Non-Stationary Structural Reliability Analysis in the Changing Climate

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Structural Safety in the Changing Climate

- A rapid pace of climate change is evident by increasing frequency and intensity of weather extremes
  - Record breaking heat waves, wildfires, rain/snow storms, and hurricanes in many parts of the world.
- Increasing severity of weather extremes is threatening the safety and functionality of existing infrastructure systems
  - Transportation systems, electrical networks, and water infrastructure suffer damaged and disrupted
- Design codes and standards must adapt to changing climate to maintain a high level of safety

Design in the Changing Climate

New models and methods are needed to account for climate change effects in the design of infrastructure systems

Design and Time-Variant Reliability Analysis

General Idea

- A structure is expected to face a sequence of loads of uncertain magnitude, arriving randomly over the service life, $(0, t)$
- The structure must withstand all such loads while ensuring the safety of occupants (many limit states of performance)
- A sequence of uncertain load events can be naturally modelled as a stochastic Point Process

A Practical Approach

- The time-variant stochastic analysis can be replaced by a time-invariant analysis
- The concept of ”extreme value distribution” plays a key role
- A structure is safe over its lifetime, if it can withstand the maximum of all load events that could occur over its lifetime

Reliability Analysis: Definition

Key Elements

A structure of uncertain strength, R, is exposed to a stochastic load process over a time interval, $(s, t]$,

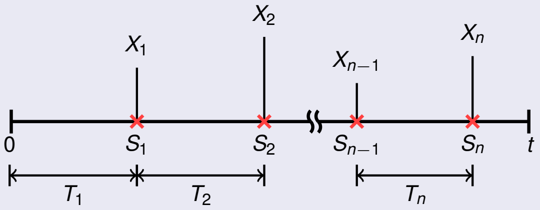
- The distribution of maximum load, $X_{max}(s, t)$, generated by the process over the interval , $(s, t]$
- Computation of the probability of failure, $P_f(s, t)$

$$P_f(s, t) = \mathbb{P}\left[R - X_{max}(s, t) \leq 0\right] \tag{1}$$

- How to derive the distribution of maximum load under non-stationary conditions?
- Other items of interest
  - Calibration of load and resistance factors
  - Target reliability considerations
  - Degradation of structural strength

A Stochastic Load Model

Shock Process (Marked Point Process)

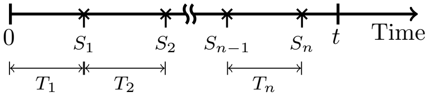


- Random components of the model: (T, X) vectors of RV
  - Inter-arrival times, $T_1, T_2, \ldots$
  - Load magnitudes, $X_1, X_2, \ldots$
  - Time-dependent frequency (rate)
  - Time-dependent loads

Return Period: Definitions

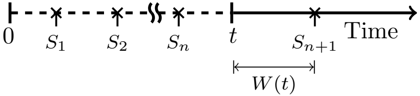
- ”Return period” is a widely used term in design codes

(1) Inter-Arrival Time



- Average time between two consecutive events
- There are **multiple return periods**: $\mathbb{E}[T_1], \mathbb{E}[T_2], \ldots, \mathbb{E}[T_n]$

(2) Waiting Time to Next Event



- Mean waiting time to next event at time $t = \mathbb{E}[W(t)]$

Stationary Climate: Basis of Current Design

- Current design codes are based on the stationary climate conditions
- **Homogeneous Poisson Process (HPP)**
  - A common model of climate loads in structural reliability
  - The load occurrence rate, a constant (λ)
  - Inter-arrival times, $T_1, T_2, \ldots$, *IID* exponential RVs
  - Load magnitudes, $X_1, X_2, \ldots$, IID with a common DF
- **Return Period in Stationary Climate**
  - Average inter-arrival times are all EQUAL, $\mathbb{E}\left[T_n\right] = \frac{1}{\lambda}, \forall\, n > 0$
  - There is a SINGLE return period $\rightarrow$ convenient in design
- **Extreme Load Distribution**

$$\mathbb{P}\left[X_{max}(t) \leq x\right] = e^{-\lambda t(1-F_X(x))}$$

- It depends on the length of the interval only (owing to stationary process)

Stationary Reliability Analysis

- Annual extreme load distribution from HPP model

$$\mathbb{P}\left[X_{Amax} \leq x\right] = e^{-\lambda(1-F_X(x))}$$

  - Asymptotic extreme value distributions are also used (Gumbel distribution)
- Annual probability of failure is the same for every year in the service life

$$P_{fA} = \mathbb{P}\left[R - X_{Amax} \leq 0\right]$$

- Annual probability of failure is kept below a target level (by code calibration)

Time-Invariance

All reliability measures are time-invariant in the stationary climate

Non-Stationary Climate

- The climate change is causing temporal variations in the frequency and intensity of weather extremes
- Sustained global warming over a long period of time is likely to introduce some dependence among weather extremes
- Increasing concentration of greenhouse gases with time will continue to amplify non-stationary effects
- **Non-stationary processes** are required to model such temporal changes in the climate variables

Key Points

- Stationary load processes will not be valid in the changing climate
- All reliability measures will become time-variant quantities

Research Approach

- Non-stationary reliability analysis to model climate change effects
- Scope of research

1. Analytical developments in stochastic analysis
2. Statistical estimation of model parameters
3. Numerical parametric study
4. Practical data analysis and modelling
5. Design code development issues

Non-Stationary Point Processes

Load Arrivals

1. **Non-Homogeneous Poisson process** (NHPP)
   - A natural extension of the homogeneous Poisson process
   - The occurrence rate is time dependent, $\lambda(t)$
   - Arrival times $(T_1, T_2, \ldots)$ are dependent RVs
   - "Independence" property still holds
     - Number of events in an interval are independent of any other disjoint interval
     - This property is also a limitation of the model
2. **Non-Homogeneous Birth Process**
   - Includes all NHPP properties, but "Independence" property relaxed
   - Number of events in an interval depends on the history of the process

Load Magnitudes

- A function of the time of load arrival

$$X(s_k) = \phi_1(s_k) + \phi_2(s_k) X_k$$

- $\phi_1(s_k), \phi_2(s_k)$ are time-dependent amplification functions

Non-Homogeneous Poisson Process

- Fully analytically tractable model
- The probability distribution of the number of events, $N(s,t)$, $s < t$,

$$f_N(k; s,t) = \frac{(\Lambda(s,t))^k}{k!} e^{-\Lambda(s,t)}, \quad (0 \le k < \infty) \tag{2}$$

- NHPP is defined by the Mean Value Function, $\Lambda(s,t) = \mathbb{E}[N(s,t)]$.
- The occurrence rate (or frequency), $\lambda(t) = \frac{\mathrm{d}\Lambda(t)}{\mathrm{d}t}$
- If the rate is increasing, inter-arrival times periods will be decreasing, and $\mathbb{E}[T_n] < \cdots < \mathbb{E}[T_2] < \mathbb{E}[T_1]$
- Analytical results were derived for all necessary quantities required for structural reliability analysis
  - Return period, mean waiting time
  - Extreme value distribution with time-dependent loads

Pandey, M.D., and Lounis, Z. (2023). Stochastic modelling of non-stationary environmental loads for reliability analysis under the changing climate. Structural Safety, 103, 102348, pp.1-11.

NHPP Model: Some Results

- Distribution of inter-arrival time, T_n (complementary CDF)

$$\overline{F}_{T_n}(t) = \int_0^\infty f_{T_1}(t+u)\frac{[\Lambda(u)]^{n-1}}{(n-1)!}\,\mathrm{d}u$$

- The mean inter-arrival time or n^{th} return period

$$\mathbb{E}\left[T_n\right] = \int_0^\infty \overline{F}_{T_1}(s)\frac{[\Lambda(s)]^{n-1}}{(n-1)!}\,\mathrm{d}s, \quad (n \geq 1)$$

  - The return period is no longer a constant, rather it changes with n

Non-Stationary Reliability Analysis

- The extreme load distribution in a time interval, $(t_1, t_2]$, $t_1 < t_2$:

$$F_{max}(x, t_1, t_2) = e^{-\Lambda(x, t_1, t_2)}$$

where the mean value function is given as,

$$\Lambda(x, t_1, t_2) = \int_{t_1}^{t_2} \overline{F}_X(\psi(x, s))\, \lambda(s) \mathrm{d}s, \quad \text{and} \quad \psi(x, s) = \frac{x - \phi_1(s)}{\phi_2(s)}$$

- Probability of failure in a time interval

$$P_f(t_1, t_2) = \mathbb{P}\left[R - X_{max}(t_1, t_2) \leq 0\right]$$

- Computation using FORM or simulations

Non-Stationary Climate

- Time-invariance property of all the reliability measures is lost
- All the measures must be defined with reference to a time interval

Non-Homogeneous Birth Process

- **Conditional Intensity**
  Conditional probability of an event given the history of the process, $\lambda_C(n, t)$

$$\mathbb{P}\left[N(t + \mathrm{d}t) - N(t) = 1 | N(t) = n\right] = \lambda_C(n, t)\mathrm{d}t + o(\mathrm{d}t),$$

  - Form of intensity function, $\lambda_C(n, t) = h(n)\lambda(t)$
  - Includes dependence on the number of past events (n)
  - Also depends on the time (t) elapsed since the start of the process
- Various types of birth processes depending on the cond. intensity
  - Homogeneous form, $\lambda(t) = \lambda$
  - Yule process, Generalized Polya process, ...
  - NHPP is a special case, $\lambda_C(n, t) = \lambda(t)$
- Analytical solutions
  - Homogeneous birth process - all analytical solutions have been derived
  - Non-homogeneous case - analyzed for a linear form of intensity function

Linear Extension of the Yule Process (LEYP)

- Conditional intensity

$$\lambda_C(n,t) = \underbrace{(an+b)}_{\textit{past history}} \times \underbrace{\lambda(t)}_{\textit{time elapsed}} \quad , \quad (a \geq 0, b > 0)$$

  - **History dependence:** A linear form, $h(n) = (a\,n + b)$
  - **Temporal effect:** A time-dependent rate, $\lambda(t)$
  - Parameter, $b = 1$, fixed to ensure the model identification
  - Scale parameter, a, controls the influence of the process history

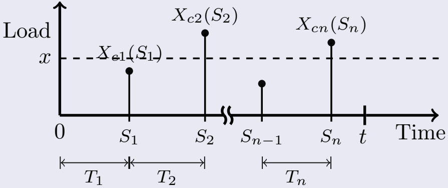
- **Negative binomial distribution** for the number of events

$$\boxed{\mathbb{P}\left[N_t = n\right] = \frac{\Gamma\left(\alpha + n\right)}{\Gamma(n+1)\Gamma\left(\alpha\right)} \left(\beta_t\right)^{\alpha} \left(1 - \beta_t\right)^{n}}$$

Pandey, M.D., and Mercier, S. (2024). Stochastic Modelling of Non-Stationary and Dependent Weather Extremes for Structural Reliability Analysis in the Changing Climate. Structural Safety (under review)

General Non-Stationary Load Model

LEYP Shock Process



- **Non-stationary components**
  - Frequency of occurrence of load events
  - Dependence on the number of events occurring over time
  - Intensity (magnitude) of loads are also time-dependent
- **Analysis Results**
  - Expressions for any n^{th} return period, mean waiting time to the next event
  - Correlation coefficient between the number of events in two intervals (dependence measure)
  - Distribution of maximum loads in $(s, t]$

Reliability Analysis

- Time-dependent loads, $X(s_k) = \phi_1(s_k) + \phi_2(s_k)X_k$
- Distribution of maximum load in a given time interval

$$\boxed{F_{max}(x, t_1, t_2) = \left[1 + \frac{\mu_{12}}{\alpha} - aq^*(x, t_1, t_2)\right]^{-\alpha}}$$

where $$q^*(x, t_1, t_2) = \int_{t_1}^{t_2} \frac{\lambda(s)}{\beta_s} F_X(\psi(x, s))\mathrm{d}s$$

$$\text{with} \quad \psi(x, s) = \frac{x - \phi_1(s)}{\phi_2(s)}, \quad \beta_s = e^{-a\Lambda(s)}, \text{ and } \Lambda(s) = \int_0^s \lambda(u)\,\mathrm{d}u$$

- Probability of failure in a given time interval

$$\boxed{P_f(t_1, t_2) = \mathbb{P}\left[R - X_{max}(t_1, t_2) \le 0\right] = \int_0^{\infty} \overline{F}_{max}(x, t_1, t_2) f_R(x)\mathrm{d}x}$$

Inter-Arrival Times

- n^{th} return period, $T_n = S_n - S_{n-1}, n > 1, S_0 = 0$
- CCDF of T_n:

$$\overline{F}_{T_n}(u) = \frac{a}{B\left(\frac{b}{a}, n-1\right)} \times \int_0^\infty \lambda(s)\, e^{-g(u,s)} \left(1 - e^{-a\Lambda(s)}\right)^{n-2} ds$$

where

$$g(u,s) = a(n-1)\,\Lambda(s, u+s) + b\Lambda(u+s)$$

- An n^{th} return period by integration: $\mathbb{E}[T_n] = \int_0^\infty \overline{F}_{T_n}(u)\mathrm{d}u$

Variable Return Periods!

In a non-stationary process, $\mathbb{E}[T_1], \mathbb{E}[T_2], \ldots, \mathbb{E}[T_n]$, are all DISTINCT values

A Measure of Dependence

- **Correlation coefficient**
  Between the number of events in two adjacent intervals, $(t_1, t_2]$ and $(t_2, t_3]$ with $0 \leq t_1 < t_2 < t_3$
- Definition

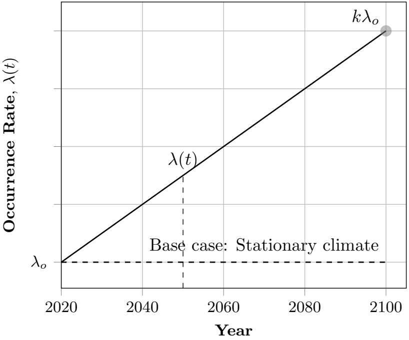
$$\rho[N_{12}, N_{23}] = \frac{\mathrm{COV}\,[N_{12}, N_{23}]}{\sigma_{12}\,\sigma_{23}}$$

- A general expression

$$[\rho\,(N_{12}, N_{23})]^2 = \left(\frac{\mu_{12}}{\alpha + \mu_{12}}\right)\left(\frac{\mu_{23}}{\alpha + \mu_{23}}\right)$$

Numerical Examples

Linear Rate Function



- The reference period for the analysis is 2020 - 2100, ($t_e = 80$ years)
- The base case is the stationary climate with a rate, $\lambda_o = 1$ event/year
- An overall, linear increase in the occurrence rate is $k\lambda_o$ over t_e years

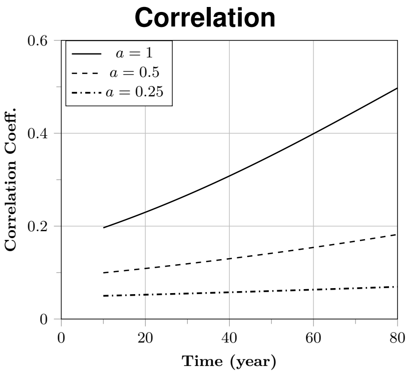
- **Climate Amplification Factors**
  Increase as a multiple of the base case (stationary climate)
  Increase in the frequency: $k = k_F$
  Increase in the load magnitude: k_L

Parametric Study

- Investigation of the effect of various model parameters
  - Correlation coefficient (dependence)
  - Expected number of events
  - Return periods
  - Mean waiting time
  - Probability of Failure
- Model parameters
  - Scale parameter of the history function
  - Base rate
  - Liner increase in the rate and load magnitude
  - Amplification factors for the rate and the load

Parameter	Notation	Values
Scale	a	0.1 - 1
Base rate	λ_0	0.02 - 1
Amplification Factors	-	-
Frequency (rate)	k_F	1 - 4
Load	k_L	1 - 1.4

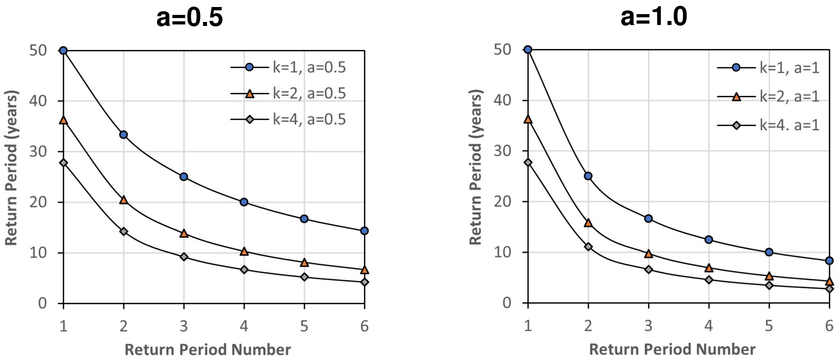
Dependence in LEYP



- The decadal correlation coefficient between the number of events ($t \pm 10$)
- Results for homogeneous birth process
- A constant rate, $\lambda = 0.02$/year

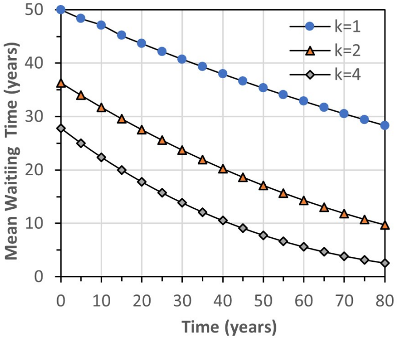
- Corre. coeff. increases continuously as time increases from 10 - 80 years
- The rate and the magnitude of this increase are controlled by "*a*"

Example: Inter-Arrival Times



- The first 6 return periods (RPs) are plotted
- A dramatic decrease in subsequent RPs of extreme events
- **Frequency amplification and the scale (a) parameter have significant influence on the inter-arrival times**

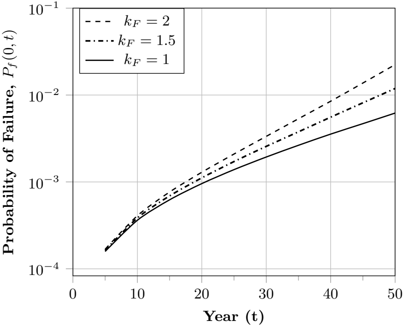
Mean Waiting Time to the Next Event



- Mean waiting time decreases with the passage of time
- Results are shown for $a = 0.5$
- Mean waiting time decreases with an increase in the frequency amplification

- The mean waiting time (MWT) is a more useful measure than mean inter-arrival time
- MWT does not require any information about the history of the process

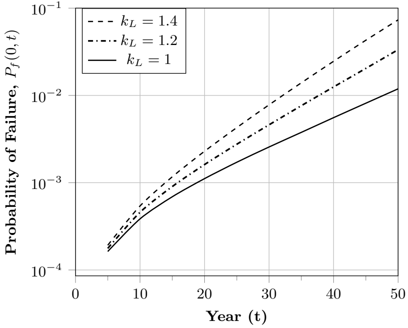
Probability of Failure: Increasing Frequency



- Cumulative probability of failure in $P_f(0, t)$
- Scale, $a = 0.5$, $\lambda_0 = 0.1$
- 50 to 100% increase in frequency over 50 years
- Time-invariant load

- The impact of frequency amplification on the probability of failure
- An order of increase in k_F has a modest effect on $P_f(0, t)$

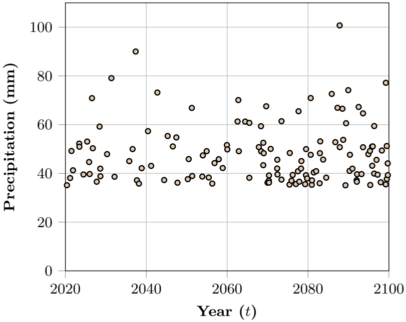
Probability of Failure: Increasing Intensity



- Cumulative probability of failure in $P_f(0, t)$
- Scale, $a = 0.5$, $\lambda_0 = 0.1$
- Frequency amplification, $k_F = 1.5$, fixed
- load intensity increase 20 to 40% over 50 years

- The impact of load amplification on the probability of failure
- A modest increase in k_L has a significant effect on $P_f(0, t)$

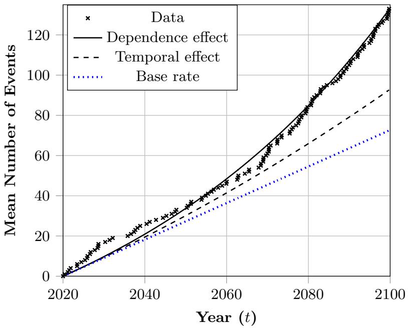
Application: Extreme Precipitation in Future



- Future climate forecast by the Canadian Earth System Model (5.03)
- Simulation data for heavy precipitation events (>35 mm/day)
- Scenario: SSP 5-8.5 (Fossil-fuelled Development)
- Expected increase of 4.4°C in the the mean global temperature by 2100

- Data for a spatial grid-box of of 6×10 km^2 size in Toronto, Ontario

Precipitation Data: LEYP Model



- Expected number of events in $(0, t)$
- Base rate $\lambda_0 = 0.91$
- Frequency amplification, $k_F = 1.55$
- Intensity amplification is absent, $k_L = 1$

- Three stochastic effects are present
  1. A stationary rate, $\lambda_o = 0.91$, implies 73 events expected in 80 years
  2. Frequency amplification by 55% over 80 years increases to 92 events
  3. Dependence effect increasing this to 132 events
- Modest dependence, decadal corre. coeff. increases from 0.07 to 0.17
- In spite of this, dependence has a discernible effect

Conclusions

- Non-stationary stochastic load models are developed for structural reliability analysis in the changing climate
  - NHPP and LEYP models are analysed in detail
  - The ”order statistics property” is a key to derive analytical results
- Analytical results are derived for various elements of reliability analysis framework
- The LEYP model overcomes a major limitation of the classical Poisson process by including the statistical dependence among extreme events
  - Even a mild dependence can lead to a significant increase in the frequency of extremes and the probability of failure
- **Path forward**
  - Code calibration in non-stationary climate
  - Target reliability considerations
  - Degradation of structural strength
  - Stochastic load combination rules

Q & A

Thank you for your attention!

Non-Stationary Modelling: Current Approach

- In the literature, very limited use of non-stationary stochastic processes for modelling the climate change effects
- The Gumbel (or GEV) distribution with time dependent parameters is the state-of-the-art
- Parameter estimation using annual maxima data obtained through climate simulation models
- Using this, the annual probability of failure is computed
- The annual maxima distribution (and probability of failure) in a given year is assumed to be ”Independent” of all other years
- This ”Independence” implies that extremes are generated by an NHPP
- **The current approach is quite restrictive**
  - It does not allow to investigate other aspects of the problem (inter-arrival times, dependence, intensity and frequency amplification)