JCSS Workshop on Time-Variant Reliability Analysis:
Old challenges and new solutions
December 2-3, 2024, Technical University of Munich

同濟大學
TONGJI UNIVERSITY

Time-variant global reliability of concrete structures under multihazards

Jie Li, Jianbing Chen

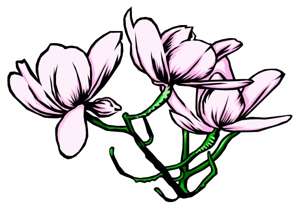
Munich
December 2, 2024

- Background and Challenge
- Principle of Load Coincidence
- Combination of Structural Load Effects

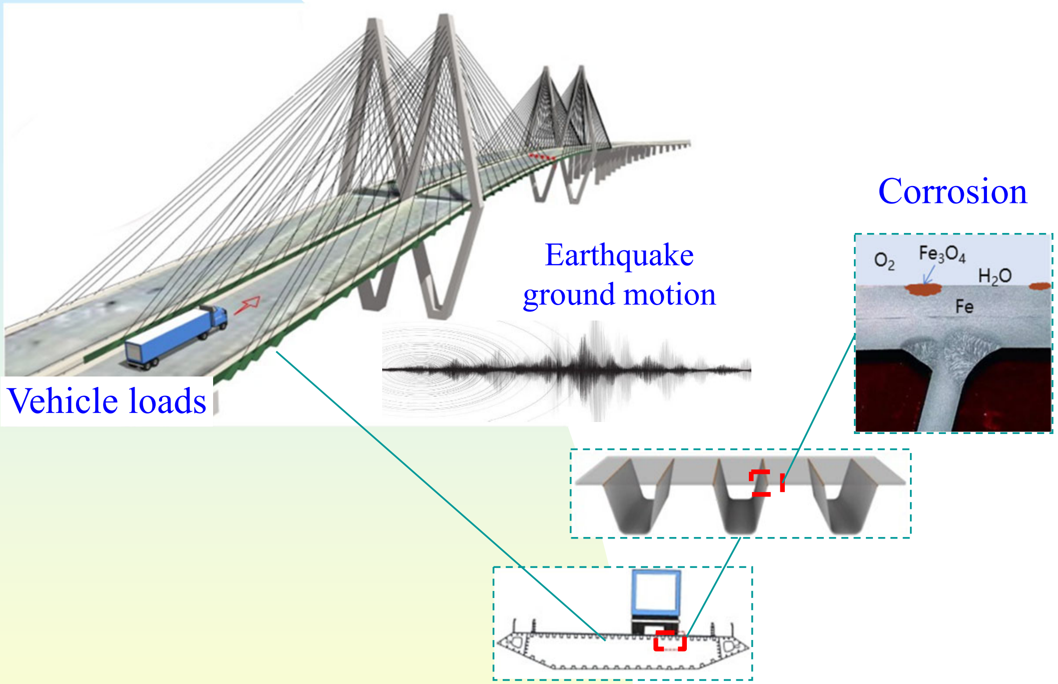
  Nonlinear structural analysis

- Global Reliability Analysis of Structures under Multi-loads and Disastrous Actions
- Typical Engineering Applications
- Conclusions

1. Background

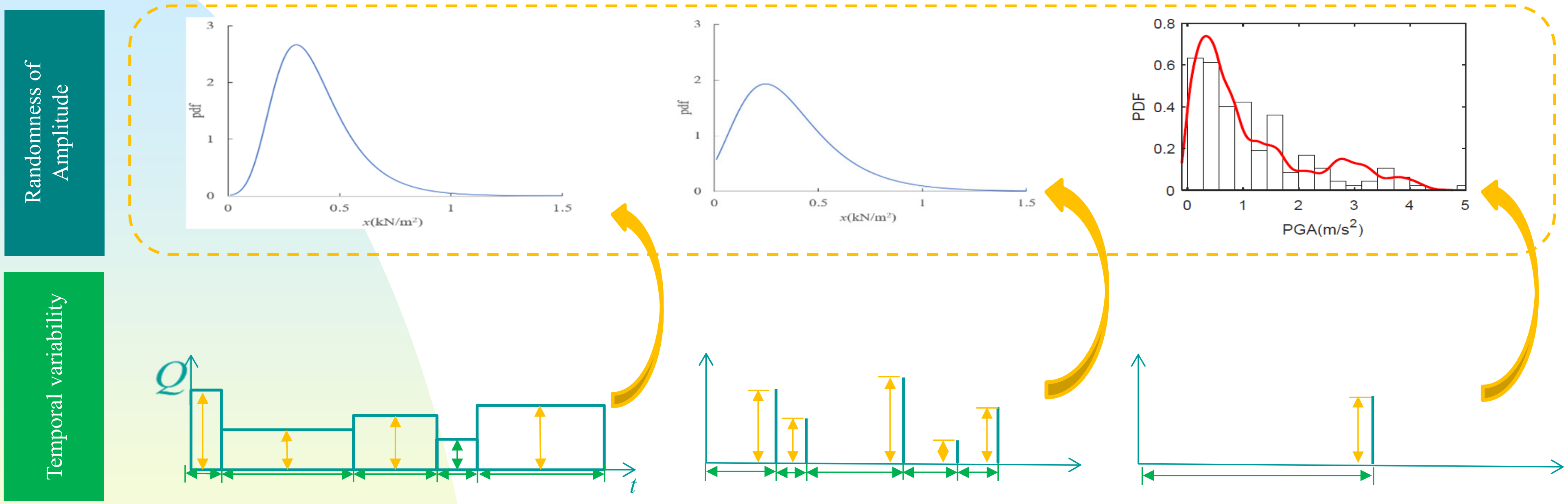


Multiple hazards: Typhoons, earthquakes, huge waves, wind-waves-currents…



Temporal and spatial uncertainty quantification, integrated risk modeling and function simulation of the structures?

Randomness of Structural Loads and Disastrous Actions



Sustained load

Extraordinary load

Wind, earthquakes or huge waves

Poisson process model to characterize loads or disastrous actions

Basic Problems

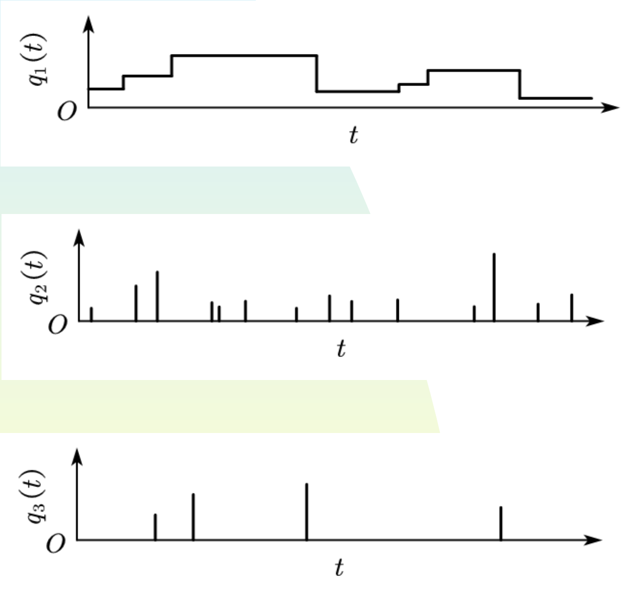
How to Simulate the Lifecycle Function of Structures?

How to describe the random coincidence of load processes

Sustained load

Extraordinary load

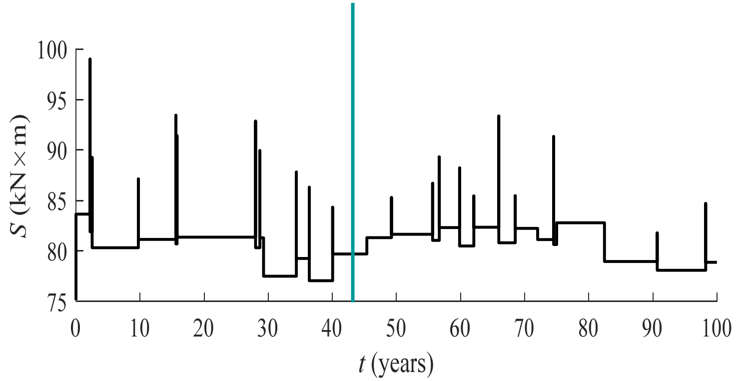
Earthquake excitation



Multiple loading processes

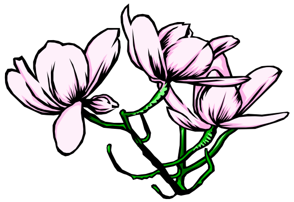
Engineering structure

How to analyze the load combination effect of structures



Load combination effect

2. Principle of Load Coincidence



Linear superposition principle and load effect combination

Due to the existence of linear superposition principle, the load effect combination is converted to load combination.

$$S_{\max}(0,T)=\max_{0\le t\le T}\left[S(t)\right]=\max_{0\le t\le T}\left[\sum_{l=1}^{N_Q} c_l Q_l(t)\right]$$

$$\boxed{c_1=c_2=\cdots=c_l=c}$$

$$\Downarrow$$

$$S_{\max}(0,T)=c\max_{0\le t\le T}\left[\sum_{l=1}^{N_Q} Q_l(t)\right]=c\max_{0\le t\le T}\left[Z(t)\right]=cZ_{\max}(0,T)$$

The traditional load combination can be categorized into two groups:

1. **Intuition-based load combination method**
2. **Rational approximation (analytical) load combination method**

Intuition-based load combination method

Turkstra (1970)

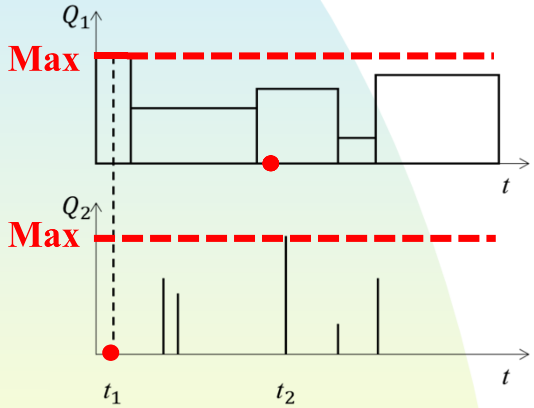
Ferry Borges & Castanheta (1971)

JCSS (1976)

Rackwitz & Flessler (1978)

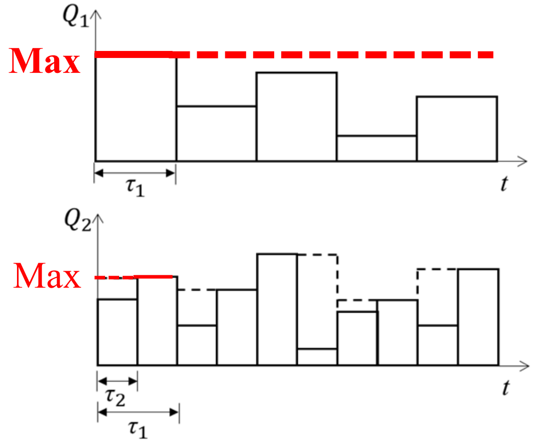
…

Turkstra combination rule



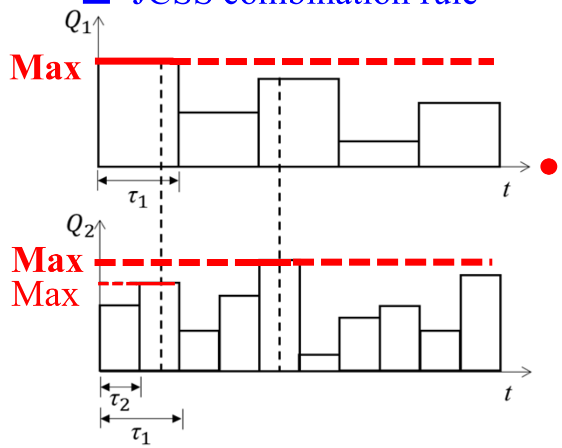
$$Z_{\max}(T) = \max\left\{\begin{matrix} \max\limits_{0\le t\le T}\left[Q_1(t)\right]+Q_2(t_1) \\ Q_1(t_2)+\max\limits_{0\le t\le T}\left[Q_2(t)\right] \end{matrix}\right\}$$

FBC combination rule



$$Z_{\max}(T) = \max_{0\le t\le T}\left\{Q_1(t)+\max_{0\le t\le \tau_1}\left[Q_2(t)\right]\right\}$$

JCSS combination rule



$$Z_{\max}(T) = \max\left\{\begin{matrix} \max\limits_{0\le t\le T}\left[Q_1(t)\right]+\max\limits_{0\le t\le \tau_1}\left[Q_2(t)\right] \\ Q_1(t)+\max\limits_{0\le t\le T}\left[Q_2(t)\right] \end{matrix}\right\}$$

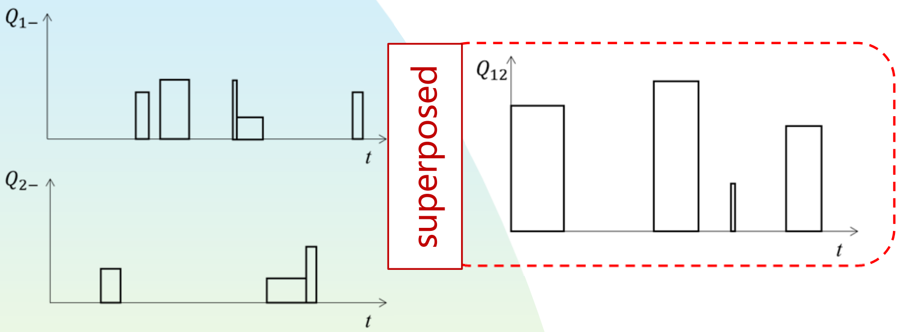
To employ stationary binomial processes and give an intuitive judgment to determine the maximum combination load.

Rational approximation (analytical) load combination method

Hasofer (1974) Gaver & Jacobs (1978) Gong & Zhao (2001) Mori et al. (2003) Pandey et al. (2021)

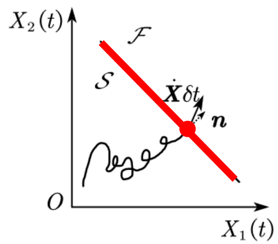
The classical research lacks the analysis on the Principle of Load Coincidence.

$$v_{12}(a) \le \int_{-\infty}^{\infty} v_1(u) f_2(a-u)\,\mathrm{d}u + \int_{-\infty}^{\infty} v_2(u) f_1(a-u)\,\mathrm{d}u$$



$$F_{Z\max} = \exp\left\{-\lambda_1 T\left[1-F_1(x)\right] - \lambda_2 T\left[1-F_2(x)\right] - \lambda_{12} T\left[1-F_{12}(x)\right]\right\}$$

Beyond crossing



$$v_{\mathcal{S}} = \int_{\partial\mathcal{S}} E\left(\dot{X}_n \,\middle|\, \boldsymbol{X} = \boldsymbol{x}\right)^{+} f_{\boldsymbol{X}}(\boldsymbol{x})\,\mathrm{d}\boldsymbol{x}$$

Breitung & Rackwitz (1982)

Wen (1977) Larrabee &Cornell (1978)

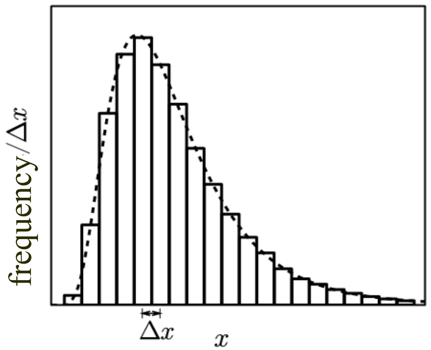
Taking the compound Poisson process as the load probability model, and introducing approximate assumptions to provide analytical solutions for load combinations.

Load combination: analysis of basic problems

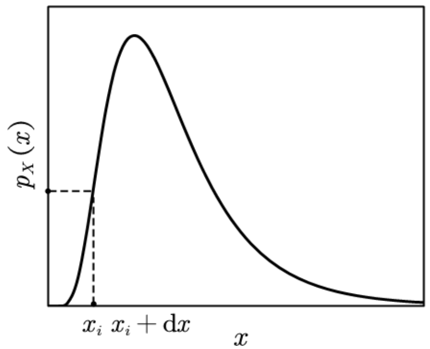
Occurrence probability of random events

1. A single random variable

The probability of a sample value occuring in a certain interval of the histogram is actually equal to the frequency of the interval data.



Statistical histogram



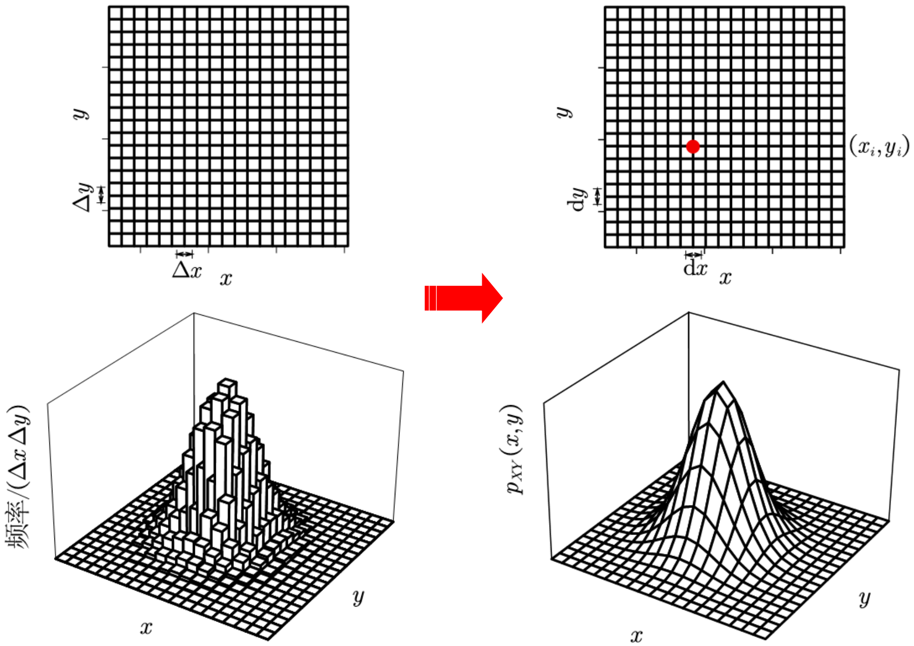
Probability density function

$p_X(x_i)$ provides a measure to the occurrence possibility of sample x_i.

Random sample	Deterministic sample
Discrete point sequence	Continuous variable
Occurrence frequency	Occurrence probability

Coincidence of Random Events

2. Two random variables (Random coincidence of samples):



Statistical histogram and Joint probability density function

$p_{XY}(x_i, y_i)$

provides a measure to the simultaneous occurrence possibility of sample x_i and y_i

x_i, y_i

is a realization of coincidence for random variables (X, Y)

$p_{XY}(x_i, y_i)\mathrm{d}x\mathrm{d}y$

is the probability of random variables (X, Y) occurring at the interval of $\left(x_i \pm \frac{1}{2}dx, y_i \pm \frac{1}{2}dy\right)$

(can be called as the coincidence probability)

Load Coincidence Principle

- **Coincidence problem of two stochastic processes**

$Q_1(t)$ $Q_2(t)$

Stochastic harmonic function expression of compound Poisson process

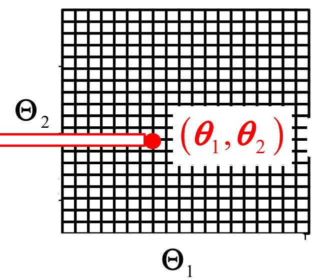
Coincidence of two stochastic process samples!

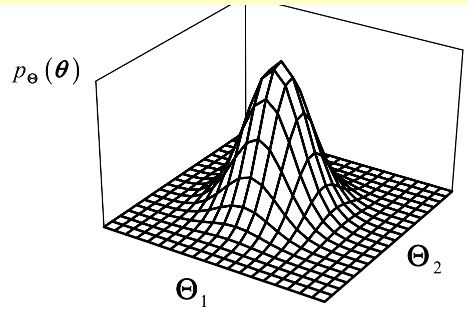
Load Coincidence Principle: The coincidence probability (density) of random load samples is equal to the joint probability density at the realized sample.

- **Coincidence for two sets of basic random variables**

$\boldsymbol{\Theta} = (\Theta_1, \Theta_2)$

The subscript represents the process of belonging



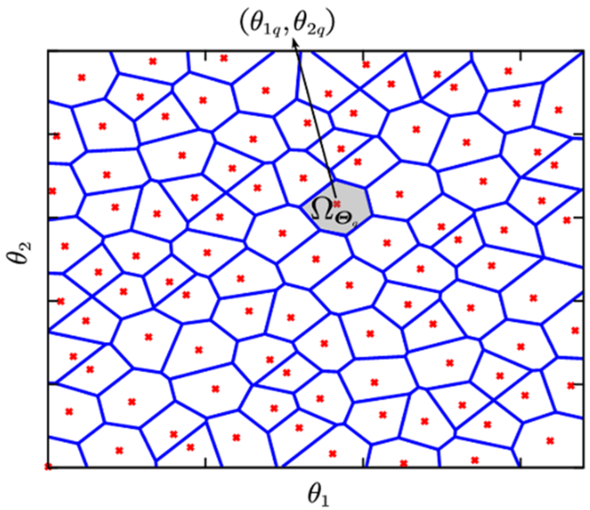


In the probability space forming by basic random variables $\Omega_{\Theta} = (\Theta_1, \Theta_2)$, $p_{\Theta}(\theta)$ provides a measure to the coincidence possibility of samples $\Theta = (\Theta_1, \Theta_2)$. Thus, it also provides a measure to the coincidence possibility of two load processes,

$Q_1(t) = Y_{Q_1}(\theta_1, t)$ **and** $Q_2(t) = Y_{Q_2}(\theta_2, t)$

Li, J. and D. Wang. On the principle of load combination of structures. Structural Safety, 2021, 89: 102046.

Probability Space Partition and Load Coincidence Probability



Example for space partition of coincidence probability in two dimensions

Probability Space Partition:

Partition subdomains: $\Omega_{\Theta_q}, q = 1, 2, \cdots, N_{\text{sel}}$

Coincidence Probability:

$$P_q = \int_{\Omega_{\Theta_q}} p_{\Theta}(\boldsymbol{\theta}) \mathrm{d}\boldsymbol{\theta}, q = 1, 2, \cdots, N_{\text{sel}}$$

N_{sel} ——**Number of subdomains**

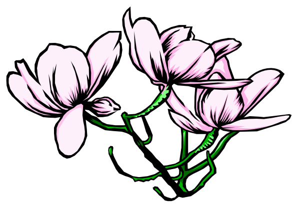
$\boldsymbol{\theta}_q$ ——**Realization of representative samples**

Through probability space partition, the deterministic expression of random coincidence and the coincidence probability can be achieved.

Li, J. and D. Wang. On the principle of load combination of structures. Structural Safety, 2021, 89: 102046.

Maximum Combined Load of Multiple Loads

Target: To compute the probability density function of the maximum combined load within a specified service life.



Extreme probability distribution of combined loads: Extreme distribution method

Define the maximum of combined load for a specified service time in the probability space Ω_{Θ}

$$Z_{\max}(\boldsymbol{\Theta},T)=\max_{0\le t\le T}\left[Z(\boldsymbol{\Theta},t)\right]=\max_{0\le t\le T}\left[\sum_{l=1}^{m}Y_{Q_l}(\boldsymbol{\Theta}_l,t)\right]$$

Probability density evolution equation of extreme load value

$$\begin{cases} Z(\boldsymbol{\Theta},\tau)=\psi\left(Z_{\max}(\boldsymbol{\Theta},T),\tau\right) \\ \dfrac{\partial p_{Z\boldsymbol{\Theta}}(z,\boldsymbol{\theta},\tau)}{\partial\tau}+h_Z(\boldsymbol{\theta},\tau)\dfrac{\partial p_{Z\boldsymbol{\Theta}}(z,\boldsymbol{\theta},\tau)}{\partial z}=0 \end{cases}$$

$\psi(\cdot)=Z_{\max}(\boldsymbol{\Theta},T)\sin(\pi\tau/2)$ —— Virtual stochastic process, satisfying: $Z(\boldsymbol{\Theta},\tau)\big|_{\tau=0}=0,\ Z(\boldsymbol{\Theta},\tau)\big|_{\tau=\tau_c=1}=Z_{\max}(\boldsymbol{\Theta},T)$

τ —— Generalized time parameter

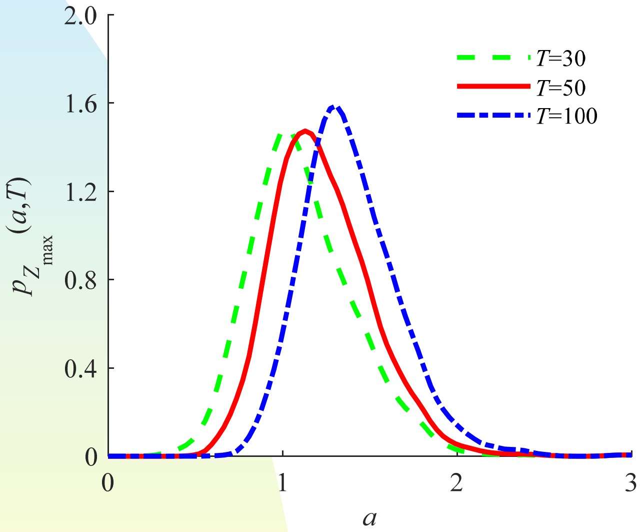
Extreme probability distribution for a given time interval

$$\begin{cases} p_Z(z,\tau)=\int_{\Omega_{\Theta}}p_{Z\boldsymbol{\Theta}}(z,\boldsymbol{\theta},\tau)\,\mathrm{d}\boldsymbol{\theta} \\ F_{Z_{\max}}(a,T)=\int_{-\infty}^{a}p_Z(z,\tau_c)\,\mathrm{d}z \end{cases}$$

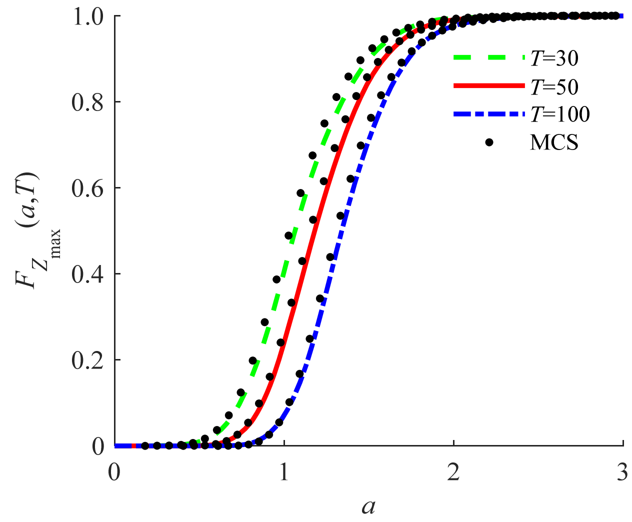
Examples: Combination of sustained and extraordinary load

$$Z(t)=Q_{\text{sus}}(t)+Q_{\text{tra}}(t)$$

$$\lambda_{\text{sus}}=0.1\ \text{year}^{-1},\ \lambda_{\text{tra}}=0.2\ \text{year}^{-1}$$



Extreme probability density function



Extreme probability distribution function

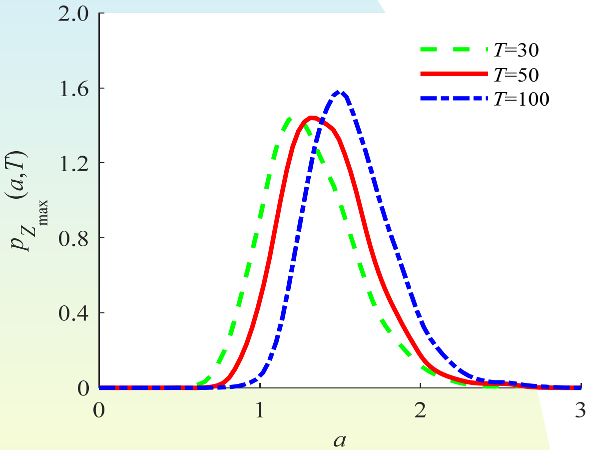
Extreme distribution method: The distribution probability of extreme load is variant for different service years

Examples: Combination of sustained load and two extraordinary loads

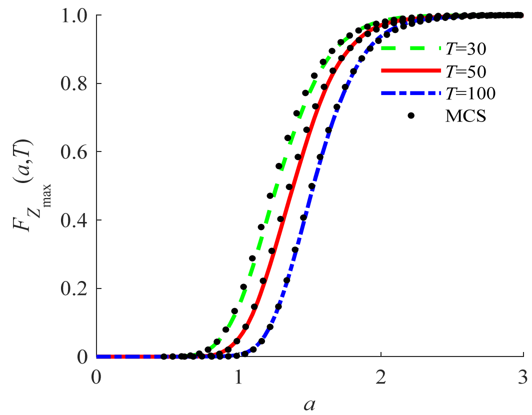
$$Z(t) = Q_{\text{sus}}(t) + Q_{\text{tra},1}(t) + Q_{\text{tra},2}(t)$$

$$\lambda_{\text{sus}} = 0.1\ \text{year}^{-1},\ \lambda_{\text{tra},1} = 0.2\ \text{year}^{-1},\ \lambda_{\text{tra},2} = 0.3\ \text{year}^{-1} \qquad \mu_{\text{tra},1} = 0.246\ \text{kN/m}^2,\ \alpha_{\text{tra},1} = 0.190\ \text{kN/m}^2$$

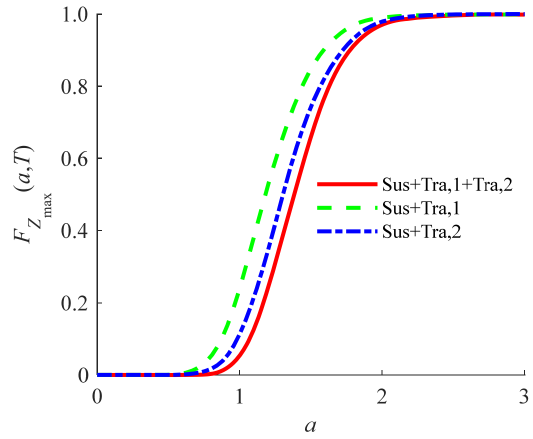
$$\mu_{\text{sus}} = 0.306\ \text{kN/m}^2,\ \alpha_{\text{sus}} = 0.139\ \text{kN/m}^2 \qquad \mu_{\text{tra},2} = 0.244\ \text{kN/m}^2,\ \alpha_{\text{tra},2} = 0.199\ \text{kN/m}^2$$



Probability density function of extreme value



Probability distribution function of extreme value



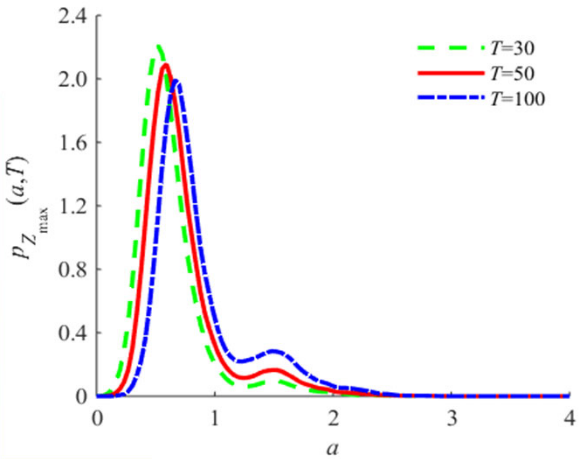
Comparison of probability distribution function for combined extreme values

For different service life, the probability density function of the maximum combined load changes significantly.

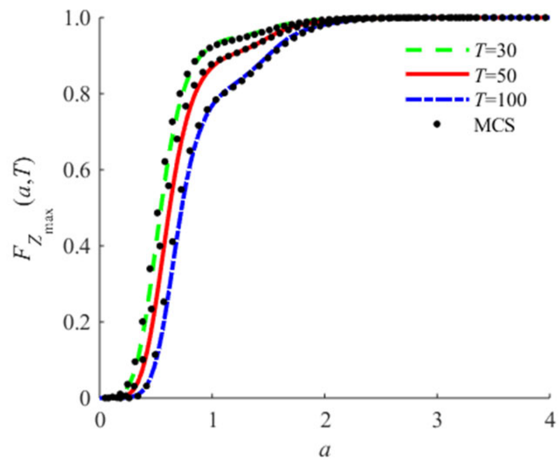
Examples: Combination of sustained load and earthquake excitation

$$Z(t) = Q_{\text{sus}}(t) + E(t)$$

Extreme distribution method



Extreme probability density function

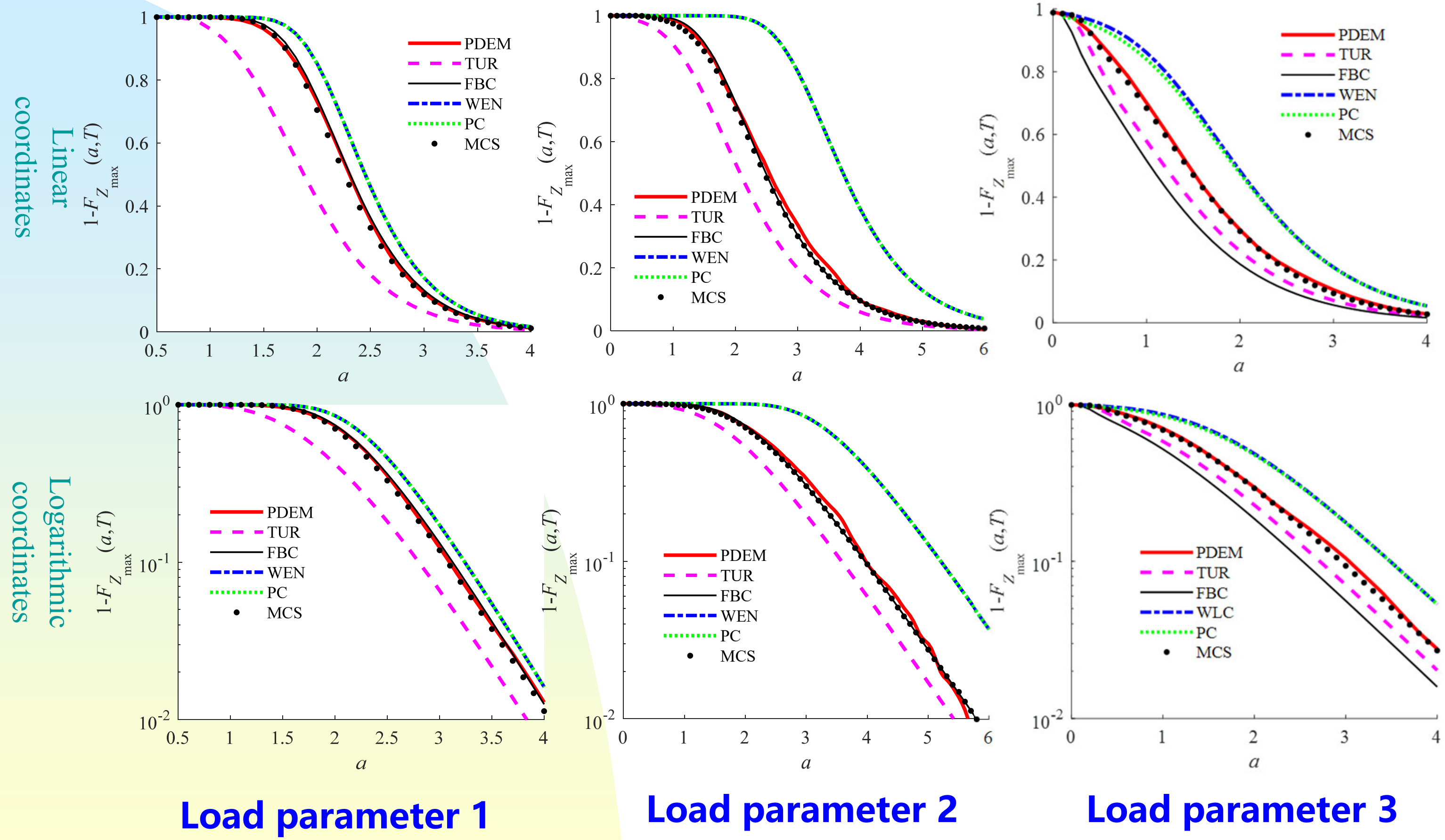


Extreme probability distribution function

Average occurrence rate of earthquake $\lambda_{\text{e}} = 0.002\ \text{year}^{-1}$ **Earthquake amplitude:** $\mu_{\text{e}} = 4.9\ \text{m/s}^2$, $\sigma_{\text{e}} = 0.49\ \text{m/s}^2$

Using the load coincidence principle, the extreme probability distribution of combined loads at a given service period can be quantitatively derived.

Comparison of different load combination methods



Threshold	1.5	2	2.5
MCS	0.9694	0.7056	0.3332
PDEM	0.68%	2.18%	0.37%
TUR	21.96%	**28.25%**	15.04%
FBC	0.70%	3.82%	3.17%
WEN	2.64%	14.81%	12.87%
PC	2.62%	14.76%	12.86%

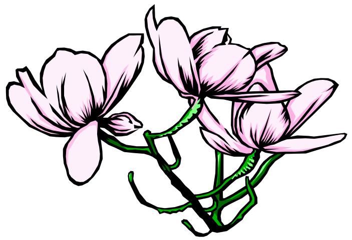
Threshold	2	3	4
MCS	0.7042	0.3004	0.0958
PDEM	1.63%	**3.55%**	0.3%
TUR	17.22%	10.36%	3.62%
FBC	2.21%	0.21%	0.06%
WEN	29.27%	**52.24%**	29.17%
PC	29.25%	**52.12%**	29.14%

Threshold	1	2	3
MCS	0.6863	0.2915	0.0917
PDEM	1.67%	0.41%	1.31%
TUR	10.49%	6.35%	2.08%
FBC	**16.7%**	10.51%	3.56%
WEN	17.7%	19.2%	8.5%
PC	15.56%	18.4%	8.39%

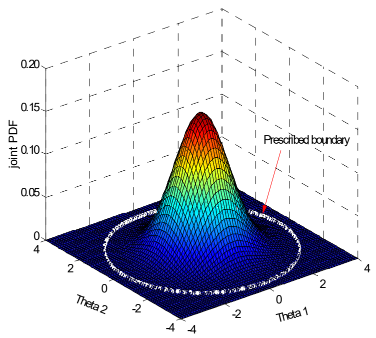
Compared with the MC method, the error of the recommended method is less than 4%, while other classical methods have an error up to 17%-52%.

3. Combination of Structural Load Effects

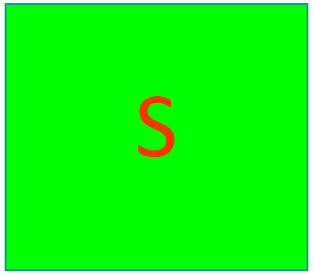
Nonlinear structural analysis



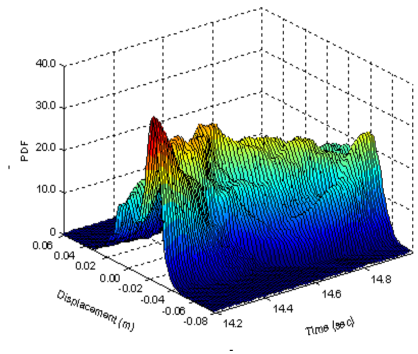
Probability Density Evolution Theory



The initial random source



The physical system

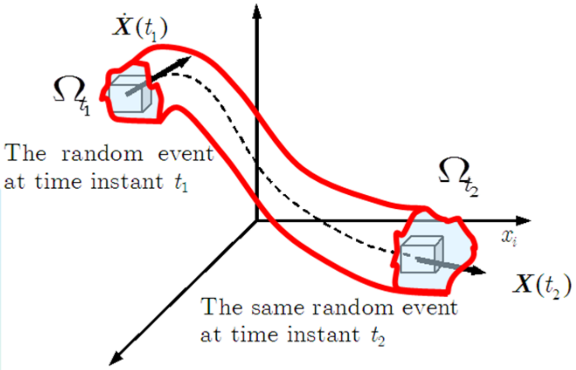


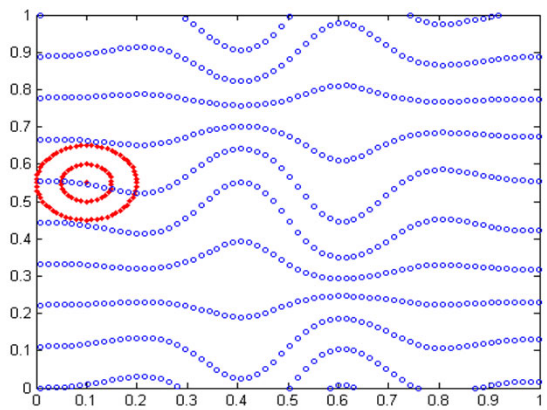
The probability density of the system

Propagation of randomness in physical systems!

Li Jie, 2005. **Some basic viewpoints on research of physical stochastic systems.** ***Scientific Report in Tongji University.***

Principle of preservation of probability





On the random event description, the principle can be described as: ***the probability measure will be preserved in a set of sample trajectories, as long as neither new random factors arise nor existing factors vanish in the physical process.***

$$\Pr\{(\mathbf{X}(t+dt),\boldsymbol{\Theta})\in\Omega_{t+dt}\times\Omega_\theta\}=\Pr\{(\mathbf{X}(t),\boldsymbol{\Theta})\in\Omega_t\times\Omega_\theta\}$$

Jie Li & Jian-Bing Chen, 2008, “The Principle of Preservation of Probability and the Generalized Density Evolution Equation ”, *Structural Safety*, 30(1), 65-77

Jie Li & Jian-Bing Chen, “Stochastic Dynamics of Structures”, John Willey, 2009.

Generalized Probability Density Evolution Equation (PDEE)

Without loss of generality, consider a general stochastic physical system

$$\mathscr{L}(\mathbf{Y}, \partial^{(j)}\mathbf{Y}, \Theta, \tau, x, t) = 0$$

where Θ is a random vector, $\mathscr{L}(\bullet)$ is a general operator.

If we regard τ as an evolution parameter like time, then the joint PDF of is governed by the following probability density evolution equation

$$\frac{\partial p_{Y_l\Theta}(y, \theta, \tau)}{\partial \tau} + \sum_{l=1}^{m} \dot{Y}_l \frac{\partial p_{Y_l\Theta}(y, \theta, \tau)}{\partial y} = 0$$

Jie Li. 2016, Probability density evolution method: Background, significance and recent developments. Probabilistic Engineering Mechanics, 44: 111-117.

Example: Elasto-plastic stochastic dynamical system

Physical Equation of Structures

$$\nabla \bullet \sigma + \mathbf{b} = \rho \ddot{\mathbf{u}} + \eta \dot{\mathbf{u}}$$ Equilibrium equation

$$\varepsilon = \frac{1}{2}\left(\nabla \mathbf{u} + \nabla^T \mathbf{u}\right)$$ Geometric equation

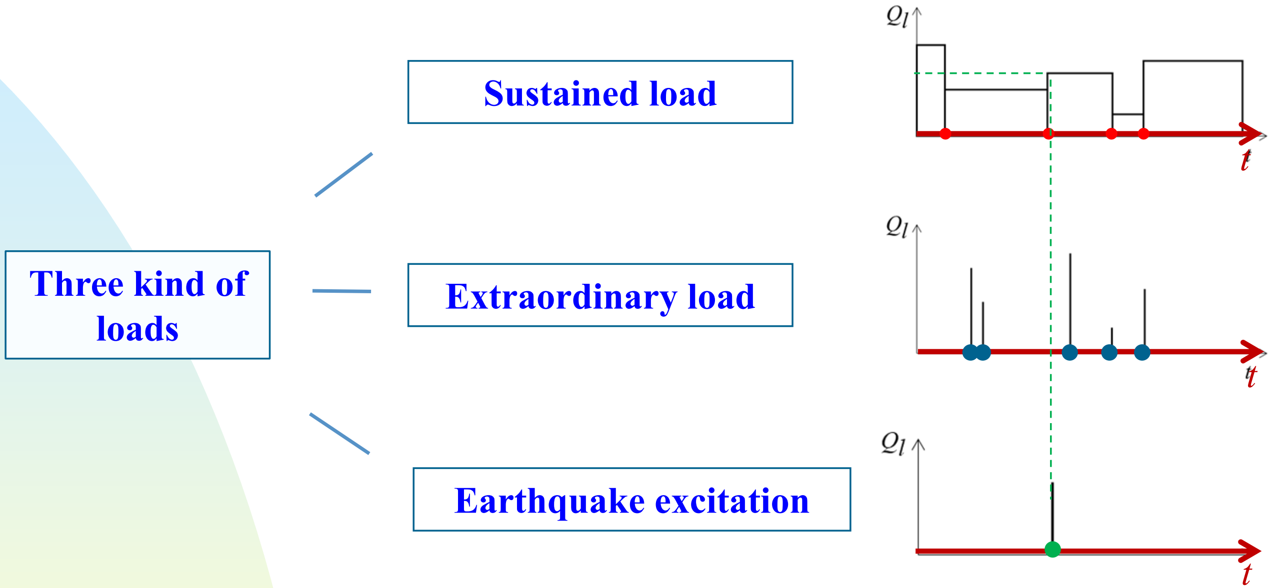
$$\sigma = \left(\mathbf{I} - \mathbf{D}\right) : \mathbf{C}_0 : (\varepsilon - \varepsilon^p)$$ Constitutive equation

Probability Density Evolution Equation

$$\frac{\partial p_{\sigma\Theta}(\sigma,\theta,\tau)}{\partial \tau} + \sum_{i=1}^{m} \dot{\sigma}_i \frac{\partial p_{\sigma\Theta}(\sigma,\theta,\tau)}{\partial \sigma_i} = 0$$

Combining dynamic equilibrium equation, geometric equation, constitutive equation and generalized probability density evolution equation, stochastic nonlinear response analysis of complex structures can be easily implemented.

Load effect combination of nonlinear structure



Generalized expression of nonlinear load effects of structures

$$\boxed{S(t) = f_S\left[Q_0, Y_{Q_1}(\boldsymbol{\Theta}_1, t), Y_{Q_2}(\boldsymbol{\Theta}_2, t), Y_{Q_3}(\boldsymbol{\Theta}_3, t)\right] = S(\boldsymbol{\Theta}, t)}$$

$$\Rightarrow \begin{cases} \nabla \cdot \boldsymbol{\sigma} + \mathbf{b} = \rho \ddot{\mathbf{u}} + \eta \dot{\mathbf{u}} \\ \boldsymbol{\varepsilon} = \frac{1}{2}\left(\nabla \mathbf{u} + \nabla^T \mathbf{u}\right) \\ \boldsymbol{\sigma} = \left(\mathbf{I} - \mathbf{D}\right) : \mathbf{C}_0 : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p) \end{cases}$$

Probability density evolution analysis

Basic Analytical Equations

$$\left.\begin{cases} \nabla \cdot \boldsymbol{\sigma} + \mathbf{b} = \mathbf{0} \\ \boldsymbol{\varepsilon} = \dfrac{1}{2}\left(\nabla \mathbf{u} + \nabla^{T} \mathbf{u}\right) \\ \boldsymbol{\sigma} = (\mathbf{I} - \mathbf{D}) : \mathbf{C}_{0} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{p}) \end{cases}\right\} S(\boldsymbol{\Theta}, t)$$

$$\frac{\partial p_{S\Theta}(s,\boldsymbol{\theta},t)}{\partial t} + \dot{S}(\boldsymbol{\theta},t)\frac{\partial p_{S\Theta}(s,\boldsymbol{\theta},t)}{\partial s} = 0$$

$$p_{S\Theta}(s,\boldsymbol{\theta},t)\Big|_{t=t_0} = \delta(s - s_0)\, p_{\Theta}(\boldsymbol{\theta})$$

Probability density function of combined load effects:

$$p_S(s,t) = \int_{\Omega_\Theta} p_{S\Theta}(s,\boldsymbol{\theta},t)\mathrm{d}\boldsymbol{\theta}$$

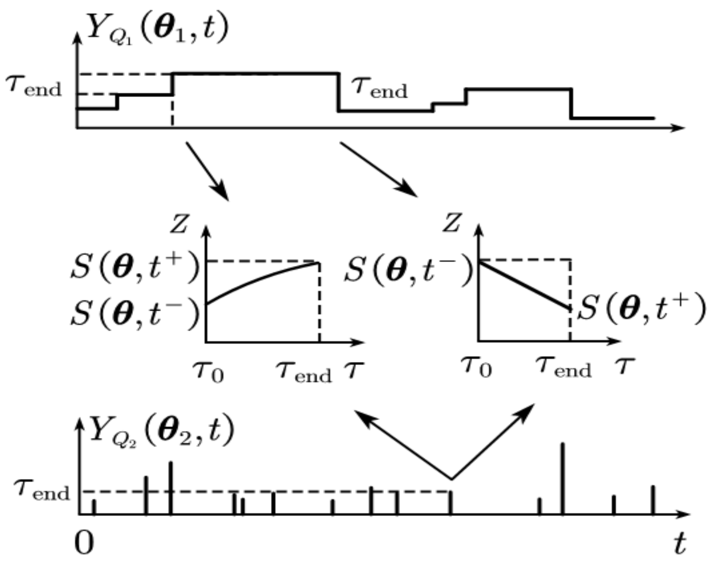
Probability distribution of extreme value for combined load effects:

$$F_{S_{\max}}(a,T) = P\{S(t) \le a, \forall t \in [0,T]\}$$

Probability density evolution analysis

Being a Discrete Events Dynamic System, it requires to solve physical equations and probability density evolution equation in segments based on the change of system states. Herein, the introduction of multiscale time variables have to be introduced.

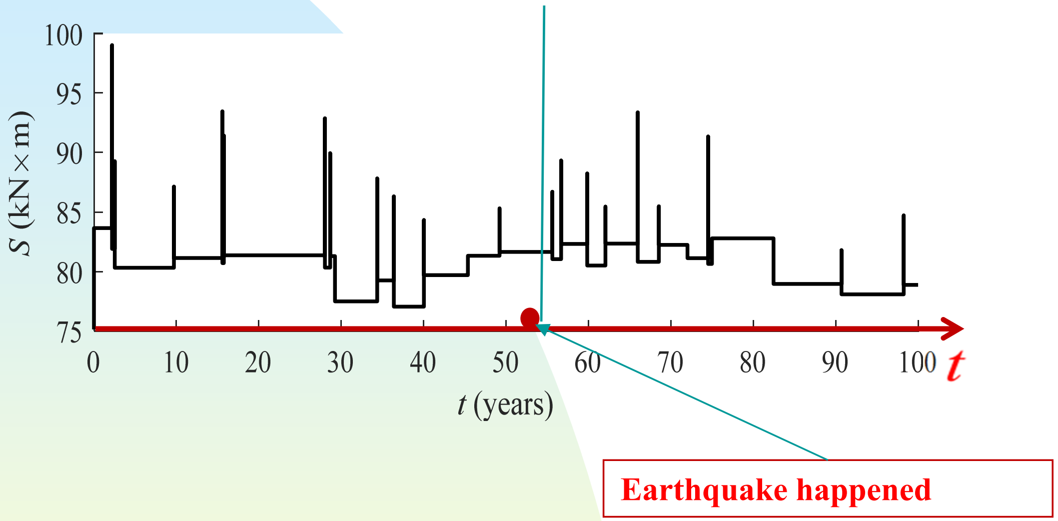
1. Gravity load time history



At time point *t*, the change of combined load effect is induced for the moment of loading and unloading.

$$
\begin{cases}
S(\Theta, t, \tau) = f_S(\tau - \tau_0) \\
\dfrac{\partial p_{S\Theta}(s, \boldsymbol{\theta}, t, \tau)}{\partial \tau} + \dot{S}(\boldsymbol{\theta}, t, \tau)\dfrac{\partial p_{S\Theta}(s, \boldsymbol{\theta}, t, \tau)}{\partial s} = 0 \\
\tau \in [\tau_0, \tau_{\text{end}}], \tau_0 = Y_{Q_l}(\Theta_l, t^-), \tau_{\text{end}} = Y_{Q_l}(\Theta_l, t^+) \\
p_{S\Theta}(s, \boldsymbol{\theta}, t, \tau_0) = p_{S\Theta}(s, \boldsymbol{\theta}, t^-) \\
p_{S\Theta}(s, \boldsymbol{\theta}, t) = p_{S\Theta}(s, \boldsymbol{\theta}, t^+) = p_{S\Theta}(s, \boldsymbol{\theta}, t, \tau_{\text{end}}) \\
S(\Theta, t, \tau_0) = S(\Theta, t^-),\ S(\Theta, t, \tau_{\text{end}}) = S(\Theta, t^+)
\end{cases}
$$

2. Earthquake ground motion



$$
\begin{cases}
S(\Theta,t,\tau_{\mathrm{e}}) = f_S(\tau_{\mathrm{e}}-\tau_0);\ \tau_{\mathrm{e}}\in[\tau_0,\tau_{\mathrm{end}}] \\
\dfrac{\partial p_{S\Theta}(s,\boldsymbol{\theta},t,\tau_{\mathrm{e}})}{\partial \tau_{\mathrm{e}}} + \dot{S}(\boldsymbol{\theta},t,\tau_{\mathrm{e}})\dfrac{\partial p_{S\Theta}(s,\boldsymbol{\theta},t,\tau_{\mathrm{e}})}{\partial s} = 0 \\
p_{S\Theta}(s,\boldsymbol{\theta},t,\tau_0) = p_{S\Theta}(s,\boldsymbol{\theta},t^-) \\
p_{S\Theta}(s,\boldsymbol{\theta},t) = p_{S\Theta}(s,\boldsymbol{\theta},t^+) = p_{S\Theta}(s,\boldsymbol{\theta},t,\tau_{\mathrm{end}}) \\
S(\Theta,t,\tau_0) = S(\Theta,t^-),\ S(\Theta,t,\tau_{\mathrm{end}}) = S(\Theta,t^+)
\end{cases}
$$

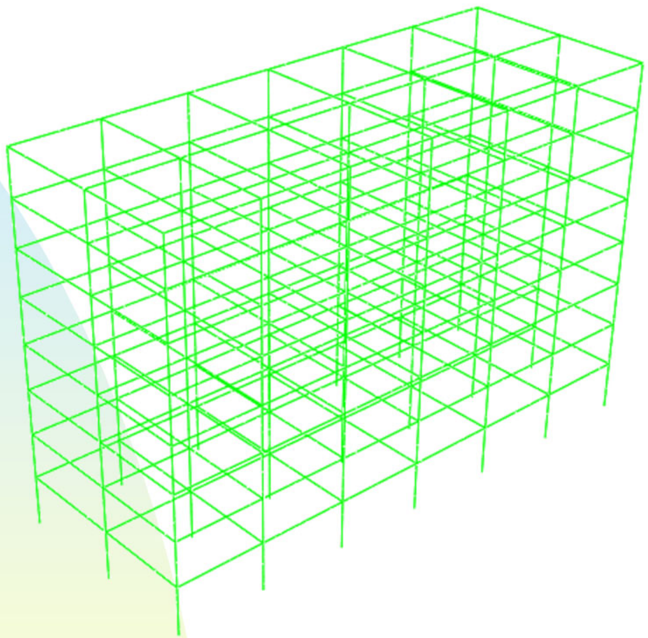
τ_0 **Starting time of seismic ground motion**

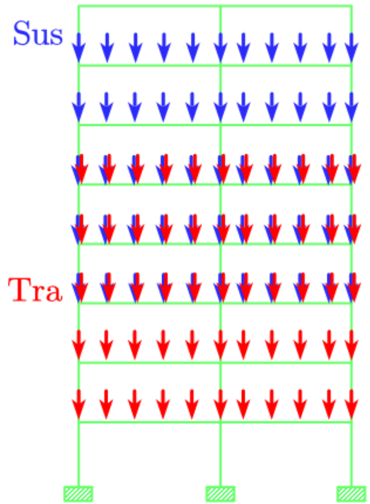
τ_{end} **Ending time of seismic ground motion**

Probability density function of combined load effects

$$p_S(s,t) = \int_{\Omega_\Theta} p_{S\Theta}(s,\boldsymbol{\theta},t)\,\mathrm{d}\boldsymbol{\theta}$$

Examples: Combined load effect analysis of structures

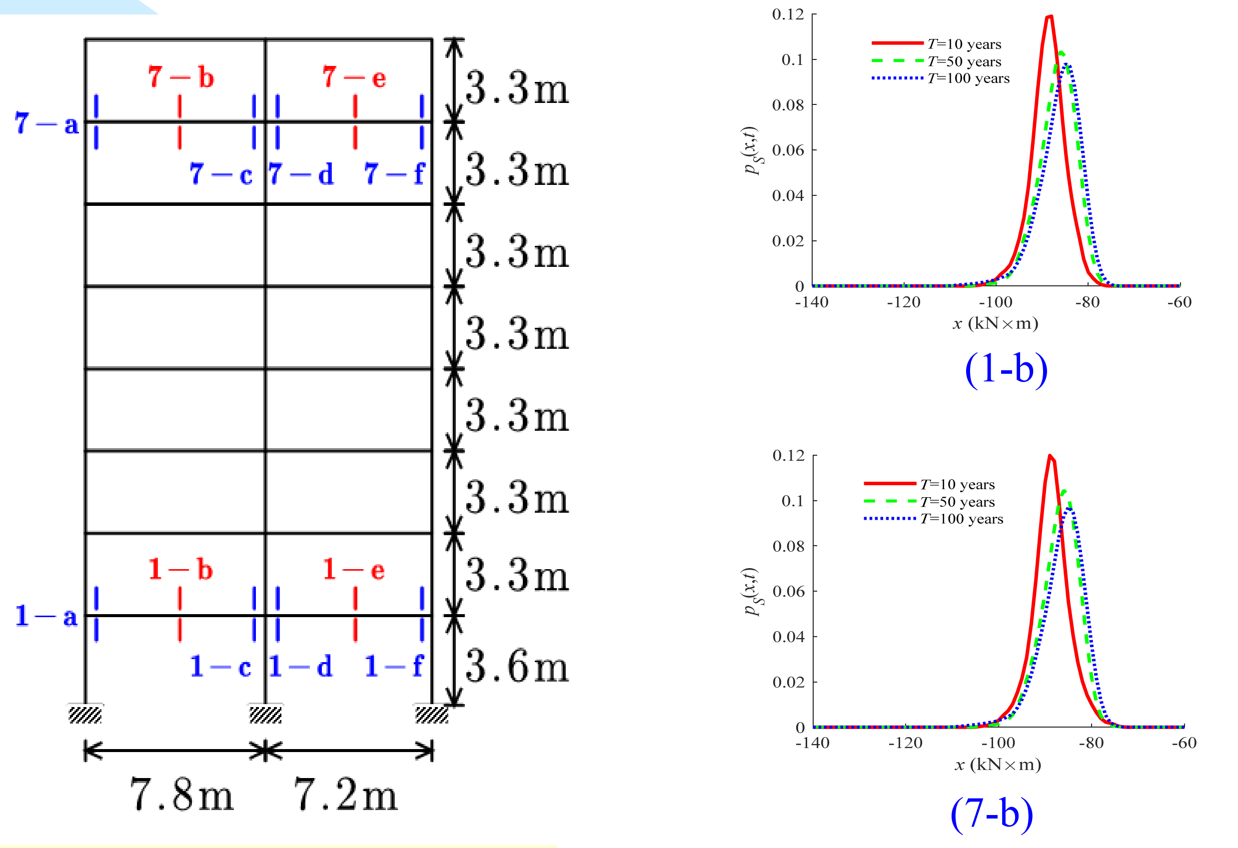




$\lambda_{\text{sus}} = 0.1\ \text{year}^{-1}$, $\lambda_{\text{tra}} = 0.2\ \text{year}^{-1}$

8-story reinforced concrete frame structure, with a total height of 26.7 m and a plan size of 39.6 m × 15 m. Describe the floor sustained and extraordinary load using the Poisson square wave process and Poisson point process, respectively, and calculate the probability distribution of load combination effects.

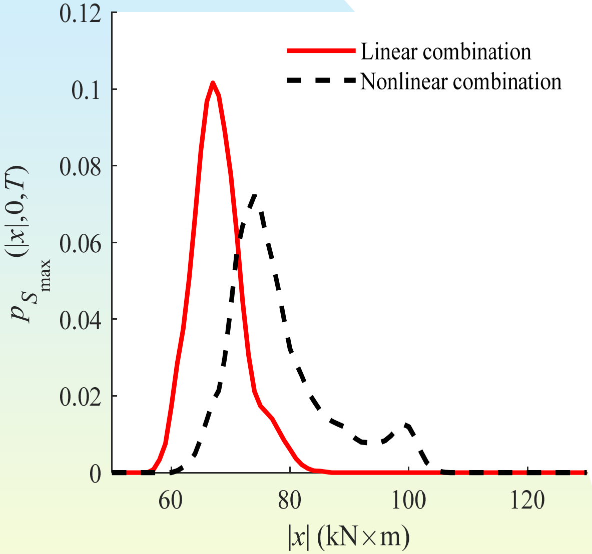
Examples: Instantaneous probability distribution of bending moment at critical sections under the combined action of gravity load



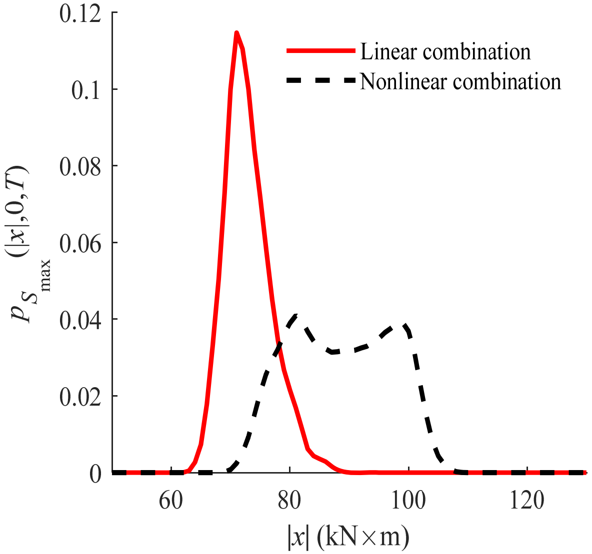
The probability density function of sectional bending moment changes with service life!

Comparison of linear and nonlinear load combination effects

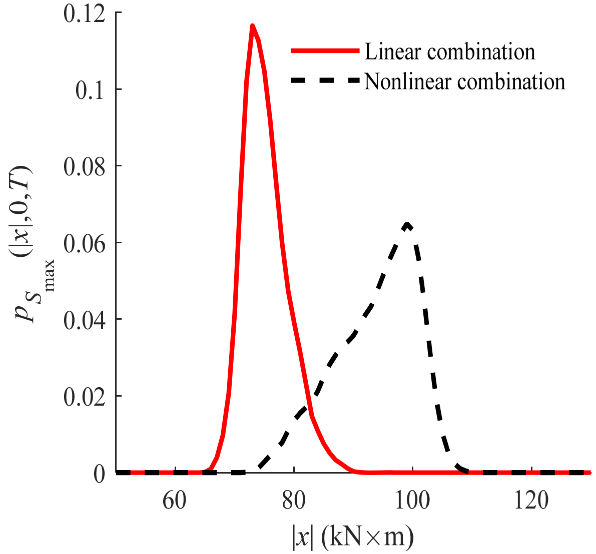
Probability density function of the maximum bending moment at a typical section



Service period at 10 years



Service period at 50 years



Service period at 100 years

The probability distribution of the maximum sectional bending moment has significant difference for the linear and nonlinear load combinations.

4. Global Reliability Analysis of Structures under Multi-loads and Disastrous Actions

Structural Global Reliability Analysis: Physical synthesis method

$$\begin{cases} \nabla \cdot \boldsymbol{\sigma} + \mathbf{b} = \rho \ddot{\mathbf{u}} + \eta \dot{\mathbf{u}} \quad \text{with } \boldsymbol{\sigma} \cdot \mathbf{n} = \mathbf{p} \\ \boldsymbol{\varepsilon} = \frac{1}{2}\left(\nabla \mathbf{u} + \nabla^T \mathbf{u}\right) \text{ with } \mathbf{u} = \bar{\mathbf{u}} \\ \boldsymbol{\sigma} = (\mathbf{I} - \mathbf{D}) : \mathbf{C}_0 : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p) \\ \frac{\partial p_{U_p\Theta}(u_p, \boldsymbol{\theta}, t)}{\partial t} + \dot{U}_p(\boldsymbol{\theta}, t) \frac{\partial p_{U_p\Theta}(u_p, \boldsymbol{\theta}, t)}{\partial u_p} = -\mathscr{H}\left[f(\mathbf{u}(\boldsymbol{\theta}, t)) \right] p_{U_p\Theta}(u_p, \boldsymbol{\theta}, t) \\ p_{U_p\Theta}(u_p, \boldsymbol{\theta}, t)\Big|_{t=t_0} = \delta(u_p - u_{p0})\, p_\Theta(\boldsymbol{\theta}) \end{cases}$$

Physical Equation

Propagation Equation

Physical failure criteria

Screening operator

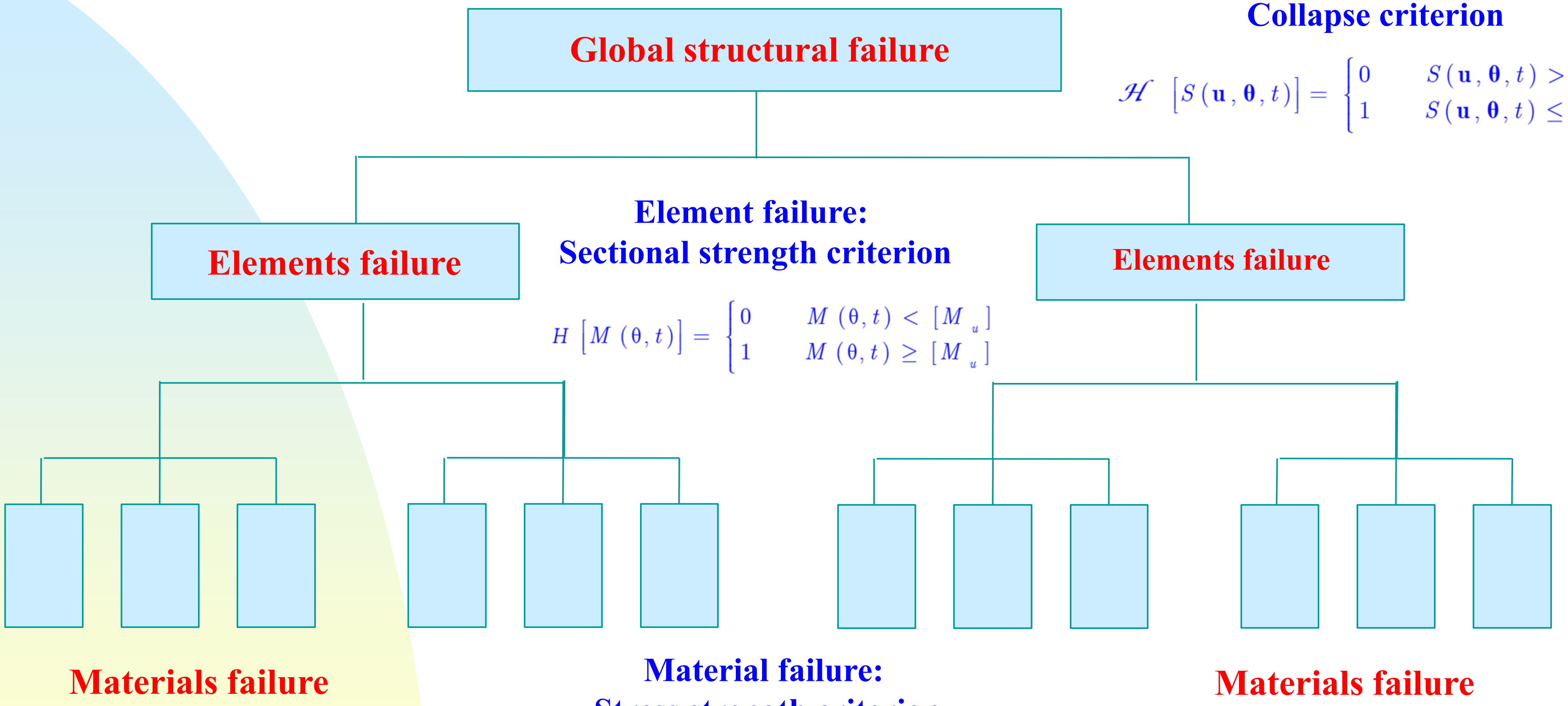
Global Reliability of Structures

$$R(\tau) = \int_{-\infty}^{+\infty} p_{U_p}(u_p, \tau)\, du_p$$

$$p_{U_p}(u_p, \tau) = \int_{\Omega_\theta} p_{U_p\Theta}(u_p, \theta, \tau)\, d\boldsymbol{\theta}$$

Li J. Advances in global reliability analysis of engineering structures. *China Civil Engineering Journal, 51(8): 1-10*. 2018

Structural global reliability analysis: Physical synthesis method



Structural global failure:
Collapse criterion

$$\mathscr{H}\left[S(\mathbf{u},\boldsymbol{\theta},t)\right] = \begin{cases} 0 & S(\mathbf{u},\boldsymbol{\theta},t) > 0 \\ 1 & S(\mathbf{u},\boldsymbol{\theta},t) \le 0 \end{cases}$$

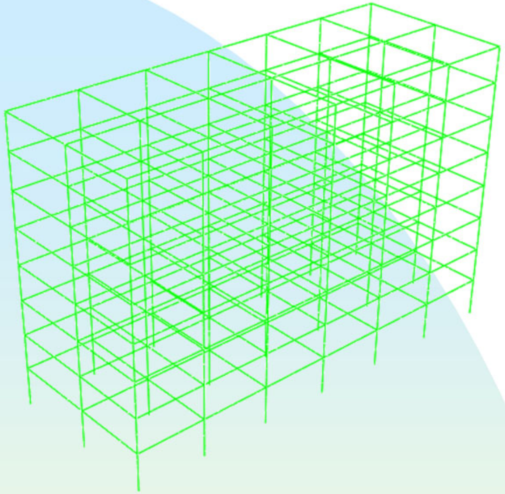
Element failure:
Sectional strength criterion

$$H\left[M(\theta,t)\right] = \begin{cases} 0 & M(\theta,t) < [M_u] \\ 1 & M(\theta,t) \ge [M_u] \end{cases}$$

Material failure:
Stress strength criterion

$$\mathscr{H}\left[\sigma(\boldsymbol{\theta},t)\right] = \begin{cases} 0 & \sigma(\boldsymbol{\theta},t) < [\sigma] \\ 1 & \sigma(\boldsymbol{\theta},t) \ge [\sigma] \end{cases}$$

Example: Sample analysis (Multiple live loads + Earthquake)



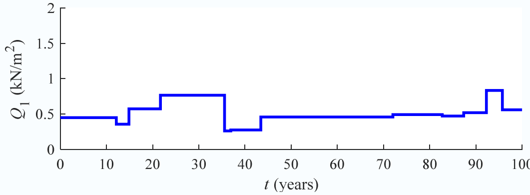
$$\lambda_{\text{sus}} = 0.1\ \text{year}^{-1},\ \lambda_{\text{tra}} = 0.2\ \text{year}^{-1},\ \ \lambda_{\text{E}} = 0.002\ \text{year}^{-1}$$

$$\mu_{\text{sus}} = 0.504\ \text{kN/m}^2,\ \sigma_{\text{sus}} = 0.162\ \text{kN/m}^2$$

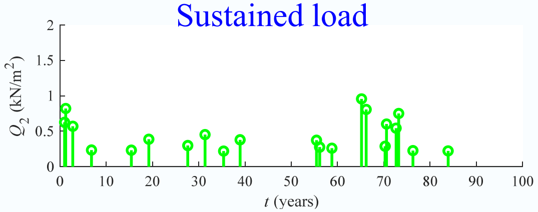
$$\mu_{\text{tra,1}} = 0.468\ \text{kN/m}^2,\ \sigma_{\text{tra}} = 0.253\ \text{kN/m}^2$$

$$\mu_{\text{E}} = 0.433\ \text{m}/s^2,\ \sigma_{\text{E}} = 0.598\ \text{m}/s^2$$

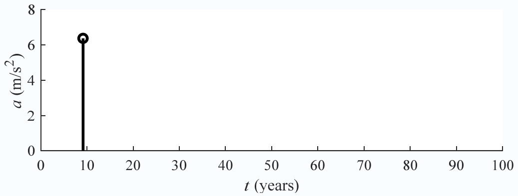
- Typical load sample q=133



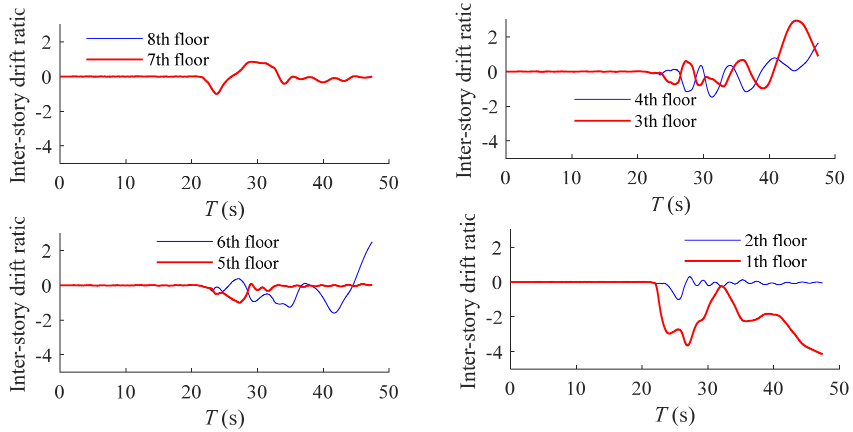
Sustained load



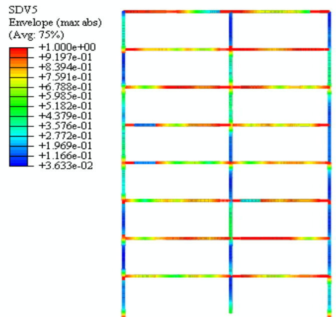
Extraordinary load



Earthquake



Structural response

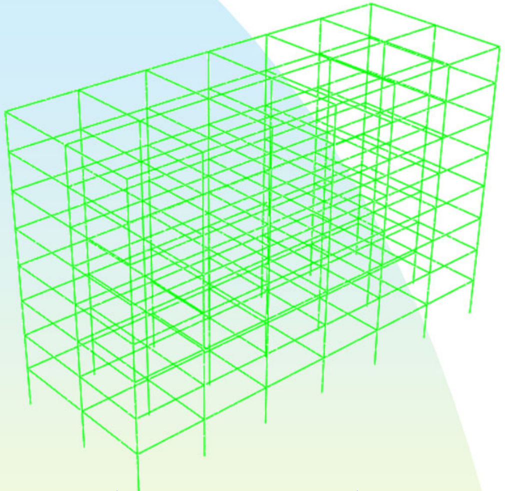


Structural damage distribution

The simulation of life-cycle performance of the structure!

Example: Structural Reliability (Element failure criterion)

The repairable probability for structures for different service years under combinations of live loads and earthquake ground motions.

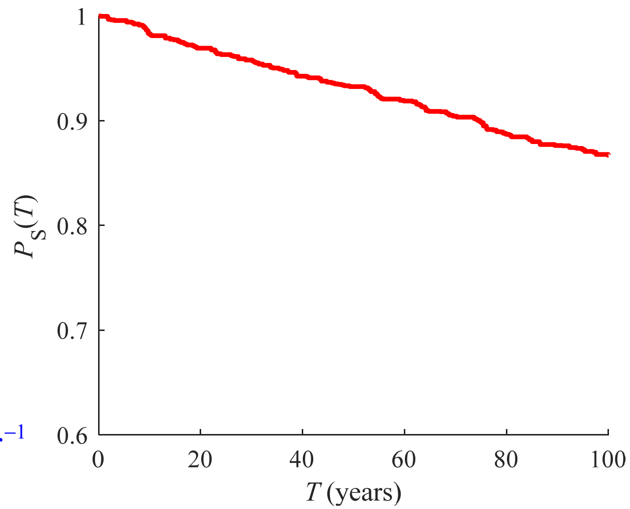


$$\lambda_{\mathrm{sus}} = 0.1\ \mathrm{year}^{-1},\ \lambda_{\mathrm{tra}} = 0.2\ \mathrm{year}^{-1},\quad \lambda_{\mathrm{E}} = 0.002\ \mathrm{year}^{-1}$$

$$\mu_{\mathrm{sus}} = 0.504\ \mathrm{kN/m^2},\ \sigma_{\mathrm{sus}} = 0.162\ \mathrm{kN/m^2}$$

$$\mu_{\mathrm{tra,l}} = 0.468\ \mathrm{kN/m^2},\ \sigma_{\mathrm{tra}} = 0.253\ \mathrm{kN/m^2}$$

$$\mu_{\mathrm{E}} = 0.433\ \mathrm{m}/s^2,\ \sigma_{\mathrm{E}} = 0.598\ \mathrm{m}/s^2$$



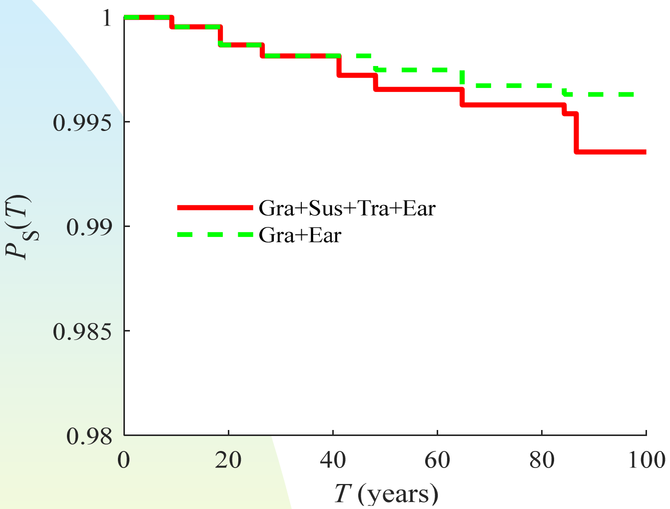
Change of structural repairable probability with the service year

Service years	**Repairable** probability
10	0.9831
20	0.9691
30	0.9578
40	0.9424
50	0.9324
60	0.9186
70	0.9042
80	0.8869
90	0.8769
100	0.8667

As the structural service year increases, the probability of structures encountering earthquakes increases, and the repairable probability decreases.

Example: Structural Global Reliability （Global failure criterion）

The non-collapse probability for structures at different service years under combinations of gravity loads and earthquake ground motions.

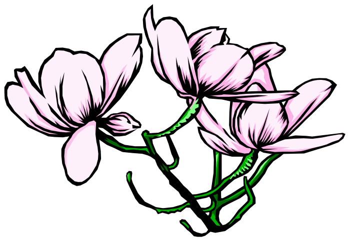


Structural non-collapse probability
(Criterion: The inter-storey drift ratio at any floor reaches at 0.05)

Service years	Non-collapse probability	
	Threshold value 0.02	Threshold value 0.05
10	0.9995	0.9995
20	0.9987	0.9987
30	0.9982	0.9982
40	0.9982	0.9982
50	0.9965	0.9965
60	0.9956	0.9965
70	0.9949	0.9958
80	0.9949	0.9958
90	0.9926	0.9935
100	0.9926	0.9935

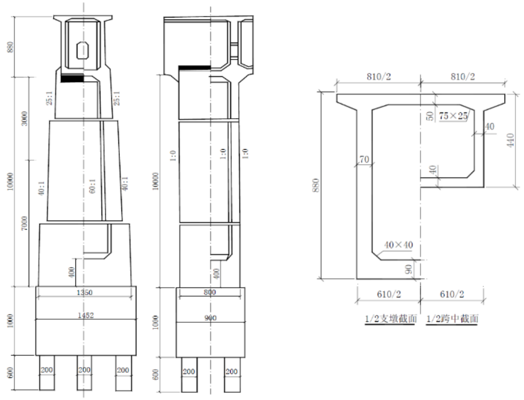
- **Structural collapse probability increases with the extension of service years**
- **As the structural service year extends, the impact of live loads on the structural seismic reliability becomes increasingly apparent.**

5. Typical Engineering Applications



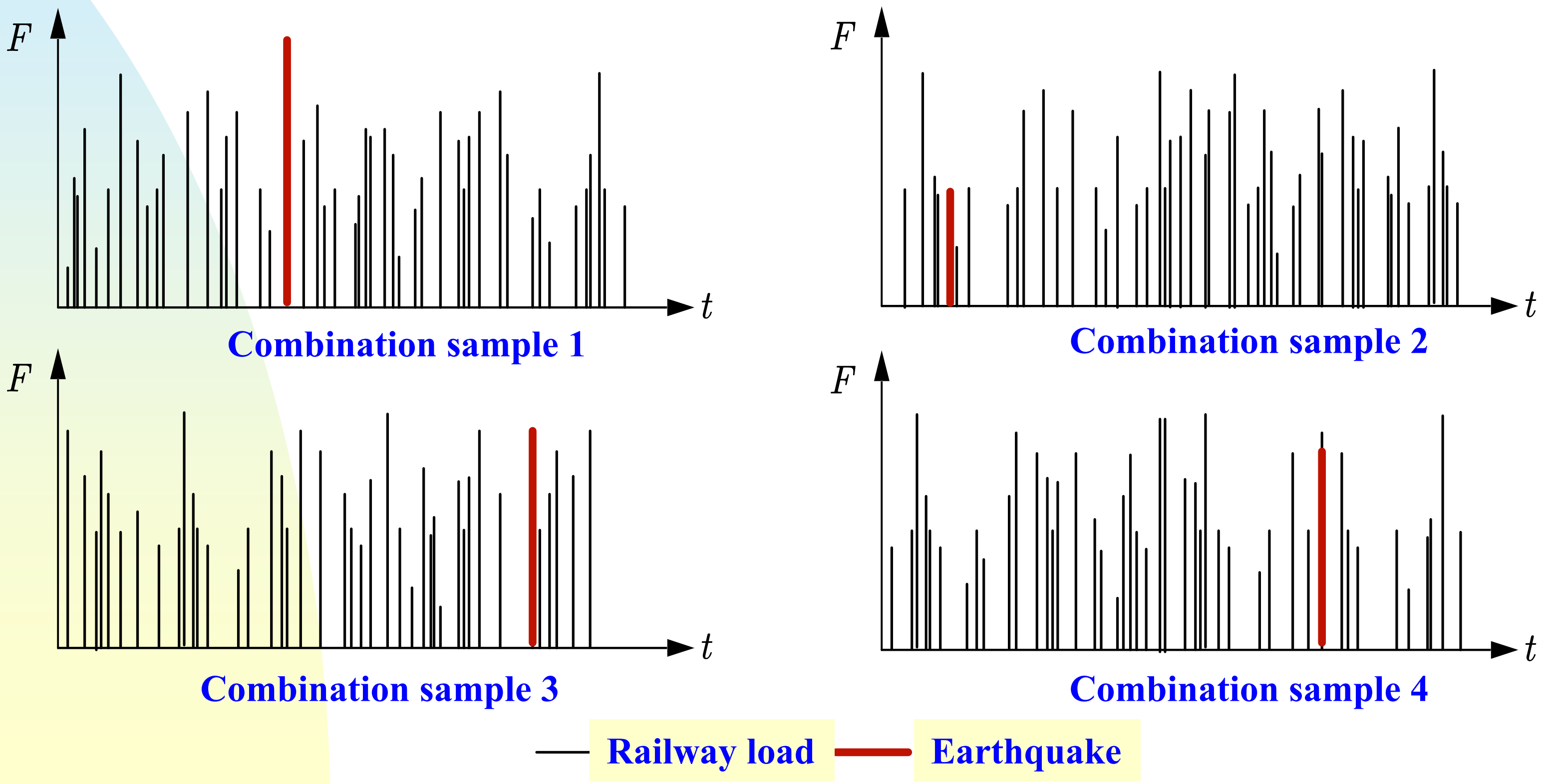
1. Railway bridge (railway loads and earthquakes)

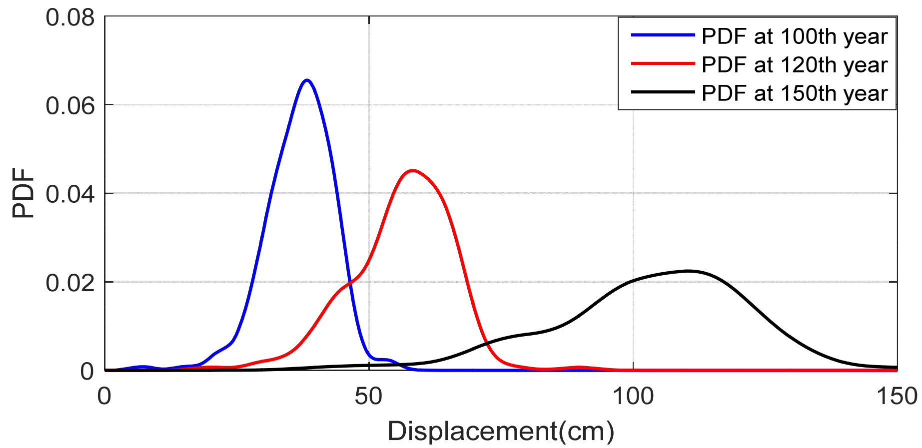
Qingshuihe bridge on the Nan-kun Railway Line of China. A heavy load railway concrete bridge. The total length of the bridge is 360.5 m, with a height of 183 m from the riverbed to the bridge deck. The main span is 272 meters (72+128+72), and the main pier has heights of 86 meters and 100 meters, respectively. The concrete grade is C50, and the design life is 100 years.



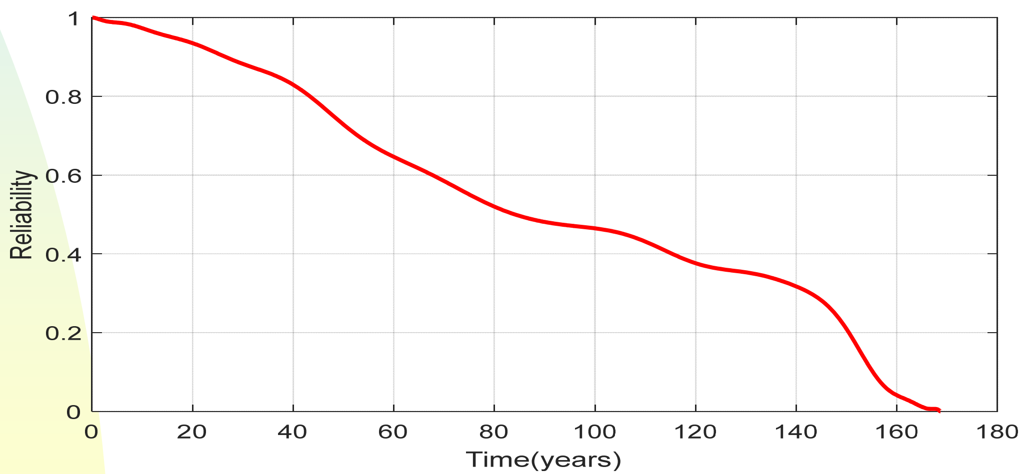
Combination of railway loads and earthquakes

The compound Poisson process is adoped to simulate railway loads and earthquakes, and the probability partition method is employed to determine the load coincidence sample.



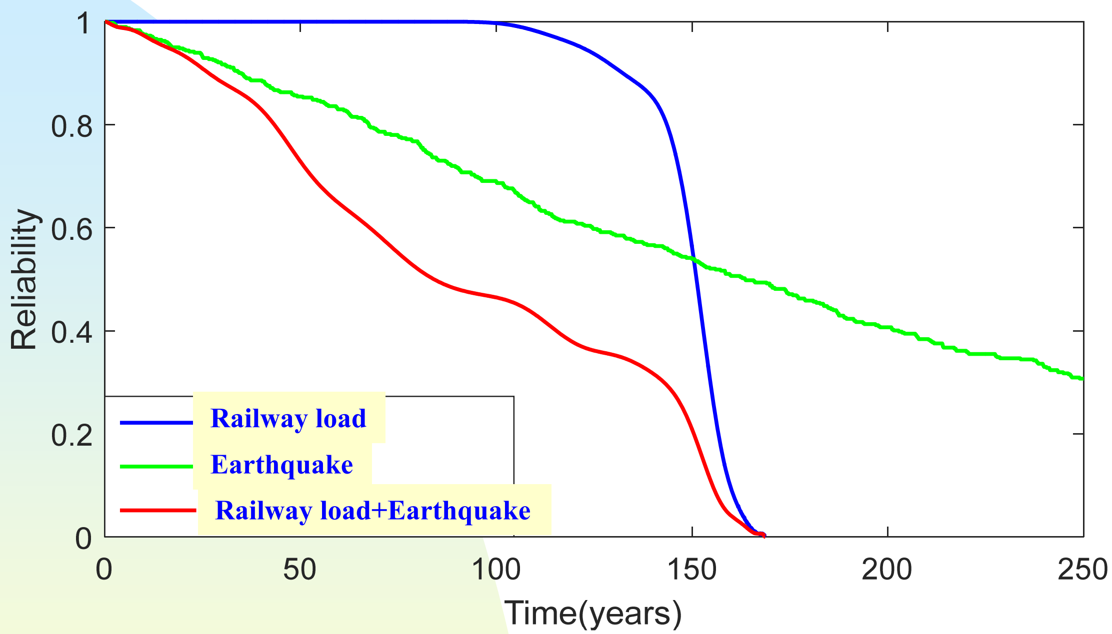


Probability density evolution of displacement at the middle span



Time-varying reliability under combination of railway loads and earthquakes

Structural global reliability under different load combinations



服役期（年）	Railway loading	Railway loading + Earthquake
20	1.0000	0.9351
40	1.0000	0.8295
60	1.0000	0.6464
80	1.0000	0.5198
100	0.9971	0.4646
120	0.9561	0.3761
140	0.8522	0.3172
160	0.0929	0.0409

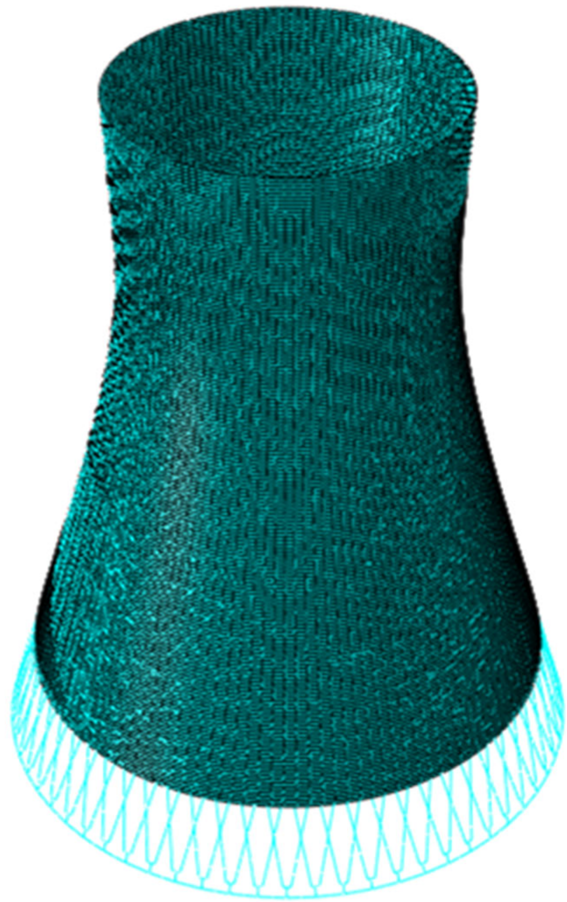
If considering railway loading only, the structural reliability begins to decline when the service life approaches 100 years. However, if considering the combined railway loads and earthquakes simultaneously, the structural reliability will decrease at the beginning of its service life. The structural failure probability is as high as 55% when the service life reaches 100 years!

2. Cooling Tower of Power plant **(Typhoon and earthquakes)**

The cooling tower has **a total height of 249 meters**, with the shell portion being 220 meters high and the bottom inlet height being 29 meters. The shell has a top diameter of 118 meters, a bottom diameter of 186 meters, and a throat diameter of 113 meters. The thickness of the shell gradually decreases from bottom to top, with the thickest part being 1.8 meters and the thinnest part being 0.42 meters. The bottom cross-braces are 30.8 meters long, with a rectangular cross-section measuring 1.0m x 1.7m, totaling 132 braces.

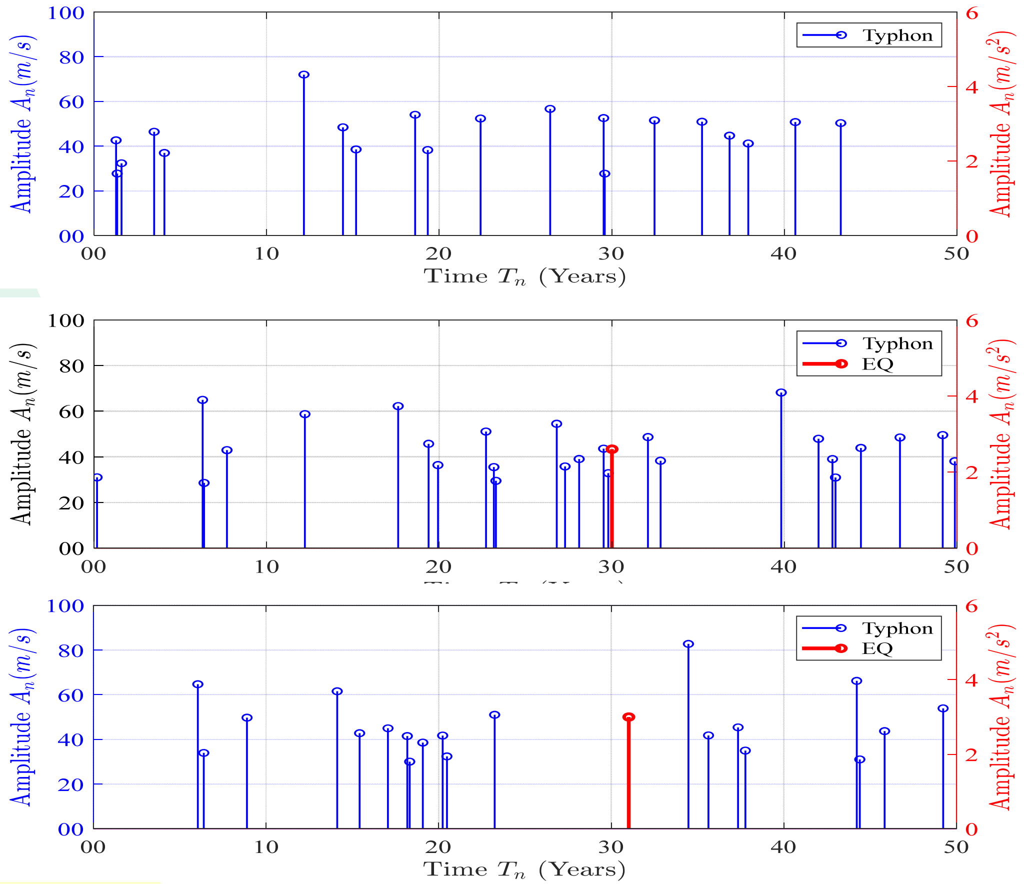
In the finite element model (as shown in the diagram), the cross-braces are modeled using fiber beam elements, and the shell is modeled using layered shell elements. The entire structure is divided into **23,528 elements**.

Nonlinear wind-induced vibration response analysis is conducted using the **concrete elastoplastic stochastic damage model** established by our team. The analysis of stochastic wind-induced vibration response adopts the **ensemble evolutionary algorithm**.

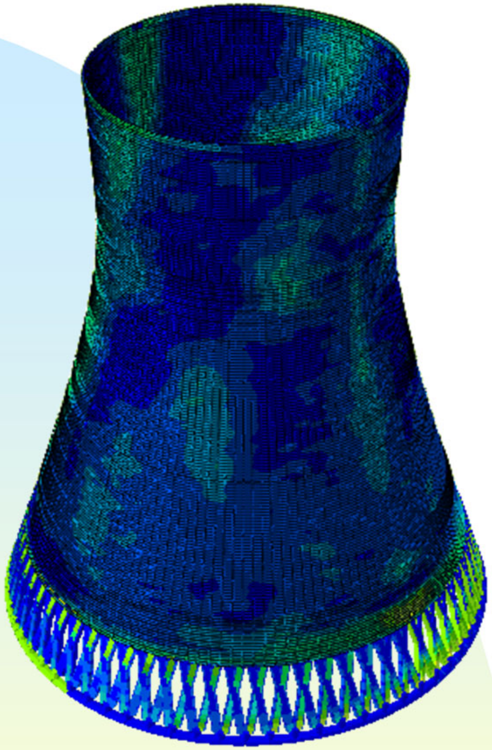


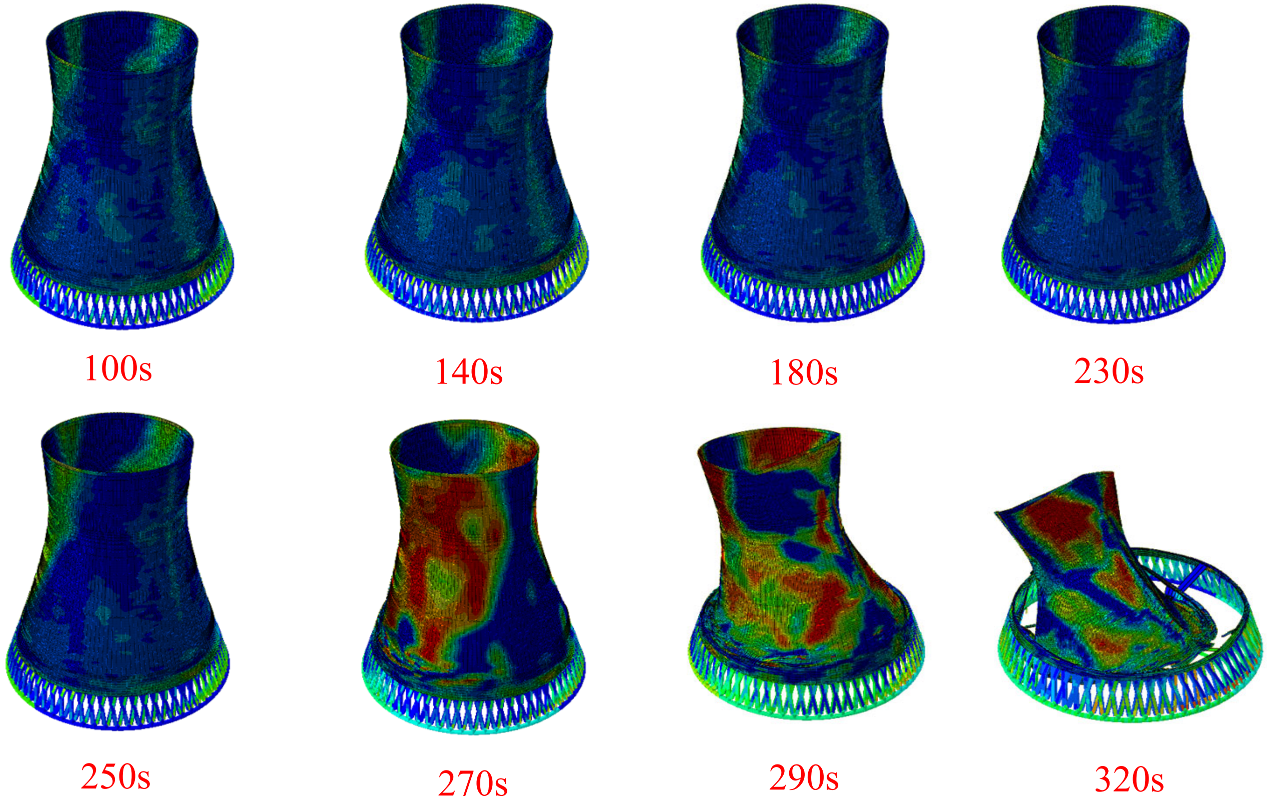
Simulation of typhoon and earthquakes sequence

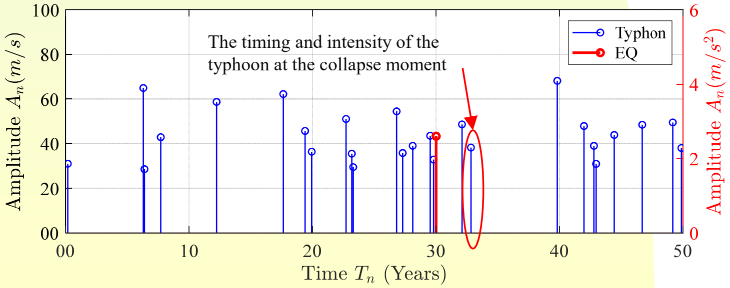
The compound Poisson process is employed to simulate typhoon and earthquakes, and the probability partition method is adopted to determine the load coincidence sample.



Structural Collapse under the Sequence Action of Typhoon and Earthquakes

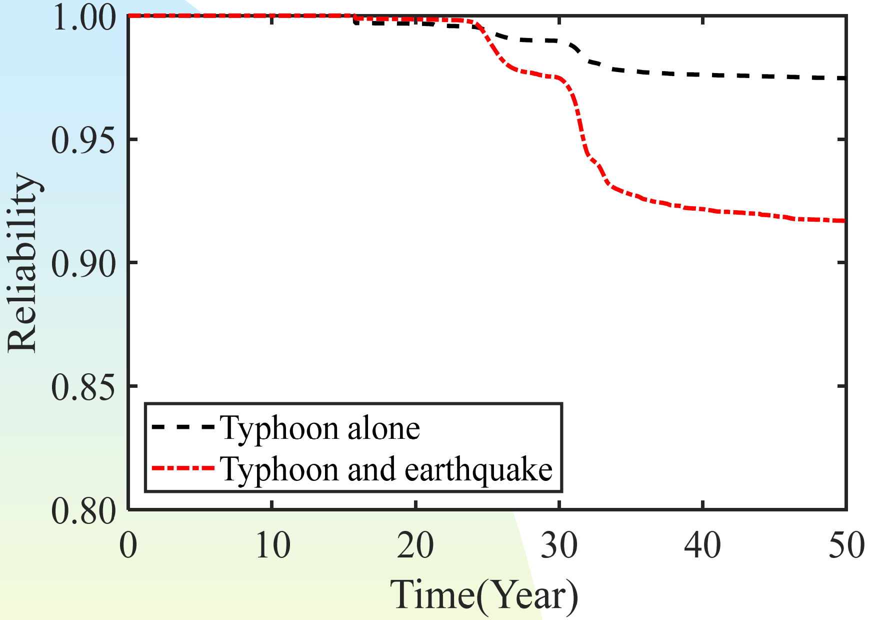






After experienced 16 typhoon and 1 earthquake (the 30th year, with a peak ground acceleration of 2.6 m/s^2), the structure still stand up to two more consecutive typhoon events with peak wind speeds of 48.64 m/s and 38.26 m/s, respectively, finally resulting in structural collapses. (The structural collapse occurred in the 32nd year).

Strctural reliability for different service life

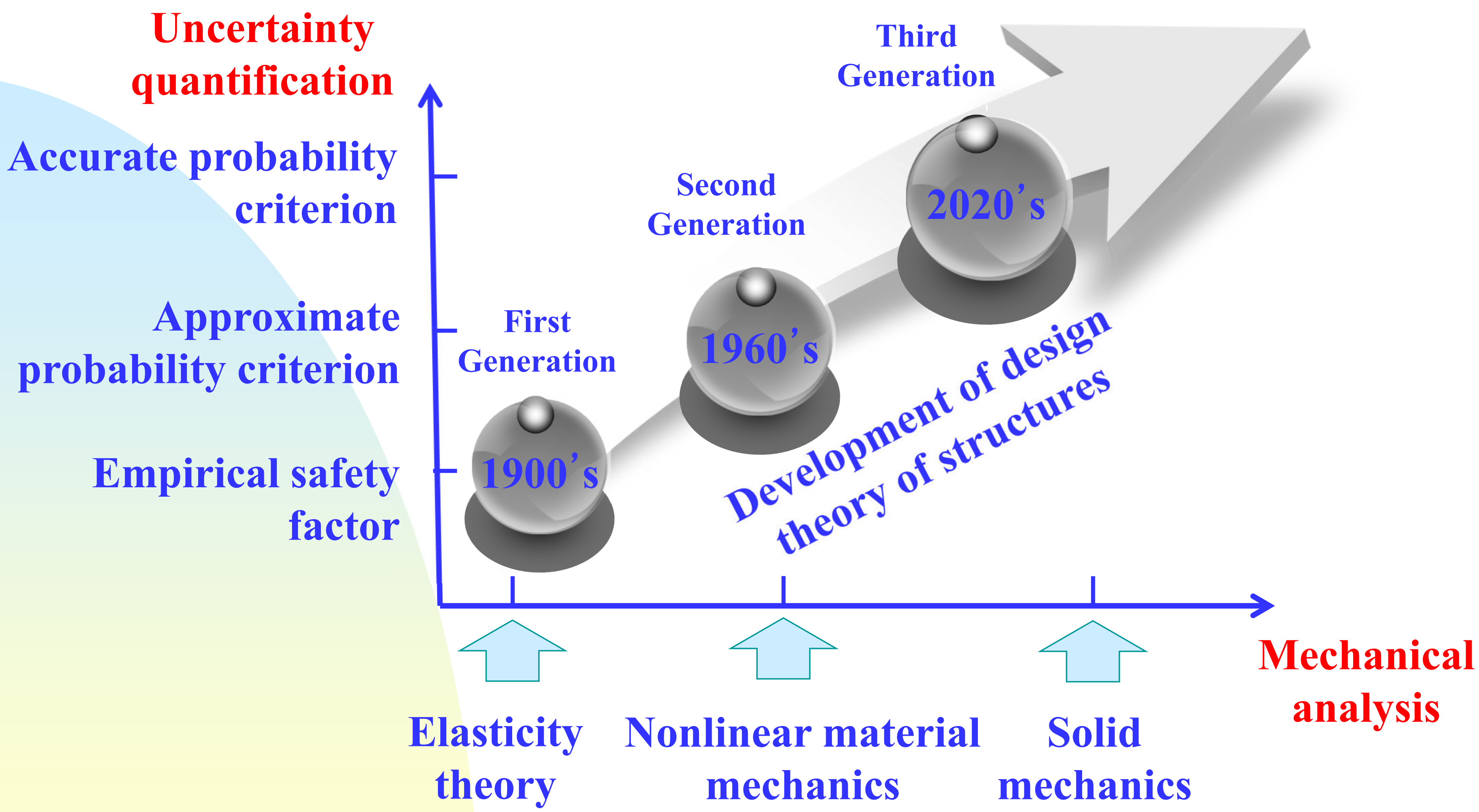


Service life \ Action	Typhoon acting alone	Sequential action of typhoon and earthquake
0 Year	1.0000	1.0000
5 Year	1.0000	1.0000
10 Year	1.0000	1.0000
15 Year	1.0000	1.0000
20 Year	0.9961	0.9963
25 Year	0.9920	0.9816
30 Year	0.9879	0.9661
35 Year	0.9774	0.9237
40 Year	0.9760	0.9183
45 Year	0.9751	0.9170
50 Year	0.9741	0.9166

For 50 years design service life, if considering typhoon only, the structural reliability is about 97%. However, if considering the sequence action of typhoon and earthquakes, the structural reliability will decrease to 92% !

Conclusions

- The simulation of lifecycle performance of structures is a basic foundation to analysis lifecycle reliability of engineering structures;
- The principle of structural load coincidence has laid a scientific foundation to solve the structural load effect combination problem;
- There is a significant difference between the combination of linear and nonlinear load effects, which should be given a special attention;
- Probability density evolution theory could reveal the uncertainty propagation law of engineering systems. It established a scientific fundation for the study of the combined effects of multiple loads and disaster dynamic effects, as well as the reliability design of structures.



Jie Li, 2017, "On the Third Generation of Structural Design Theory", Journal of Tongji University (Natural Science), 45(5), pp.617-624,632

Thanks for your attention!