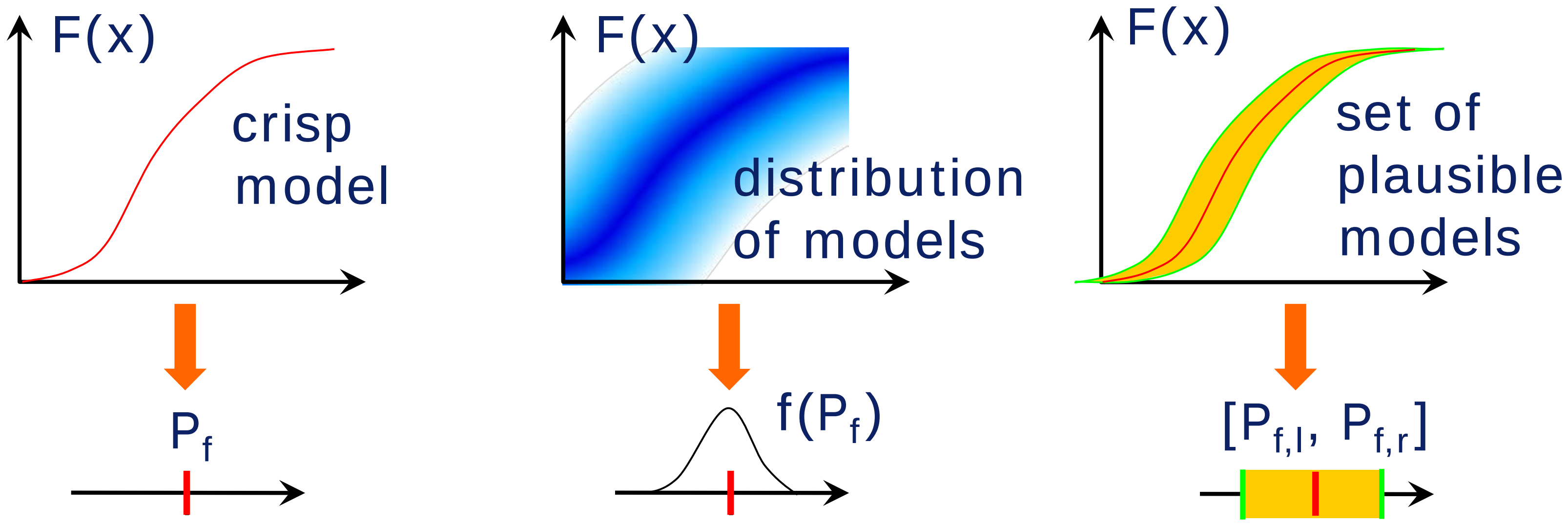
Time-dependent reliability analysis with aleatory and epistemic uncertainties

Michael Beer
Institute for
Risk and Reliability

Leibniz Universität Hannover

STOCHASTIC STRUCTURAL DYNAMICS

Uncertainties: Aleatory and Epistemic



Challenges

- appropriate model for spatial and temporal random quantities
  - ➡ physics-based covariance model
- effective quantification of vague and limited information
  - ➡ power spectrum estimation based on scarce and poor data
- efficient numerical analysis of responses
  - ➡ targeted first passage identification

MODIFIED EXPONENTIAL COVARIANCE

Karhunen-Loéve expansion

- random field $Y(x,\theta)$

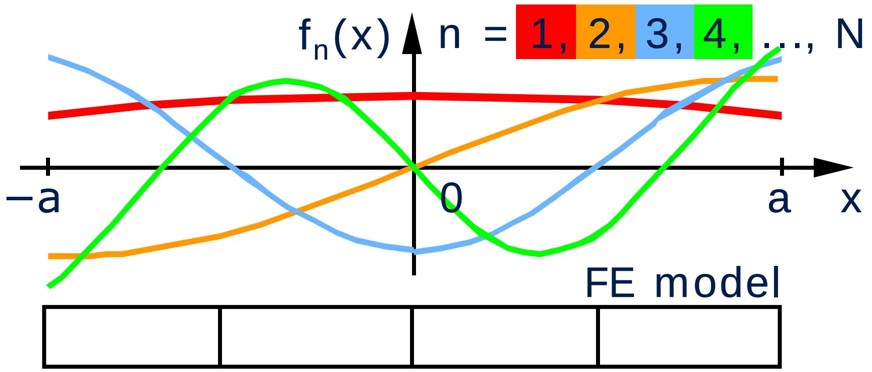
$$Y(x,\theta) = \sum_{n=1}^{\infty} \sqrt{\lambda_n} \cdot \xi_n(\theta) \cdot f_n(x)$$

- covariance function

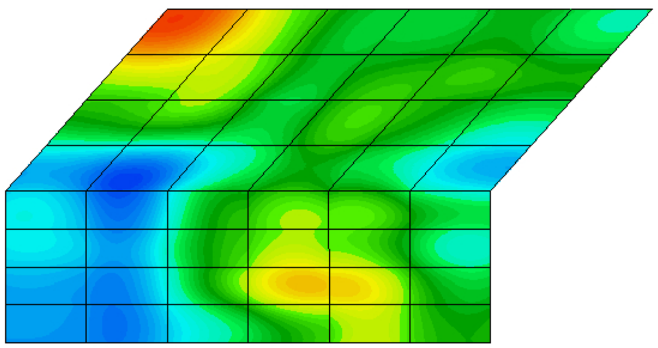
$$C(x_1, x_2) = \sum_{n=1}^{\infty} \lambda_n \cdot f_n(x_1) \cdot f_n(x_2)$$

θ – elementary events
$\xi_n(\theta)$ – random variables
λ_n – constant, positive real numbers
$f_n(x)$ – real functions, orthonormal
x – spatial / temporal coordinate

- Stochastic Finite Element Method



- realization y(x) (example)



- efficiency criterion
  - » minimum number N of $\xi_n(\theta)$ and $f_n(x)$ → larger Finite Elements, smaller number of dof

MODIFIED EXPONENTIAL COVARIANCE

Modification of the Exponential Covariance Model

- traditional

  $C(x_1, x_2) = e^{-a \cdot |x_1 - x_2|} = e^{-a \cdot |u|}$

  » $\left.\dfrac{\partial C(u)}{\partial u}\right|_{u=0} = 0$ not complied with

  » directionality of the field coordinate x (time)

  $Y(x+1) = c \cdot Y(x) + W(x)$

  (first-order process in time)

- modified

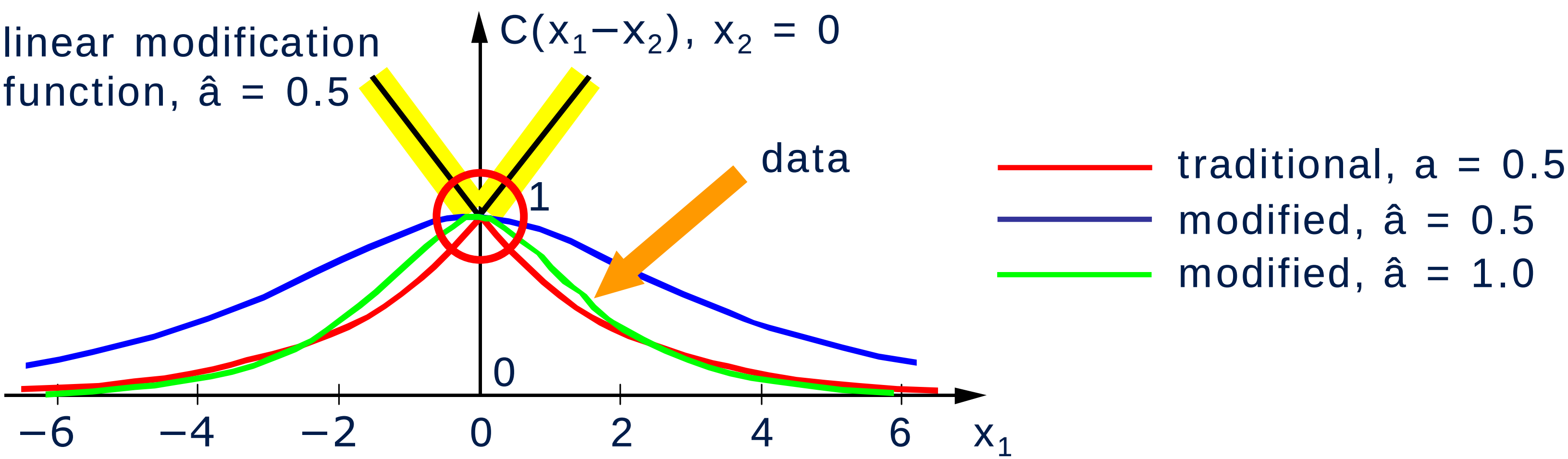
  $\hat{C}(x_1, x_2) = e^{-\hat{a} \cdot |u|} \left(1 + \hat{a} \cdot |u|\right), \quad u = x_1 - x_2$

  » $\left.\dfrac{\partial \hat{C}(u)}{\partial u}\right|_{u=0} = 0$

  » non-directionality of the field coordinate x (space)

  $Y(x) = c \cdot \left[Y(x-1) + Y(x+1)\right] + W(x)$

  (second-order process in time)



MODIFIED EXPONENTIAL COVARIANCE

Generalization: Whittle-Matèrn Kernel

$$C^{\nu}(\tau) = \frac{2^{1-\nu}}{\Gamma(\nu)}\left(\sqrt{2\nu}\frac{\tau}{b}\right)^{\nu} K_{\nu}\left(\sqrt{2\nu}\frac{\tau}{b}\right), \quad \tau = x_1 - x_2$$

ν = “smoothness”: sample paths are $\lceil\nu\rceil-1$ times differentiable

b = correlation length

- $\nu = 0.5$ ➡ traditional exponential covariance kernel

  $C^{0.5}(\tau) = e^{-|\tau|/b}$ not differentiable

- $\nu = 1.5$ ➡ modified exponential covariance kernel

  $C^{1.5}(\tau) = \exp(-|\tau|/b)(1 + |\tau|/b)$ once differentiable

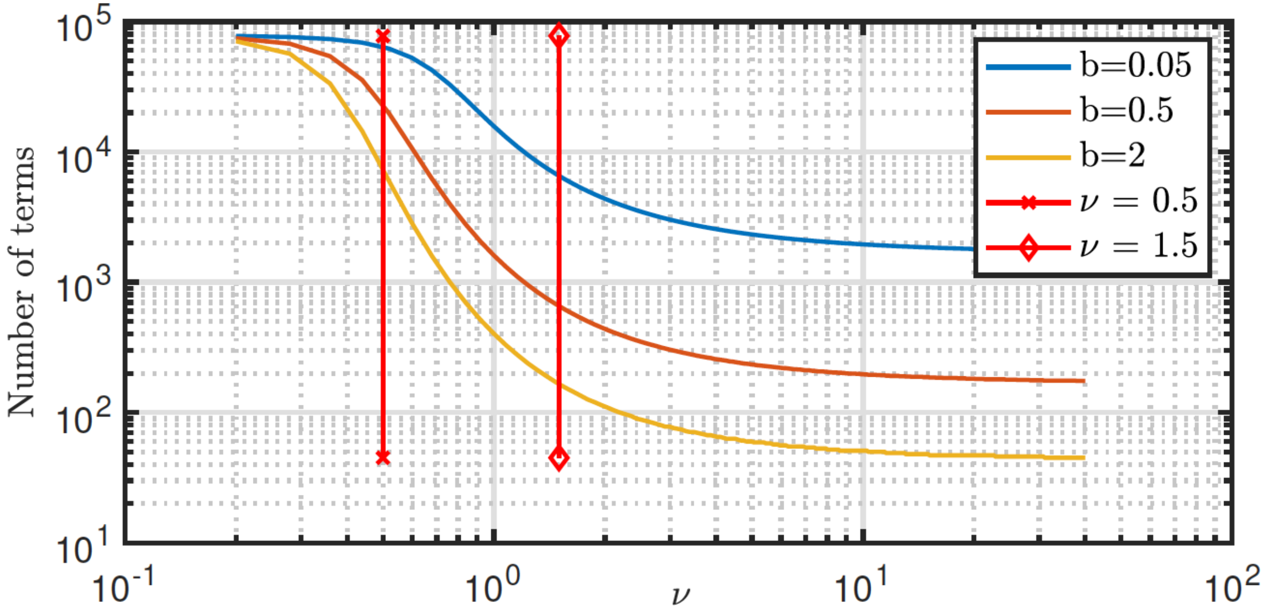
- $\nu = \infty$ ➡ squared exponential covariance kernel

  $C^{\infty}(\tau) = \exp(-\tau^2/b^2)$ infinite differentiability (unrealistic, hence limited to $\nu = 3.5$)

MODIFIED EXPONENTIAL COVARIANCE

Convergence of corresponding power spectra

- number of terms in Shinozuka-Deodatis expansion to represent 99.9% of the original signal
  - » higher-order differentiability at zero-lag causes faster convergence
  - » analytical solutions for the realizations exist for ν=0.5 and ν=1.5



Spanos, P.D.; Beer, M.; Red-Horse, J. (2007):
Karhunen-Loéve Expansion of Stochastic Processes with a Modified Exponential Covariance Kernel, ASCE Journal of Engineering Mechanics, 133(7), 773–779.
Kosheleva, O.; Beer, M. (2016):
Why Modified exponential covariance kernel is empirically successful: A theoretical explanation, Journal of Uncertain Systems, 10(1), 10–14.
Faes, M.G.R.; Broggi, M.; Spanos, P.D.; Beer, M. (2022):
Elucidating appealing features of differentiable auto-correlation functions: a study on the modified exponential kernel, Probabilistic Engineering Mechanics, 69, 103269.

JOINT TIME-FREQUENCY ANALYSIS

Excitation → System → Response

Excitation

non-stationary stochastic process

(earthquake, wind, ocean waves, blast events etc)

System

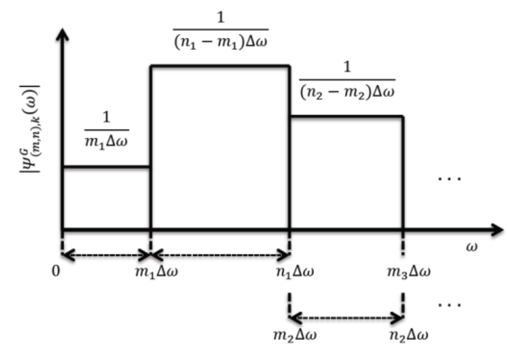

nonlinear and time-varying behavior due to severe dynamic excitation

Response

non-stationary stochastic process

- excitations with time-varying intensity & frequency content
  → joint time-frequency analysis

- generalized harmonic wavelets
  - » orthogonality, flexible window size, non-overlapping supports



$$\int_{-\infty}^{\infty}|f(t)|^2 dt = 2\sum_{m,n}\sum_{k}\left(\frac{T_o}{n-m}\left|W^G_{(m,n),k}\right|^2\right)$$

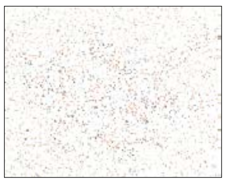
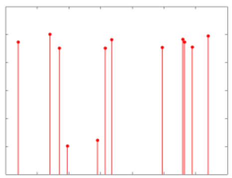
$$S(\omega_i, t_i) = \frac{T_o}{2\pi(n-m)}E\left[\left|W^G_{(m,n),k}\right|^2\right], \quad m\Delta\omega \le \omega_i < n\Delta\omega, \quad \frac{k}{n-m}T_o \le t_i < \frac{k+1}{n-m}T_o$$

Beer, M.; Gholami, A.; Kreinovich, V. (2019):
A Theoretical Explanation for the Efficiency of Generalized Harmonic Wavelets in Engineering and Seismic Spectral Analysis, Mathematical Structures and Modeling, 3(51), 97–104.

JOINT TIME-FREQUENCY ANALYSIS

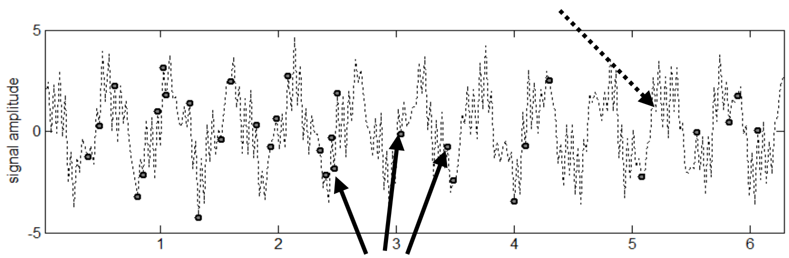
Incomplete data

- Compressive sensing approach


Assume sparsity in a known basis

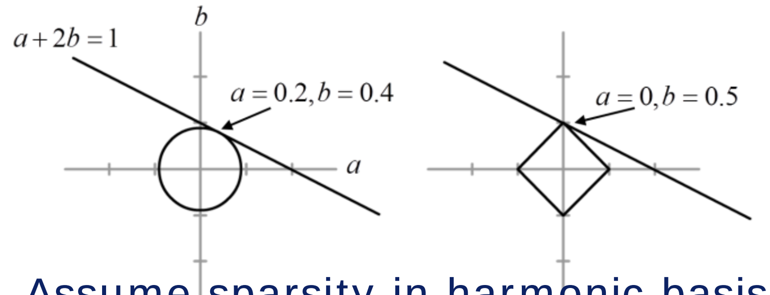
Stochastic process with 3 harmonics and white noise



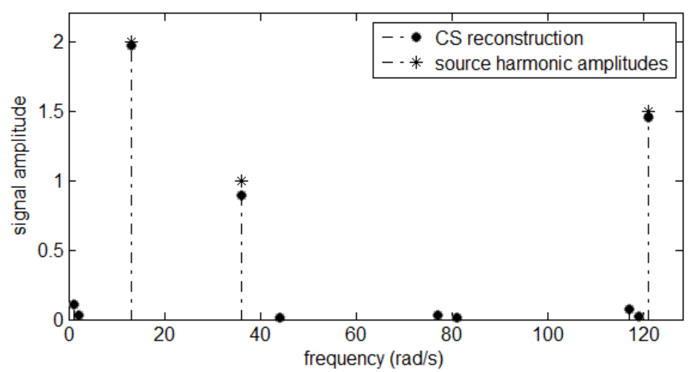
Limited number of sample points available

- Minimization of $\Sigma_i|x_i|$ in $Ax = y$ promotes sparse solutions

Least squares **L1 minimization**



Assume sparsity in harmonic basis to identify key frequencies

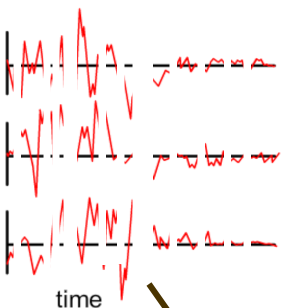


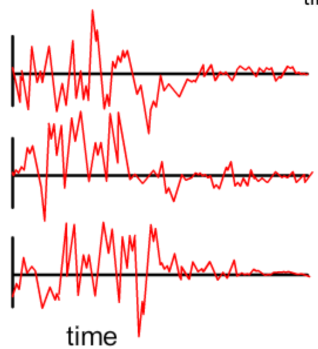
Comerford, L.; Kougioumtzoglou, I.A.; Beer, M. (2016):
Compressive sensing based stochastic process power spectrum estimation subject to missing data, Probabilistic Engineering Mechanics, 44, 66–76.
Zhang, Y.J.; Comerford, L.A.; Kougioumtzoglou, I.A.; Beer, M. (2018):
Lp-norm minimization for stochastic process power spectrum estimation subject to incomplete data, Mechanical Systems and Signal Processing, 101, 361–376.

EXAMPLE: RELIABILITY ANALYSIS

Incomplete Earthquake Records

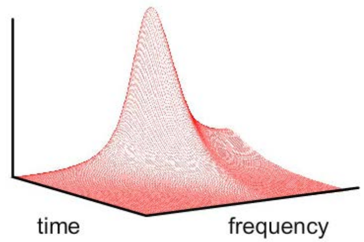
Incomplete non-stationary earthquake records



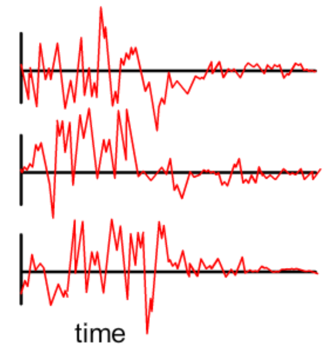


Data reconstruction

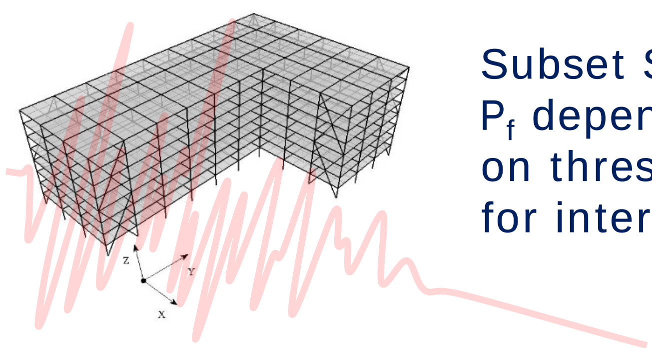
Spectrum estimation



Simulated realizations



Reliability Analysis



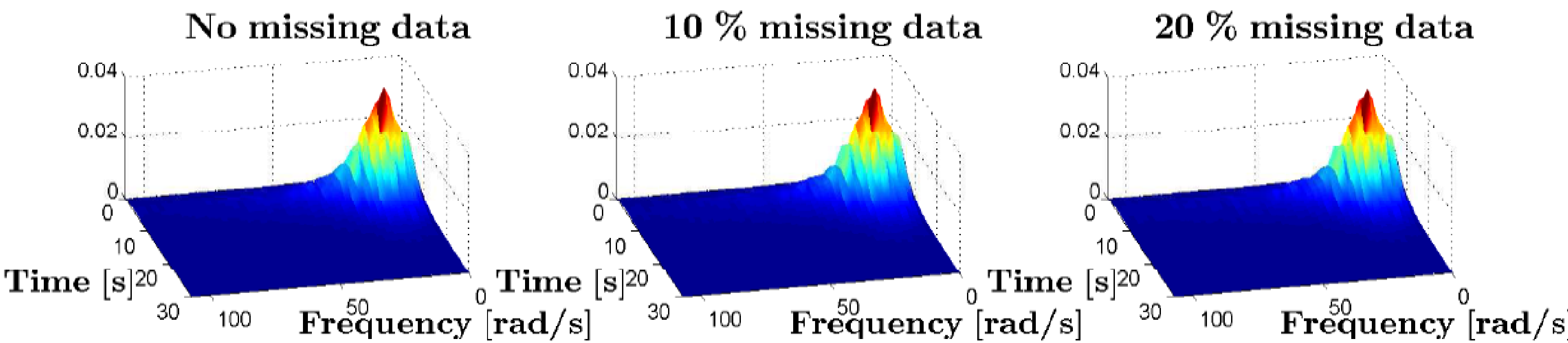
Subset Sampling; P_f depending on threshold for interstory drift

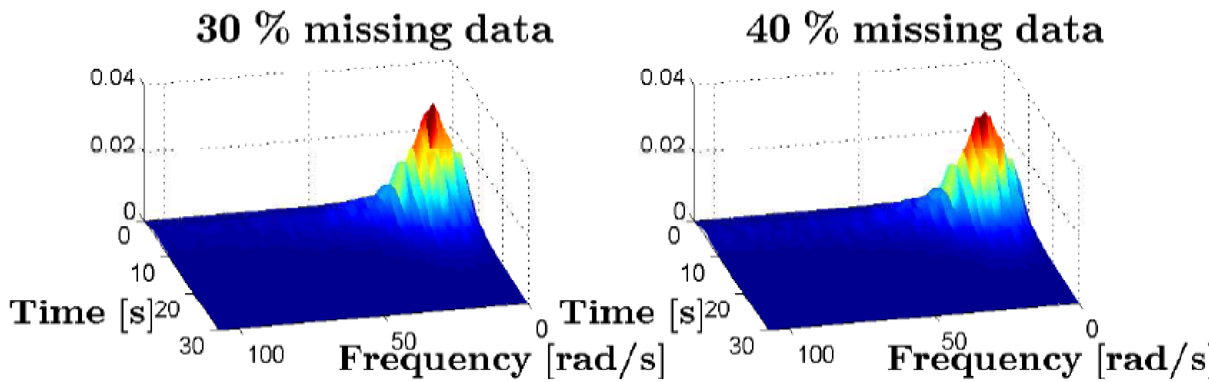
Comerford, L.; Jensen, H.A.; Mayorgab, F.; Beer, M.; Kougioumtzoglou, I.A. (2017):
Compressive sensing with an adaptive wavelet basis for structural system response and reliability analysis under missing data, Computers and Structures, 182, 26–40

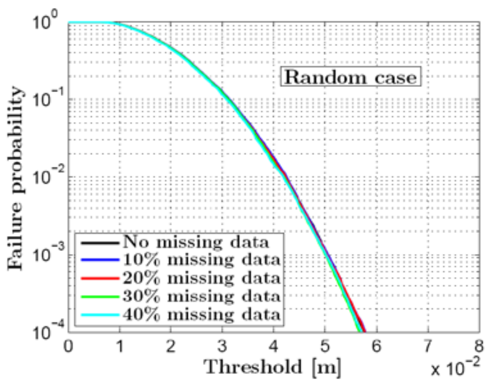
EXAMPLE: RELIABILITY ANALYSIS

Incomplete Earthquake Records

- data removed randomly (10%, 20%, 30%, 40%)



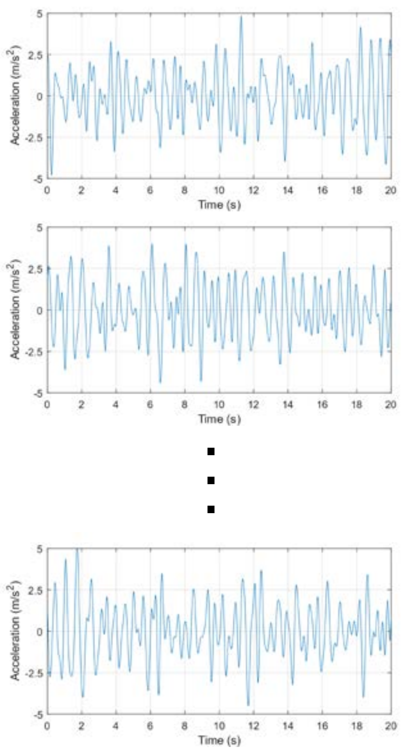




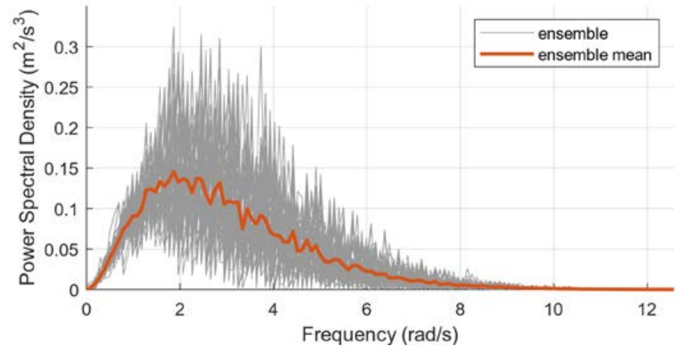
RELAXED POWER SPECTRA

Sampling Uncertainty

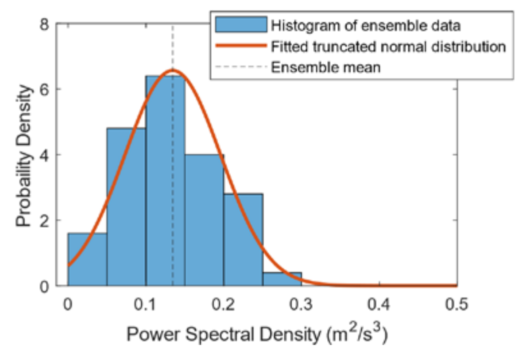
- ensemble of power spectra

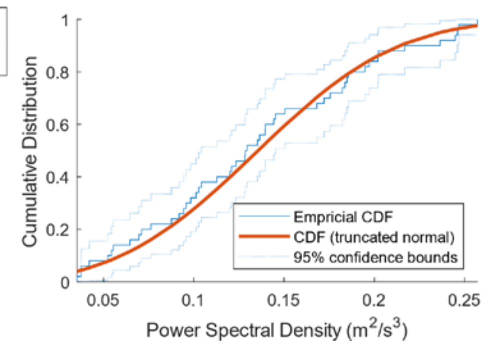


PSD estimation



confidence bounds and
truncated normal distribution for each frequency





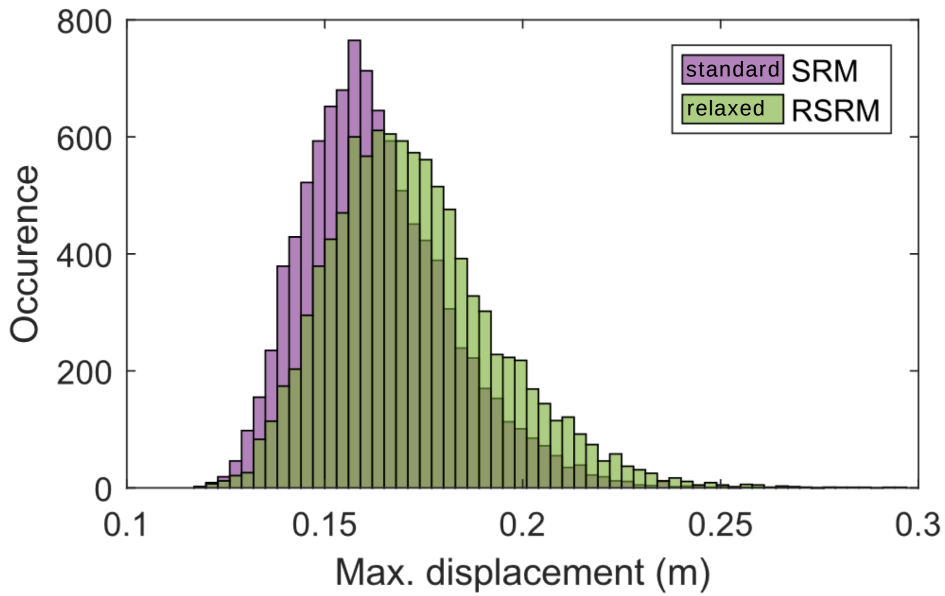
Behrendt, M.; Bittner, M.; Comerford, L.; Beer, M.; Chen, J.B. (2022):
Relaxed Power Spectrum Estimation from Multiple Data Records utilising Subjective Probabilities, Mechanical Systems and Signal Processing, 165, Article 108346.

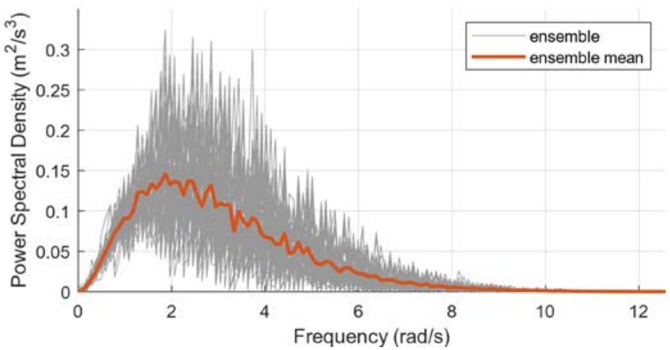
RELAXED POWER SPECTRA

Sampling Uncertainty

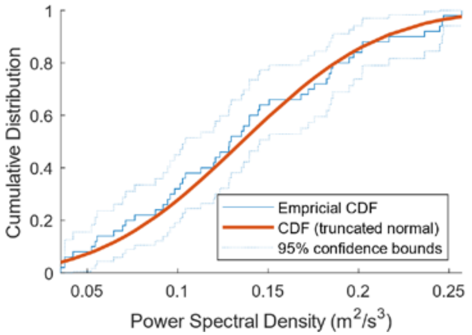
- ensemble of power spectra

example:
effect on response statistics of
SDOF oscillator





confidence bounds and
truncated normal distribution
for each frequency



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Relaxed Power Spectrum Estimation from Multiple Data Records utilising Subjective Probabilities, Mechanical Systems and Signal Processing, 165, Article 108346.

IMPRECISE POWER SPECTRA

Limited Data

- empirical bounds on power spectrum
  - » identification of the basis power spectrum of the ensemble
  - » fitting an RBF network to the basis power spectrum
  - » optimization of the weights of the basis functions to find PSD bounds

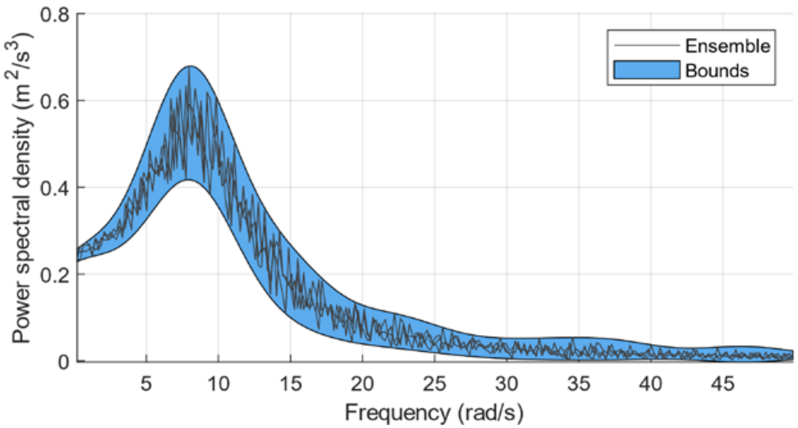
$$\overline{S_{opt}}(\omega_n; w^{up}) = \sum_{i=1}^{m} w_i^{up}\, \phi_i(\omega) + b_0$$

$$\underline{S_{opt}}(\omega_n; w^{low}) = \sum_{i=1}^{m} w_i^{low}\, \phi_i(\omega) + b_0$$

$$\min \sum_{\omega_n} \left|\overline{S_{opt}}(\omega_n; w^{up}) - \underline{S_{opt}}(\omega_n; w^{low})\right|$$

s.t.

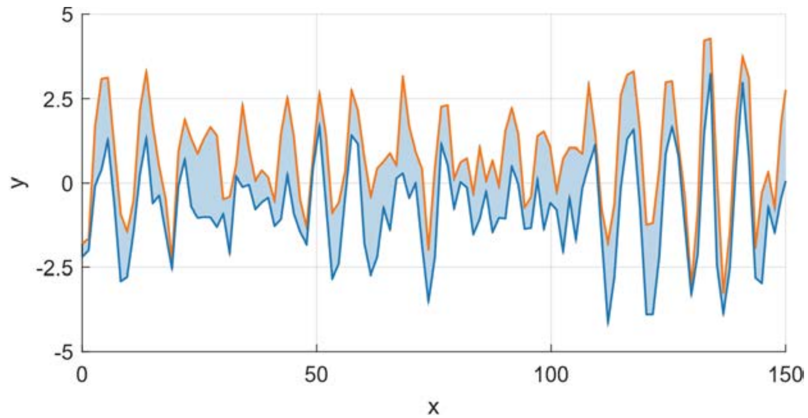
$$\overline{S_{opt}}(\omega_n; w^{up}) \geq S_{max}(\omega_n)$$
$$\underline{S_{opt}}(\omega_n; w^{low}) \leq S_{min}(\omega_n)$$
$$\underline{S_{opt}}(\omega_n; w^{low}) \geq 0$$

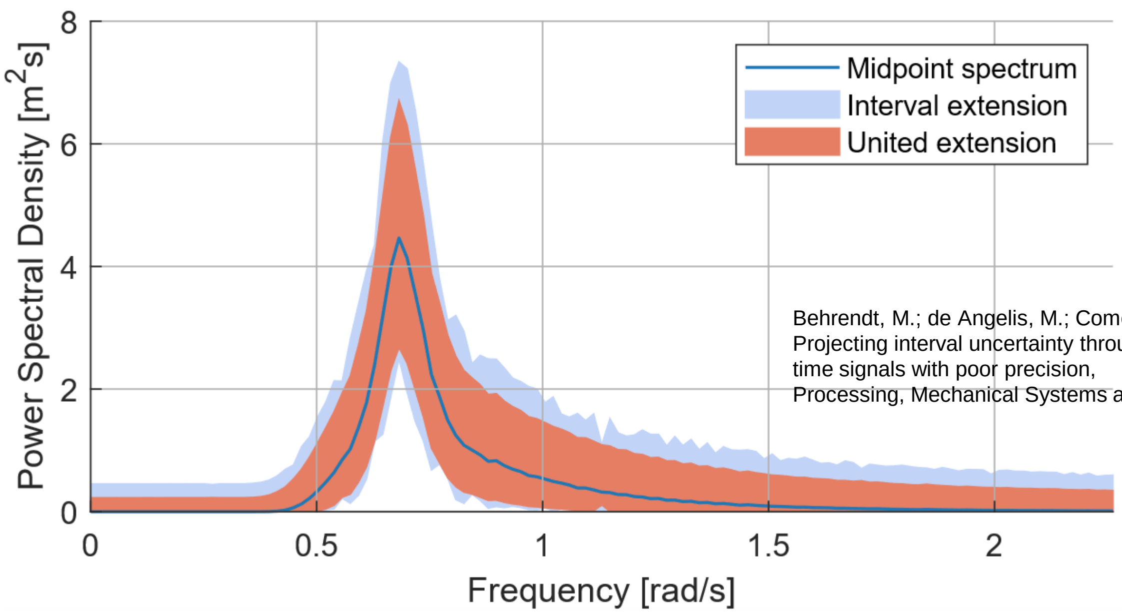


IMPRECISE POWER SPECTRA

Poor Data (interval-valued)

- rigorous bounds on power spectrum
  - » expansion of DFT to interval-DFT
  - » resolving dependability problem by intrinsic identification of extreme amplitudes for each frequency
  - » construction of bounds of PSD



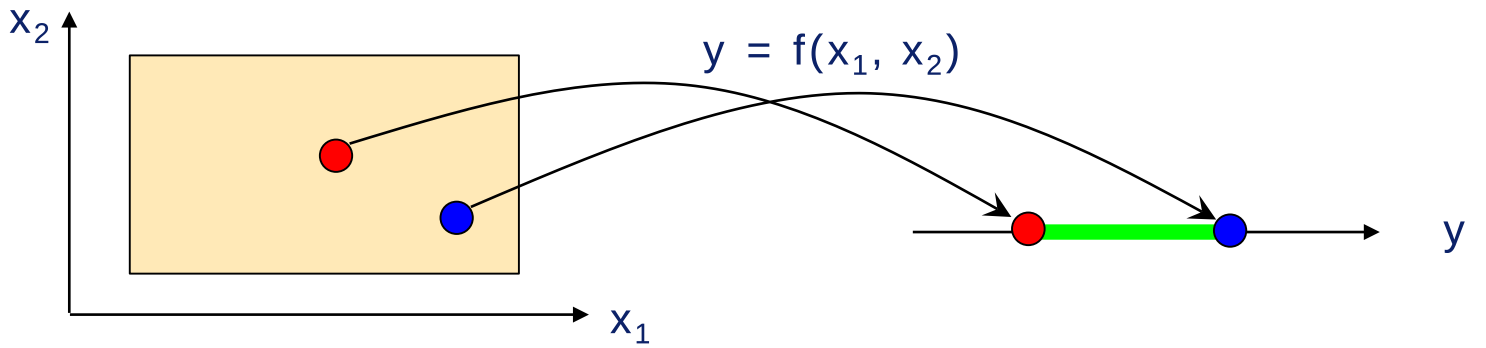


Behrendt, M.; de Angelis, M.; Comerford, L.A.; Zhang, Y.J.; Beer, M. (2022): Projecting interval uncertainty through the discrete Fourier transform: an application to time signals with poor precision, Processing, Mechanical Systems and Signal Processing, 172, Article 108920

TIME DEPENDENT RELIABILITY ANALYSIS

Analysis with interval-valued stochastic models

- explicit search for result interval bounds
  - » optimization searching the space of interval parameters



interval analysis: repeated stochastic analysis

stochastic analysis: repeated deterministic structural analysis

deterministic structural analysis

Nested loop ➡ Goal: pre-solve optimization to calculate interval result from a single efficient stochastic analysis

TIME DEPENDENT RELIABILITY ANALYSIS

First passage problem

- pre-identification of $\theta^{\bar{*}}$ such that

$$\bar{P}_f = \int_{z\in\mathbb{R}^n} I_F\left(z,\theta^{\bar{*}}\right) f_Z(z)\,dz$$

with

Operator norm

$$\theta^{\bar{*}} = \underset{\theta\in\theta^I}{arg\,max} \; \max_{i=1,\dots,n_y} \; \max_{l} \left\| A_{i,l}(\theta) \right\|_2$$

via standard optimization
on the physical model (ie FEM)
without repeated reliability analysis

- requirement:
find a continuous linear map A
that relates random input z
to random response y

$$y(\theta) = A(\theta)z$$

- operator norm theory

$$\left\| A_i(\theta)z \right\|_{p^{(1)}} \le \left| c_i(\theta) \right| \cdot \left\| z \right\|_{p^{(2)}}$$

$$\left\| y_i(t,\theta,z) \right\|_{p^{(1)}} \le \left| c_i(\theta) \right| \cdot \left\| z \right\|_{p^{(2)}}$$

➡ smallest $|c_i(\theta)|$ provides
upper bound on „amplification“

- Karhunen-Loeve expansion

A_i

$$y_i(t_k, z) = \sum_{l_1=1}^{k} \Delta t \varepsilon_{l_1} h_i\left(t_k - t_{l_1}\right) \left(\sum_{l_2=1}^{n_{KL}} \psi_{l_1,l_2} \sqrt{\lambda_{l_2}}\, z_{l_2} \right)$$

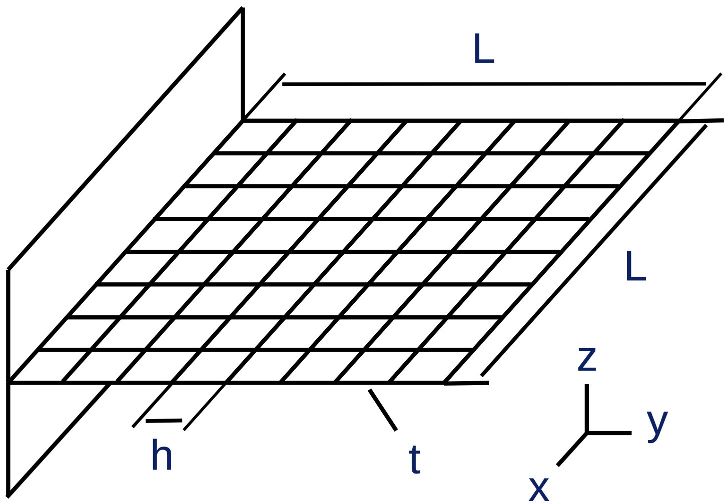
Faes, M.; Valdebenito, M.A.; Moens, D.; Beer, M. (2020):
Bounding the First Excursion Probability of Linear Structures Subjected to Imprecise Stochastic Loading, Computers and Structures 239, 106320.

Faes, M.; Valdebenito, M.A.; Moens, D.; Beer, M. (2021):
Operator norm theory as an efficient tool to propagate hybrid uncertainties and calculate imprecise probabilities,
Mechanical Systems and Signal Processing 152, 107482.

TIME DEPENDENT RELIABILITY ANALYSIS

Example: clamped steel plate

- structural model
  - » 100 shell elements, linear
  - » 110 nodes
  - » Dirichlet boundary conditions on clamp



- P_f for exceedance of displacement at corner point of 15 cm

- load model

$$F(r,\theta,z) = 1 \cdot \theta_1 \cdot sin\left(\frac{\pi}{\theta_2}\right) + \theta_3 \cdot B(\theta_4, r) \cdot z$$

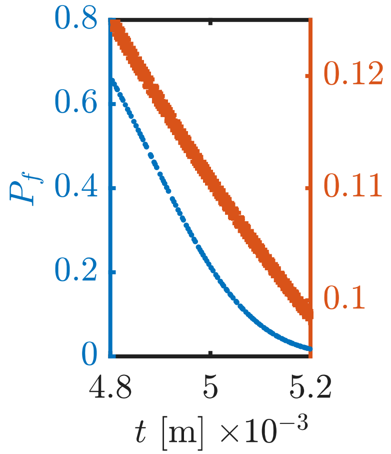
with
  - » KL-basis B
  - » 10 standard normal rv's z

- interval parameters
  - » θ_1 and θ_2 governing the expected value of random load field
  - » θ_3: standard deviation of load field
  - » θ_4: correlation length of load field
  - » E: Young's modulus
  - » t: plate thickness

TIME DEPENDENT RELIABILITY ANALYSIS

Example: clamped steel plate

- dependencies between interval parameters, operator norm and P_f



TIME DEPENDENT RELIABILITY ANALYSIS

Example: clamped steel plate

- results and numerical efficiency
  - » particle swarm optimization to evaluate operator norm
  - » FORM to compute P_f (problem linear in z and low dimensionality)
  - » comparison with vertex method and double loop solution

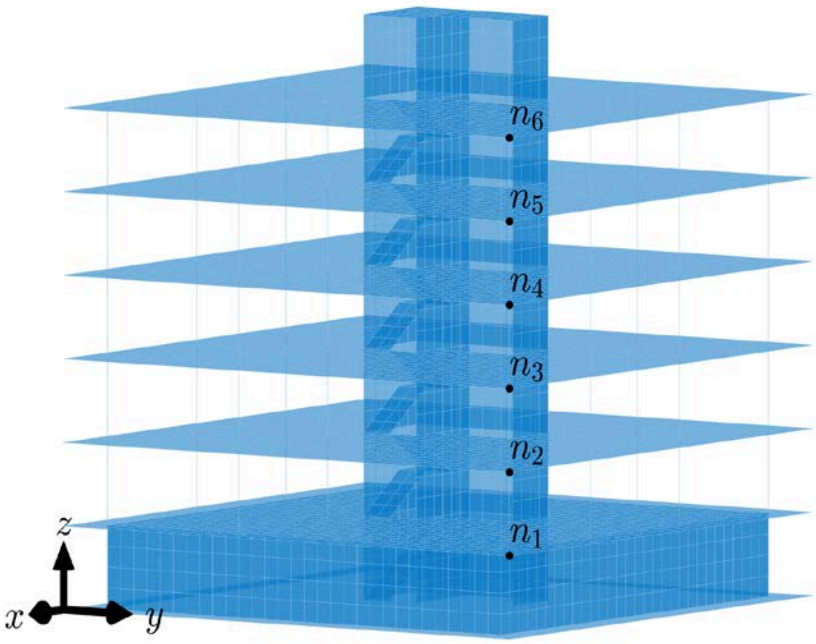
	vertex method		operator norm		double loop	
	$\underline{\theta}^*$	$\overline{\theta}^*$	$\underline{\theta}^*$	$\overline{\theta}^*$	$\underline{\theta}^*$	$\overline{\theta}^*$
operator norm	0.0208	0.0859	0.0208	0.1112	0.0208	0.1112
P_f	$8.67 \cdot 10^{-6}$	0.2907	$8.67 \cdot 10^{-6}$	0.4889	$8.67 \cdot 10^{-6}$	0.4889
FE analyses	1794		640+47	880+33	18156	26539

» numerical effort significantly reduced
» correct identification of internal optimal points

TIME DEPENDENT RELIABILITY ANALYSIS

Example: six-story building under earthquake excitation

- structural model
  - » 9500 shell and beam elements, linear
  - » reinforced concrete



- load model
  - » Gaussian stochastic process
  - » Autocorrelation governed by modulated Clough-Penzien spectrum
- interval parameters
  - » 7 parameters of the load model
  - » Young's modulus of concrete for each story

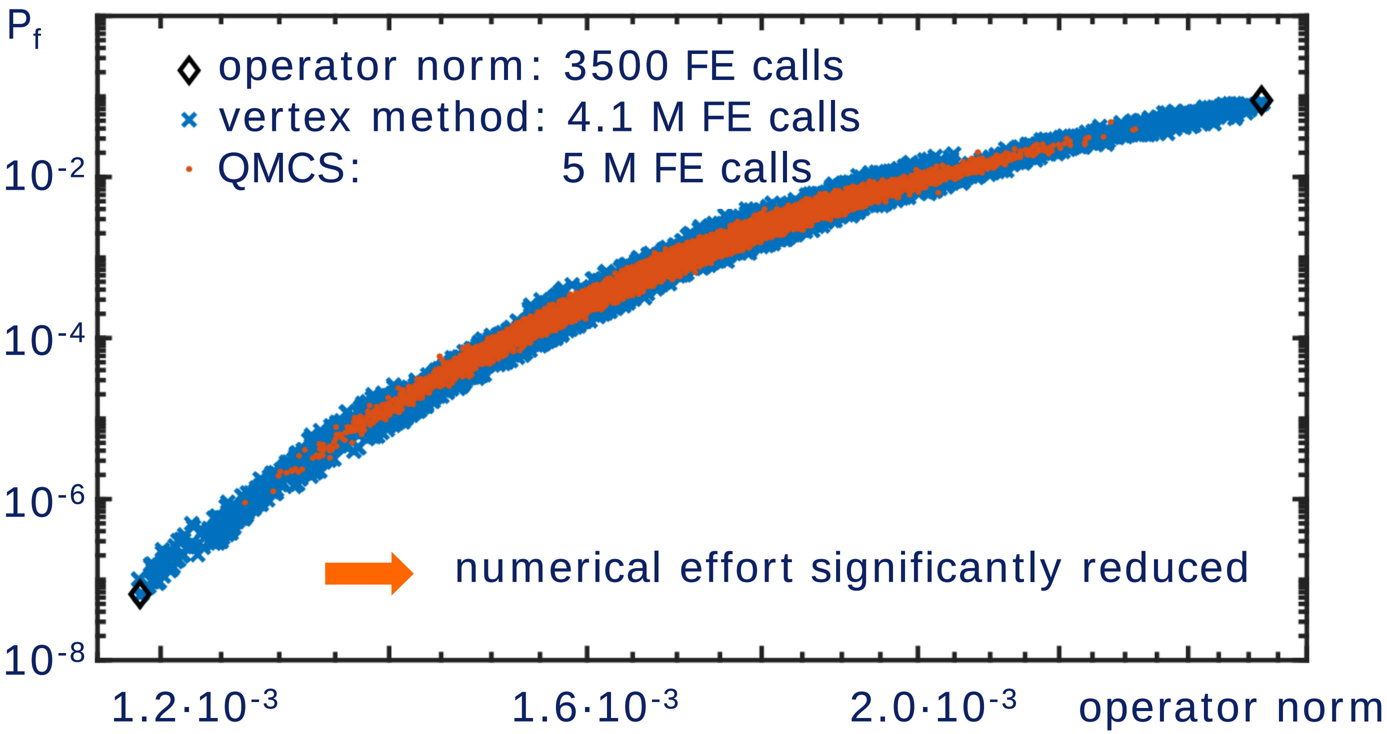
→ 13 interval parameters

- P_f for exceedance of interstory drift of $2{\cdot}10^{-3}$ times the story height

TIME DEPENDENT RELIABILITY ANALYSIS

Example: six-story building under earthquake excitation

- results and numerical efficiency
  - » particle swarm optimization to evaluate operator norm
  - » directional importance sampling to compute P_f
  - » comparison with vertex method and quasi MCS to explore intervals



RESUMÉ

Efficient and effective modeling and processing of aleatory and epistemic uncertainties

- modified exponential covariance provides realistic and efficient stochastic process model
- compressive sensing allows PSD estimation with fragmentary data
- relaxed and imprecise PSD quantify epistemic uncertainty from limited and imprecise data
- operator norm theory facilitates efficient solution of first passage problems with interval-valued stochastic models

Combinations of developments facilitate efficient and realistic stochastic dynamics analysis of structures