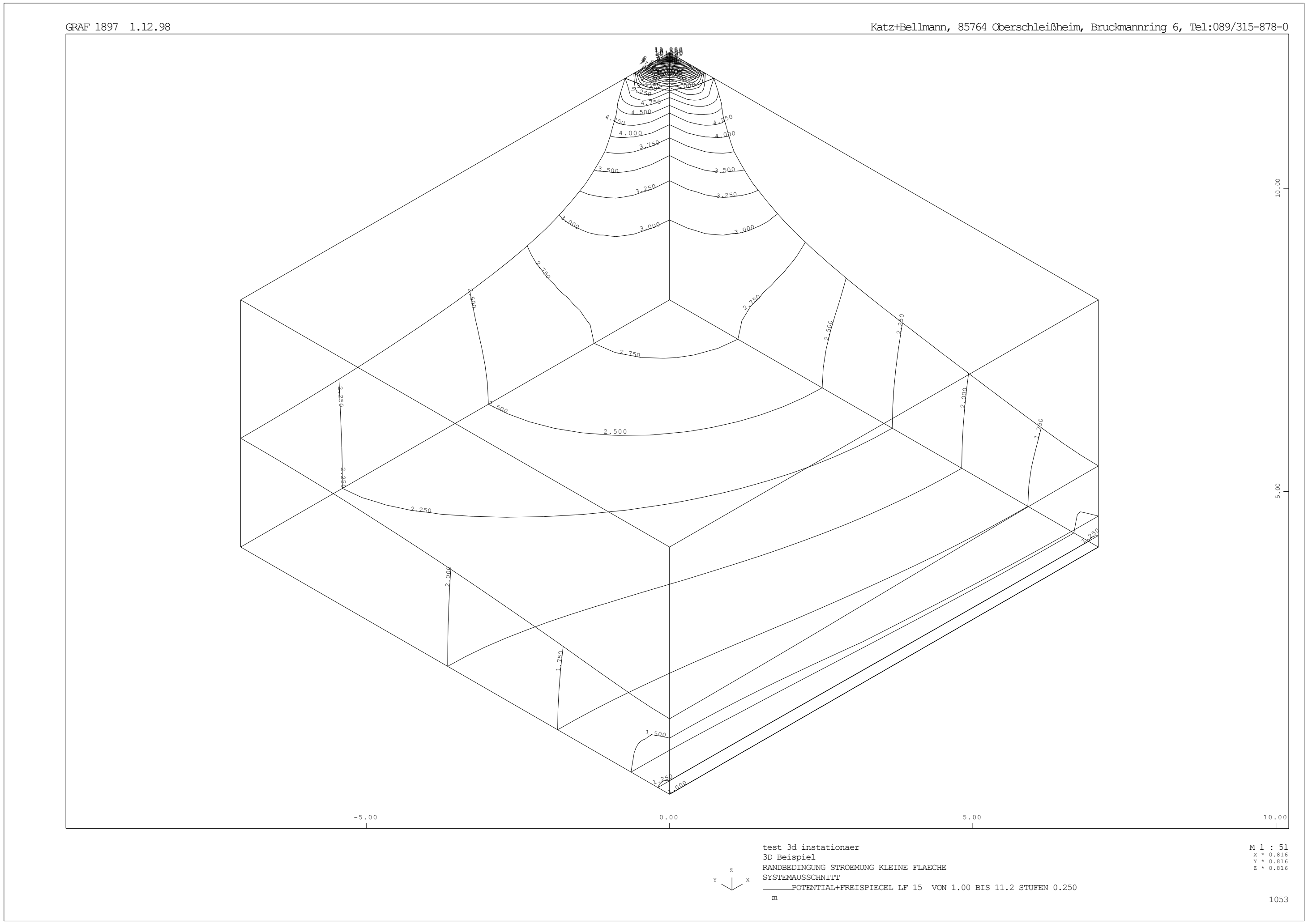
SOFiSTiK

Problem

- The point wise inflow creates a strong singularity in 3D
- As the FE-Mesh has only linear shape functions but averages the solution in an integral sense we get oscillations in our solution spoiling the non linear behaviour



Enhanced numerical approach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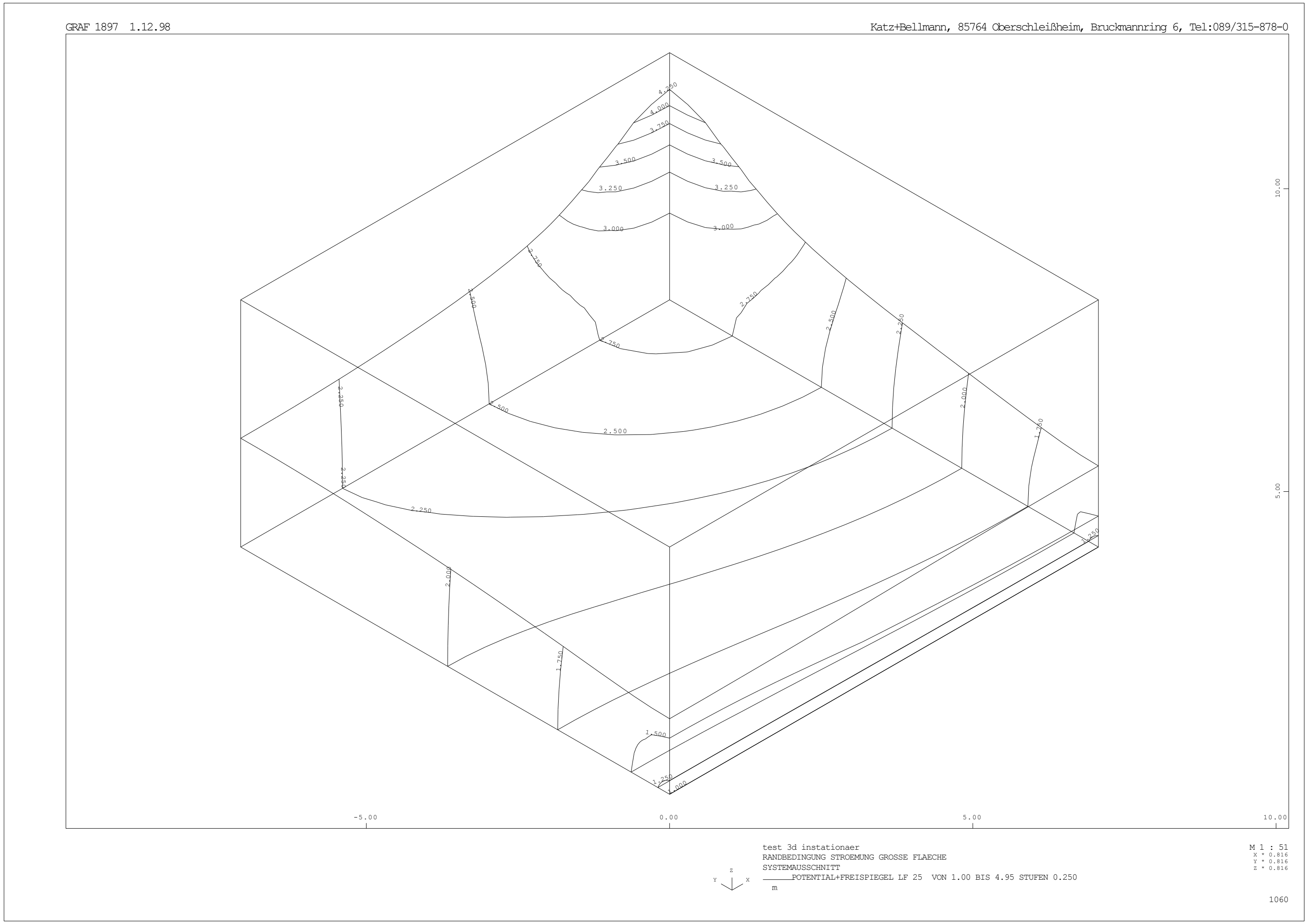
test 3d instationaer
3D Beispiel
RANDBEDINGUNG STROEMUNG KLEINE FLAECHE
SYSTEMAUSSCHNITT
———POTENTIAL+FREISPIEGEL LF 15 VON 1.00 BIS 11.2 STUFEN 0.250
m

M 1 : 51
X * 0.816
Y * 0.816
Z * 0.816

1053

Infiltration with larger area



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test 3d instationaer
RANDBEDINGUNG STROEMUNG GROSSE FLAECHE
SYSTEMAUSSCHNITT
POTENTIAL+FREISPIEGEL LF 25 VON 1.00 BIS 4.95 STUFEN 0.250
m

M 1 : 51
X * 0.816
Y * 0.816
Z * 0.816

1060

Conclusion

- Physics are rather complex
- Mathematics are rather simple
- There are a lot of numerical tools available
- However numerics are not simple
- Specialized Software has it‘s benefits
- Problems for many engineers to think in potential problems